The Population and Merger Rate of Binary Black Holes

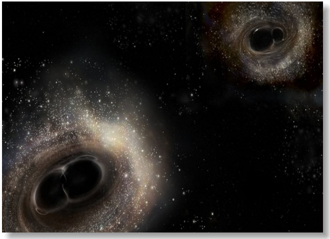
Vaibhav Tiwari

Cardiff University

tiwariv@cardiff.ac.uk

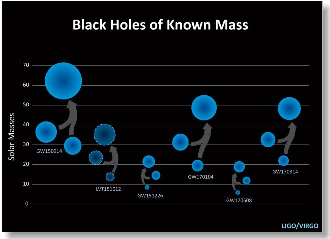
February 3, 2017

Some Major Questions

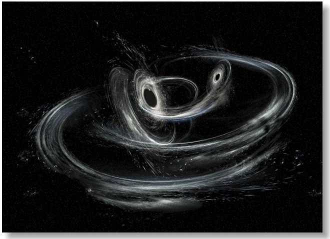


The total and parameterized merger rate of binary black holes ?

How does their distribution and merger rate evolve in redshift ?



What is the mass and spin distributions of the merging black holes ?



How to understand the dynamics or evolution of the source and its environment from the observations ?

Binary Black Hole Merger Rate

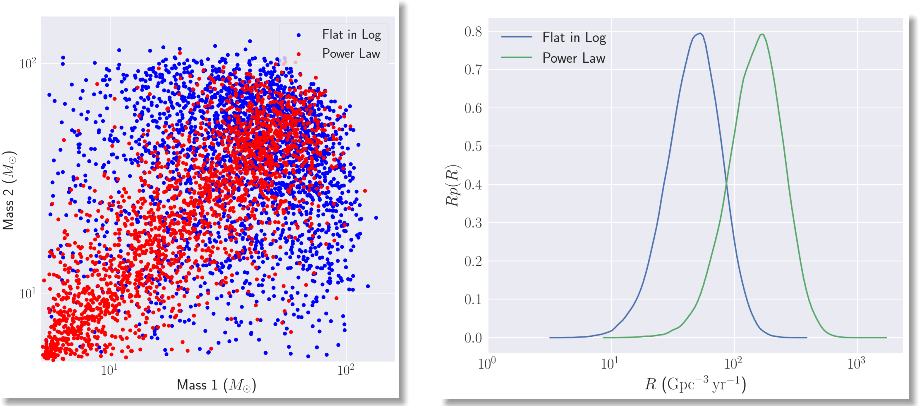


Figure 1 : left) The two mass distributions used in the estimation of the binary black hole merger rates right) The rate estimated at the end of GW170104.

So far two mass distribution of black holes have been assumed for the estimation of rates. They are orthogonal in terms of their coverage on the mass plane, hence they are expected to bracket the rate of merger well. **a)** $p(m) \propto m^{-\alpha}$, **b)** $p(m) = U(log(m_{min}), log(m_{max}))$, $\alpha = -2.35$

Binary Black Hole Mergers in the Field

Estimate how often do the binaries in the field (ones not part of clusters) merge ?

- Begin with a galaxy mass distribution and an Initial Mass Function (IMF) for the stars - Usually a power-law distribution.
- Evolve these in time using the current understanding of stellar evolution.
  - Must peg its way through present day mass and luminosity function.
- Assume a fraction of stars form binaries.
- Evolve binary through common envelop phase.
- Estimate the intrinsic rate.
- Estimate merger rates by convolving intrinsic rate, star formation rates and galaxy mass distribution
  - Sensitive to the metallicity of the galaxy as low metal stars retain a larger fraction of their masses compared to high metallic stars through their live times (metallicity increases opacity and causes increased pressure on the outer layers of the stars driving much of them out).
- Apply cosmology and observational bias - estimate the observed rate.

Population Synthesis

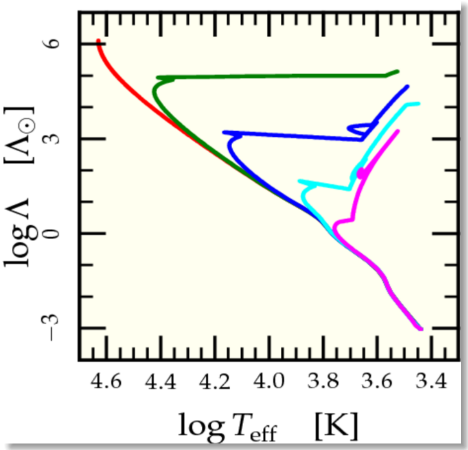


Figure 2 : Isochrone showing evolution of stars of different masses (y-axis) as time increases from red to green to blue to cyan to magenta

- Estimate the ages and masses of stellar groups (star formation history) by observing their luminosity, spectrum, colors etc.
- Can be easier for globular clusters and difficult for systems like starbursts.
- Factor dust, gas, metallicity etc. in (stars: emit light, dust: absorbs and re-radiates, gas: ionize and re-radiates).
- Needs an assumed initial mass function (IMF) which are then evolved using stellar evolutionary tracks called isochrones (analysis is much more involved than described).
- Analysis can be made more detailed (e.g. varying IMF), but, increased complexity results in increased uncertainty.

Common Envelope Evolution of a Binary

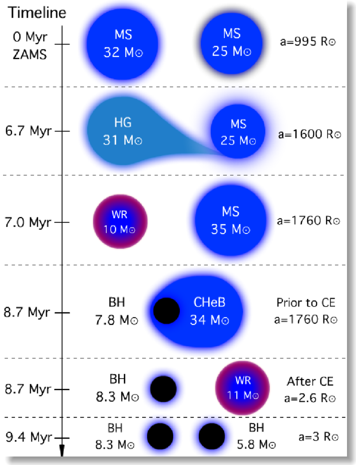


Figure 3 : Proposed mechanics that evolves a binary under a common envelope such that it merges within the Hubble time.

- Stars with zero age main sequence (ZAMS) mass of 32 and 25 solar mass respectively. Their masses are confined to their Roche-lobes.
- The heavier star burns out quickly enters the Hertzsprung gap. Its expansion initiates a mass transfer to the companion.
- Primary is almost at the verge of going supernova (WolfRayet star).
- Primary goes supernova. The secondary (CHeB - hydrogen stripped star) goes in common evolution phase with primary and transfers mass to primary - most difficult part of the evolution to model.
- Secondary is about to go supernova (SN).
- Secondary goes SN, leaving two black holes, with small enough separation, that can merge within a short period of time.

Binary Black Hole Mergers in Star Clusters

Estimate how often do the binaries in the field (ones not formed in the field) merge ?

- Black hole binaries may be rare in galactic fields, it is possible to produce such binaries through dynamical interactions in star clusters
- Estimate the per milkyway-equivalent galaxy merger rates
  - Depends heavily on cluster dynamics and star density. Studies show very few neutron star mergers.
- Assume a star formation rate.
- Assume density of galaxies.
- Convolve over the two

Binary Black Hole Mergers in Star Clusters

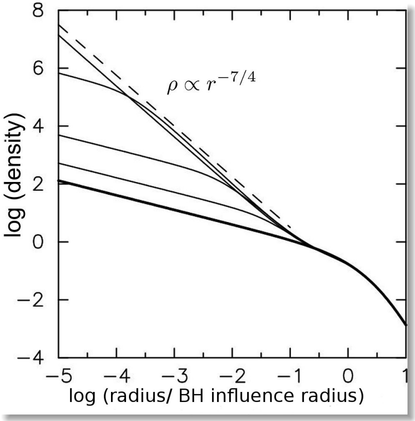


Figure 4 : Growth of Bahcall-Wolf cusp as a function of the normalized radius. The stellar density increases towards the center.

- A small fraction of stars live in star cluster, but they may be home to significant number of compact binary coalescence (CBC).
- The inner core of a star cluster could become dominated by black holes as they sink towards the center due to mass segregation.
- Two or three body interactions may give rise to binaries, tighten or loosen existing binaries and eject binaries out of the cluster.
- Its a n-body problem and no analytical solution exists. Alternatively the system is evolved by numerically integrating Fokker-Planck equations (time evolution of the probability density of the Brownian particles).
- Mergers happen in the field or within the cluster. All depends on the assumed dynamics.

Visible Volume

Visible volume is one of the primary ingredient when performing model selection and estimation of merger rate. Expanding on Equation 6, visible volume can be written as,

$$V^{vis} = \int \mathrm{d}z\mathrm{d}\theta \, \frac{\mathrm{d}V_c}{\mathrm{d}z} \frac{1}{1+z} s(\theta) f(z,\theta), \tag{1}$$

where $\mathrm{d}V_c/\mathrm{d}z$ is the differential co-moving volume, $s(\theta)$ is the distribution function for the astrophysical population and $f(z,\theta)$ is the probability of recovering a signal, with parameters θ, at a redshift z.

Equation 1 is estimated by performing Monte-Carlo integration. Simulated waveforms are injected according to distribution $s(\theta)$ and redshift distribution according to standard cosmology. Visible volume is given by,

$$V^{vis} = V_0 \frac{N_{found}}{N_{ijected}}, \qquad V_0 = \int \mathrm{d}z \, \frac{\mathrm{d}V_c}{\mathrm{d}z} \frac{1}{1+z}. \tag{2}$$

Visible Volume

The total probability (Equation 5) is a function of visible volume. Visible volume is to be calculated every time model or its parameters are changed. The approached described in the previous slide, is applicable for a fixed distribution $s(\theta)$, hence, is not a viable option. Possible solution lies in injecting simulated waveforms such that they cover the binary parameter space and then scaling the injections.

Distribution uniform in	Probability Density
Component Mass	$P(m^d) = \frac{1}{m_{\max} - m_{\min}}$
Total Mass	$P(m) = \frac{1}{(m_{\max} - m_{\min})(m_1 + m_2 - 2m_{\min})}$
Aligned Spin	$P(s_z) = \frac{1}{s_{z\max} - s_{z\min}}$
Total Spin	$P(s_z) = \frac{\log(s_{\max} - \log(\lvert s_z \rvert)}{2 s_{\max}}$
Distance	$P(d) = \frac{1}{d_{\max} - d_{\min}}$
Chirp Distance	$P(d) = \frac{\mathcal{M}_{\mathrm{BNS}}^{(5/6)}}{(\mathcal{M})^{(5/6)}(d_{\max} - d_{\min})}$

Table 1 : Different strategies used in injections runs. Strategies are independent of each other. $\mathcal{M} = (m_1 m_2)^{(3/5)}/(m + 1 + m_2)^{(1/5)}$ is the chirp mass of a binary (BNS $\equiv 1.4 M_\odot - 1.4 M_\odot$) binary.

Scaling allows visible volume to be written as,

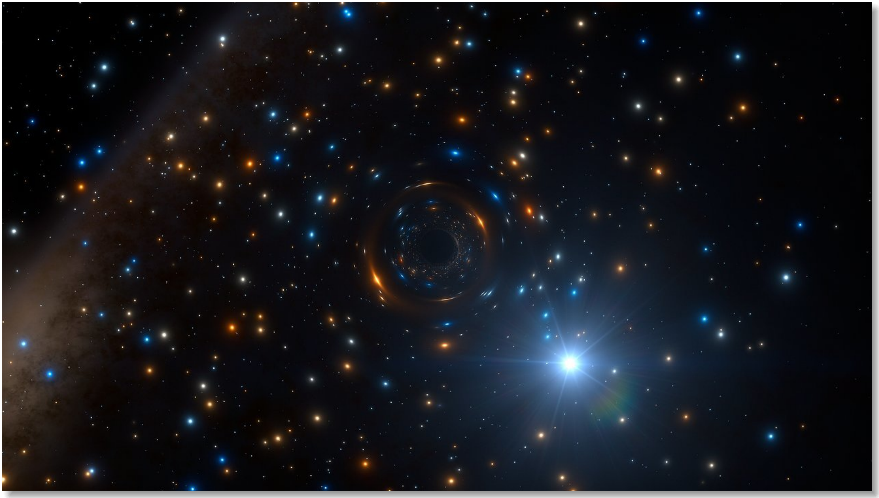
$$V = V_0^m \frac{\sum_{i \,\in\, \mathrm{Found}} w_i}{\sum_{i \,\in\, \mathrm{Injected}} w_i}, \qquad (3)$$

where,

$$w_i = \frac{p_{\mathrm{astro}}(\{m_1, m_2, s_{z,1}, s_{z,2}, d\}_i)}{p_{\mathrm{injection}}(\{m_1, m_2, s_{z,1}, s_{z,2}, d\}_i)}. \qquad (4)$$

arXiv:1712.00482 shows some results of this analysis. The method comes at no cost as generic injection runs are performed as part of the pipelines data processing.

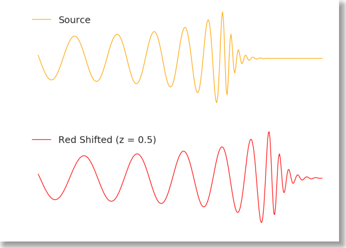
A Dichotomy of the Universe



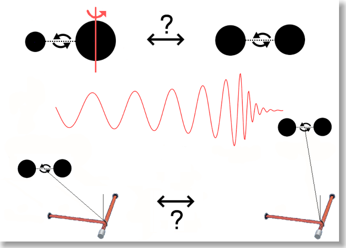
What are the physical processes that bridge these two very different systems ?

A Dichotomy in the World

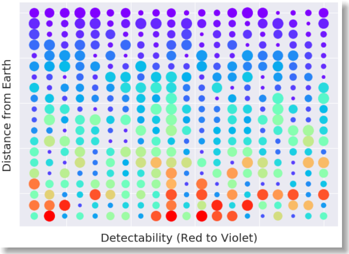
Seeing is Not Believing



Expanding universe redshifts the source waveform. Signal appears at a lower pitch.



Fundamental degeneracies allow measurement of only particular combination of parameters.



Bias towards observing powerful sources.

Always a risk of introducing systematic errors.

The Bayesian Soup for the Population Analysis

The probability that the data $\vec{d}$ comes from a population $p_{\mathrm{pop}}(\vec{\theta}|\vec{\lambda})$, defined by the parameters $\vec{\lambda}$, is given by the Bayes theorem

$$p(\vec{d}|\vec{\lambda}) = \frac{\int \mathrm{d}\vec{\theta}\; p(\vec{d}|\vec{\theta}) p_{\mathrm{pop}}(\vec{\theta}|\vec{\lambda})}{\alpha(\vec{\lambda})}, \tag{5}$$

where the equation is integrated over all possible binary parameters $\vec{\theta}$ and the evidence $\alpha(\vec{\lambda})$ is obtained by integrating over all possible selections of data from the given detector output (ensuring selected data crosses a threshold and meets requirement of an observation),

$$\begin{aligned} \alpha(\vec{\lambda}) &\equiv \int_{\vec{d}>threshold} \mathrm{d}\vec{d} \int \mathrm{d}\vec{\theta} p(\vec{d}|\vec{\theta}) p_{\mathrm{pop}}(\vec{\theta}|\vec{\lambda}) \\ &= \int \mathrm{d}\vec{\theta} \left[\int_{\vec{d}>threshold} \mathrm{d}\vec{d} p(\vec{d}|\vec{\theta}) \right] p_{\mathrm{pop}}(\vec{\theta}|\vec{\lambda}) \equiv \int \mathrm{d}\theta\, p_{det}(\vec{\theta}) p_{\mathrm{pop}}(\vec{\theta}|\vec{\lambda}). \end{aligned} \tag{6}$$

$p_{det}(\vec{\theta})$ is the detection probability for the parameters $\vec{\theta}$ and is quantifiable. $\alpha(\vec{\lambda})$ is also termed as the visible volume for the population $p_{\mathrm{pop}}(\vec{\theta}|\vec{\lambda})$.

Population Analysis - Spin Distribution

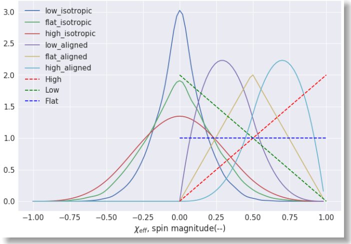


Figure 5 : Various distribution of spin magnitudes we want to compare. Models consider spins *aligned* with the orbital plane's axis or *isotropically* distributed.

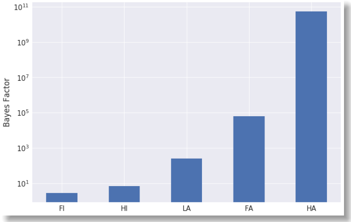


Figure 6 : Bayes Factors of low isotropic model compared to the remaining ones.

Astrophysical models that propose large spin magnitudes or alignment of spin with the orbital axis are not favoured by the current observations. This is shown by devising three spin magnitudes distributions,

$$p(\chi_i) = \begin{cases} 2(1-\chi_i) & \text{for } Low \\ 1 & \text{for } Flat \\ 2\chi_i & \text{for } High \end{cases}$$

and estimating the ratio of the total probability of the observed effective spin ($\chi_{\rm eff} = (m_1\chi_1 + m_2\chi_2)/(m_1 + m_1)$). Also termed as Bayes factor, it is defined as,

$$B = \frac{p_{\text{pop}_1}(\vec{\theta}|\vec{\lambda_1})}{p_{\text{pop}_2}(\vec{\theta}|\vec{\lambda_2})} = \Pi_{i=1}^{N} \frac{P_1(\chi_{eff,observed})}{P_2(\chi_{eff,observed})}, \quad (7)$$

where N are the number of observations.

Important Factors – Need Inclusion in the Previous Analysis

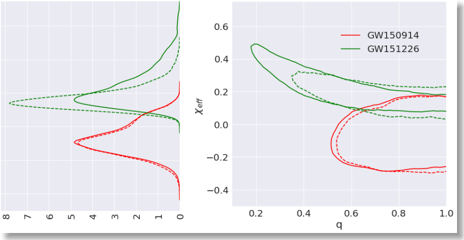


Figure 7 : Current PE priors are flat in mass distribution. Astrophysically motivated priors reduce the χ_{eff} distribution.

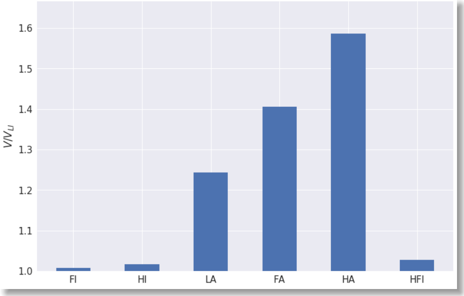


Figure 8 : Selection effects also come into play. The odds-ratio of 1st model compared to 2nd gets weighted by $(V_2/V_1)^{Nobs}$.

Population Analysis - Spin Distribution/Masses Included

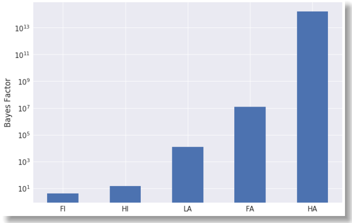


Figure 9 : Bayes Factors of low isotropic model compared to the others with mass distribution included.

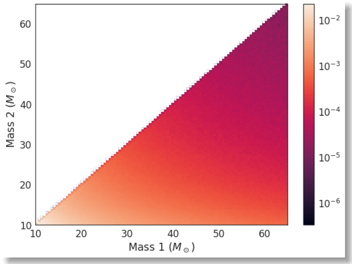


Figure 10 : Impose a realistic mass distribution: a power-law, such that, $p(m) \propto m^{-\alpha}$. $\alpha = 2$ for this figure.

The discrimination between various spin models can be improved by imposing a realistic mass distribution. The true distribution is not known, but a power-law distribution seems a realistic choice. The Bayes factors can be re-evaluated using the following equation.

$$B = \Pi_{i=1}^{N} \frac{P_1(m_1, m_2, \chi_{eff})_{observed}}{P_2(m_1, m_2, \chi_{eff})_{observed}} \left(\frac{V_2^{vis}}{V_1^{vis}} \right)^N . \tag{8}$$

Where V^{vis} is the visible volume for the population model (same mass distribution but different spin distributions) described in Slide 14. The Bayes factors increase! Primarily because incorporating mass distribution increases the discrimination due to a) visible volume's dependence on the spins b) The observed parameters don't suffer from systematic bias.

Conclusion: The current observations disfavour population models that a) propose alignment of spins with the orbital angular momentum b) large magnitude of spins.

The Mass and the Spin Distribution (preliminary)

We want a non-parametric way of estimating the mass and spin distribution of the binary black holes. However, implementation of Equation 5 is challenging. I solve it by using a histogram as the non-parametric model, such that,

$$P(m_1, m_2) = p(i, j), \quad i = \frac{m_1 - m_{minimum}}{\Delta m}, \quad j = \frac{m_2 - m_{minimum}}{\Delta m}, \tag{9}$$

where Δm is the bin width. The log likelihood for N observations is given by,

$$\log L = \sum_{k=1}^{N} log(P_k(i, j)). \tag{10}$$

The likelihood in Equation 10 does not have a finite maximum, however, it can be penalized to quantifying the conflict between smoothness and goodness of fit. I penalize it as follows,

$$\log L = \sum_{k=1}^{N} log(P_k(i, j)) - \alpha \sum_{i,j} \sum_{k} (p(i, j) - p(i + k, j + k))^2, k = +1, 0, -1, \tag{11}$$

where α is a positive smoothing parameter. The penalizing term can be interpreted as the curvature of the histogram at the point i, j. Maximum penalized is obtained by starting with a histogram, uniform in probability, which is perturbed until the log likelihood is maximized.

The Mass and the Spin Distribution (preliminary)

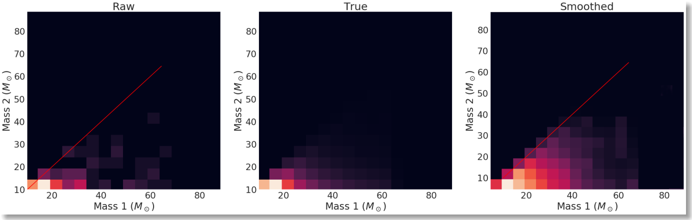
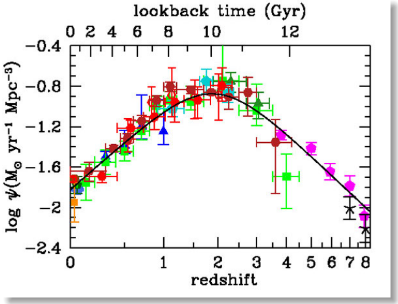
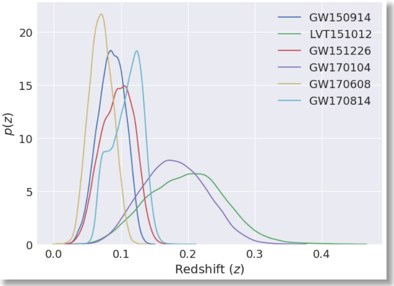


Figure 11 : Smoothing obtained by using penalized likelihood method. Its straightforward to weigh total probability by the visible volume corresponding to a binned mass distribution.

Star Formation Rate

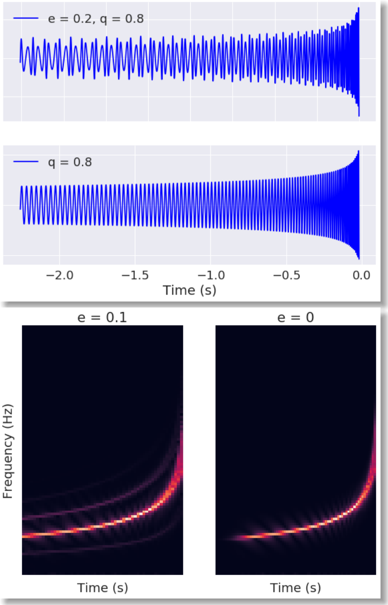




- Observations of star formation rates (SFRs) in galaxies provide vital clues to the physical nature of the Hubble sequence, and are key probes of the evolutionary histories of galaxies.
- The scaling of the SFR to continuum luminosity is a smooth function of the color of the population, and this can be calibrated using an evolutionary synthesis model.
- The merger rate can be connected with the star formation rate. However, there are multiple steps in between which are either not known or not understood.
- GW provide direct distance measurement. However due to multiple degeneracies the meaurement error in the redshift is large.

Figure 12 : The history of cosmic star formation using far Ultra-Voilet and Infrared luminosities.

Eccentric Binaries



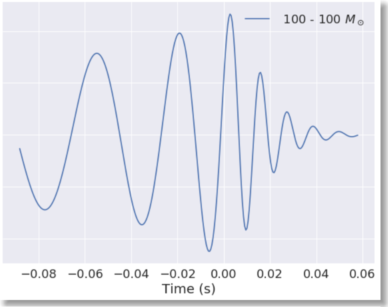
- Most of the binaries are expected to circularize by the the time their emitted Gravitational Waves enter LIGO's sensitivity band.
    - e.g. with current eccentricity of ~ 0.7, the Hulse-Taylor binary will have eccentricity of 10^{-6} by the time (2, 2) mode chirps up to 10 Hz in 300 million years.
- However if the stars were 10 times closer binary will have residual eccentricity of 0.05.
- promising source! Can tell a lot about the dynamics of galactic cores and GR in the strong field regimes.
- Can be a testing ground for the general theory of relativity.
- Current matched filtering pipelines are sub-optimal for eccentric binaries. I performed a search using excess energy method (cWB) for my PhD.

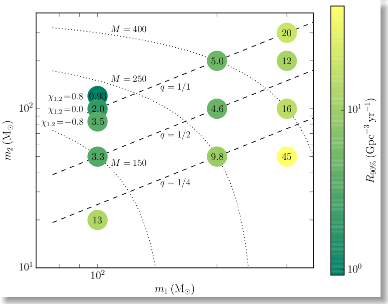
Figure 13 : top) Time series of an eccentric binary waveform, bottom) Time - Frequency plot.

Proposed Formation Channels - Eccentric Binaries

- A circular neutron star - black hole binary can gain eccentricity and emit gamma ray burst due to mass transfer from NS to BH as the NS fills BHs Roche lobe.
- It has been suggested that these supernova kicks can also render the system highly eccentric with merger following within months.
- Another proposed scenario involves hierarchical triplets modeled to consist of an inner and an outer binary. If the orbital planes of the inner and of the outer binary make mutual inclination angle large enough, then the time averaged tidal force on the inner binary may induce oscillations in its eccentricity, this is known as the Kozai mechanism.
- In the densely packed galactic cusps, there can be runaway process when two black holes have a close encounter, following which they release sufficient amount of energy in Gravitational Waves to form a bound system, and merge in time not enough for binary to circularize.

Intermediate Mass Binary Black Holes (IMBH)

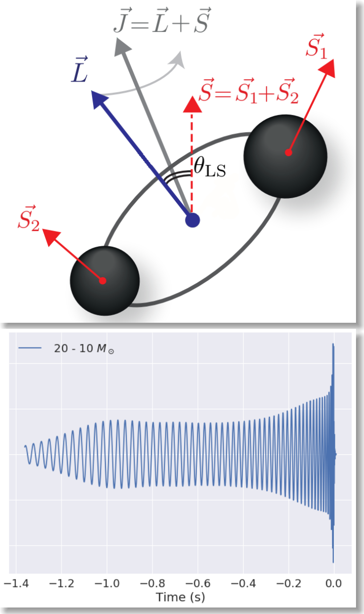




- IMBH's are categorized as black holes with mass above 100 $M_\odot$.
- GW signal from merging IMBHs has not been observed so far; some of their proposed formation channels are
  - Direct collapse of a massive population III star (th earliest stars in the universe).
  - Runaway mergers of main sequence stars within clusters.
  - Accretion of residual gas onto stellar-mass black holes.
- Their observation can help understand a) massive black hole formation b) stellar-cluster evolution c) ultra-luminous x-ray sources (i.e. sources that exceed the Eddington luminosity limit).

Figure 14 : top) The IMBH signal lasts only for a fraction of a second in the LIGO's band, bottom) The upper limits for IMBH merger rate for the first observation run.

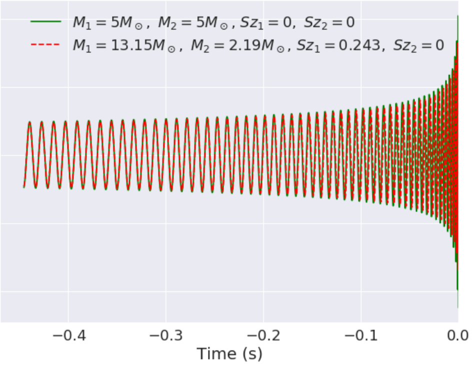
Precessing Binaries

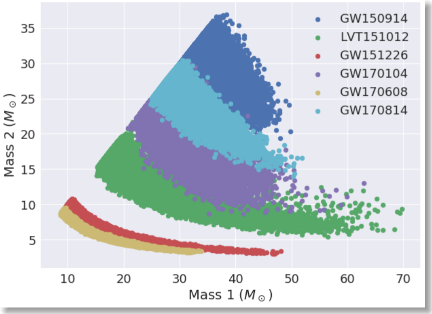


- Precession of the angular momentum vector happens when at least one of the black hole's spin is not aligned with the orbital angular momentum.
- The effect is modulation of waveform amplitude when viewed at an inclination.
- Current searches use aligned-spin waveform templates and hence are sub-optimal.
- None of the observed waveforms have shown precession (however it is hard to see even with precession present). Require more observations.
- Can inform about formation channel and dynamics of the source population (e.g. can break mass-spin degeneracy, tidal forces and stellar evolution may have direct impact on spins).

Figure 15 : top) Precession of angular momentum occurs when total angular momentum and orbital angular momentum are not aligned bottom) The waveform of a precessing binary modulates in the amplitude.

Classification Problem | Mass Gap





Due to mass-spin degeneracy, for a low mass system, it is almost impossible to identify the components that merged to produce an observed GW, i.e. whether the binary was a binary black hole or composed of a black hole and a neutron star. Degeneracy is partial because close to merger the two cases differ. However, the detector is not sensitive at this frequency. Several methods, such as, clustering observation based on astrophysics or breaking the degeneracy by using intrinsic properties, have been proposed.

Is there a mass gap between neutron star and black hole ? What is the maximum mass of a neutron star ? Is there an upper mass gap. Super massive black holes reside at the center of the galaxy but the most massive system LIGO has observed hints to a maximum mass of around 50 solar mass.

The End