Confronting computational challenges in gravitational-wave searches

Balancing computational costs and sensitivity optimizations when searching for inspiral signals

Drew Keppel

Max Planck Institute for Gravitational Physics

3 May 2013

Overview

- What are inspiral gravitational-wave (GW) signals?
- What techniques are used to perform searches for these signals?
- How are these searches optimized?
- Can low-latency searches be achieved?
- How are coherent searches performed?
- Are coherent searches more sensitive than coincident ones?

Signal Classes and Astrophysical Sources

- Stochastic Signals
  - Primordial background
  - Phase Transitions
  - Interfering Signals (e.g., Cosmic strings, Galactic White Dwarf Binaries)
- Continuous Waves
  - Mountains on rotating neutron stars
  - Quadrupole-producing oscillation modes of neutron stars
- Modeled Transients
  - Binary combinations of neutron stars and/or black holes
  - Travelling cusps on cosmic strings
  - Ringdowns of neutron stars or black holes
- Unmodeled Transients
  - Supernovae
  - ???

Inspiral Signals

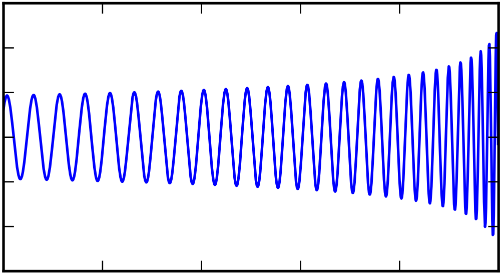
- 17 parameters determine generic inspiral waveforms:

$$\overbrace{\{D, \iota, \psi, \phi_0, \alpha, \delta, t_c\}}^{\text{extrinsic}}, \overbrace{\{m_1, m_2, \mathbf{s}_1, \mathbf{s}_2, e, \omega\}}^{\text{intrinsic}}$$

  - extrinsic parameters only affect amplitude and phase of waveform
  - e and ω from elliptical orbits
  - $\mathbf{s}_1$ and $\mathbf{s}_2$ from spinning objects
- m_1 and m_2 ($\mathcal{M}_c$ and η) are the only remaining parameters that determine the waveform for BNS signals

Inspiral Signals

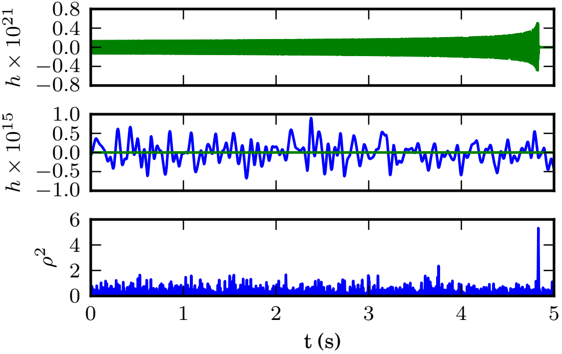
- Waveforms have "chirp" morphology
  - Objects orbit faster and emit stronger GW radiation as separation decreases
  - Post-Newtonian theory describe the waveform as a power series expansion in the orbital frequency



Search Techniques

When the signal's waveform is known, matched filtering provides the maximum likelihood estimate of the signal parameters

$$\rho(t) = (h|s)(t) := \frac{4}{\sigma} \int_0^\infty \frac{\tilde{h}^*(f)\tilde{s}(f)}{S_n(f)} e^{2\pi i f t}\, df, \quad P(\rho^2) = \frac{1}{2} e^{-\rho^2/2}$$



Template Bank Construction

Q: How do we search for a signal with unknown parameters?
A: Search for many different combinations of parameters.

Q: How do we choose which parameters?
A: Use the matched filter definition to define a metric on the parameter space.

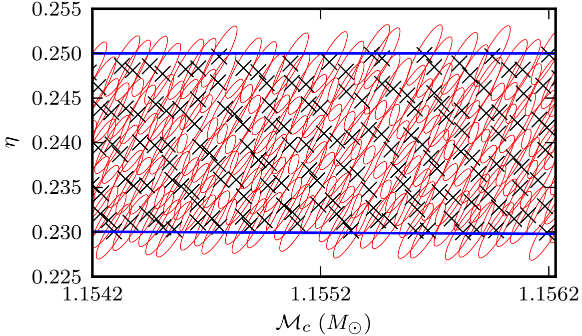
$$g_{\mu\nu} := \frac{(\partial_\mu h | \partial_\nu h)}{(h|h)}$$

$$m = \frac{1}{2} g_{\mu\nu} \Delta\lambda^\mu \Delta\lambda^\nu$$

Metric currently computed to 3.5PN for non-spinning inspiral signals.[1]

[1] Drew Keppel et al. *in preparation* (2013).

Template Bank Construction



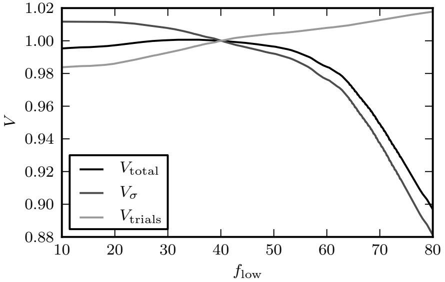
- Typical BNS search uses $N_{\mathrm{templates}} \approx 10^5$–$10^6$

$$N = \theta m^{-d/2} \int \sqrt{|g|}\, \mathrm{d}\lambda^d$$

Bandwidth

Q: How do we choose the bandwidth we will search?[2]

$$\text{“Trials factor”} \propto \int \sqrt{|g|}\, \mathrm{d}\lambda^d, \;\; \sigma^2 = 4 \int_{f_\text{low}}^{f_\text{upper}} \frac{|\tilde{h}(f)|^2}{S_n(f)}\, \mathrm{d}f$$



- Sensitivity at fixed false alarm rate

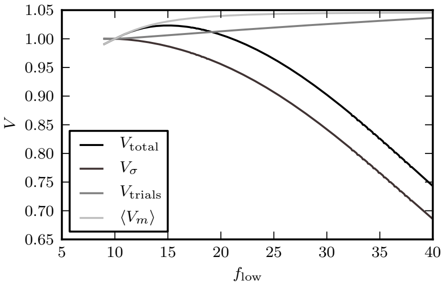
A: Reduce f_low until increasing trials factor balances increasing σ.

[2] Drew Keppel. *submitted to PRD* (2013). arXiv:1303.2005.

Bandwidth and Template Density

Q: What about computational limitations?[3]

- $\downarrow f_{\mathrm{low}} \Rightarrow \uparrow T_{\mathrm{waveform}}, \uparrow \sqrt{|g|}$
- $\uparrow m \Rightarrow \downarrow N_{\mathrm{templates}}$



A: Balance bandwidth and template density to maximize sensitivity at fixed computational cost.

[3] Drew Keppel. *submitted to PRD* (2013). arXiv:1303.2005.

Low-latency

- Can a GW event spur EM telescope observations?
- EM counterparts can fade quickly
- Minimize latency of reporting GW events

Filtering	FLOPs	Latency (s)
FIR	1.2×10^{12}	2.4×10^{-4}
FFT	2.5×10^{5}	1.1×10^{3}

LLOID Algorithm

Several techniques are used to reduce computational costs:

- Remove redundant filters computations by SVD of templates[4]
- Construct composite detection statistic (CDS) to hierarchically reconstruct physical template SNRs[5]
- Use multirate filtering to reduce computational cost of lower frequency portions of waveform[6]

---

[4] Kipp Cannon et al. *Phys. Rev. D* 82.4 (2010), p. 044025.
[5] Kipp Cannon et al. *Phys. Rev. D* 83.8 (2011), p. 084053.
[6] Kipp Cannon et al. *The Astrophysical Journal* 748.2 (2012), p. 136.

LLOID Costs

Combinations of techniques can reduce both the computational cost and latency.

Filtering	SVD	Multirate	CDS	FLOPs	Latency (s)
FIR				1.2×10^{12}	2.4×10^{-4}
FFT				2.5×10^{5}	1.1×10^{3}
FFT	X			1.2×10^{6}	1.1×10^{3}
FFT		X		4.9×10^{5}	5.2×10^{1}
FFT	X	X		2.0×10^{5}	5.2×10^{1}
FFT	X	X	X	2.2×10^{4}	5.2×10^{1}
FIR	X	X	X	9.5×10^{4}	2.4×10^{-4}

Coherent Analysis

- Signal model based on four basis waveforms $\{h_\mu\}$

$$s^Y = \mathcal{A}^\alpha h^Y_\alpha, \quad x^Y = n^Y + s^Y$$

$F^Y_{(\cdot)} h_{(\cdot)}$	F^Y_+	$F^Y_\times$
h_0	h^Y_1	h^Y_2
$h_{\pi/2}$	h^Y_3	h^Y_4

(1)

$$\mathcal{A}^\mu(D, \iota, \psi, \phi_0), \quad F^Y_{(+\times)}(\alpha, \delta)$$

- Maximize log likelihood ratio $\sum_Y (x^Y | s^Y) - \frac{1}{2}(s^Y | s^Y)$

$$\rho^2_{\mathrm{coh}} = x_\mu \mathcal{M}^{\mu\nu} x_\nu =: 2\mathcal{F}, \quad x_\mu = \sum_Y (h^Y_\mu | s^Y)$$

$$\mathcal{M}_{\mu\nu} := \sum_Y (h^Y_\mu | h^Y_\nu) = \begin{pmatrix} A & C & 0 & 0 \\ C & B & 0 & 0 \\ 0 & 0 & A & C \\ 0 & 0 & C & B \end{pmatrix}_{\mu\nu}$$

Comparing Network Analyses

- Coincident SNR

$$\rho^2_{\mathrm{coi}} := \sum_Y \rho^2_Y$$

- When mass and sky parameters of template match signal

$$\rho^2_{\mathrm{coi}} = \rho^2_{\mathrm{coh}}$$

  - Coincident SNR is $\chi^2(2D)$
  - Coherent SNR is $\chi^2(4)$

Coherent Metric Family

- The metric on $\mathcal{F}$ is given by a projected normalized Fisher matrix.[7]

$$g_{ab} = \frac{\sum_Y (\partial_a s^Y | \partial_b s^Y)}{\sum_Y (s^Y | s^Y)} \quad \Rightarrow \quad g_{ij}^{\mathcal{F}} = \frac{\mathcal{A}^\alpha \mathcal{G}_{\alpha\beta ij} \mathcal{A}^\beta}{\mathcal{A}^\alpha \mathcal{M}_{\alpha\beta} \mathcal{A}^\beta}$$

$$\begin{aligned}\mathcal{G}_{\mu\nu ij} =& h_{\mu\nu ij} - R_{\alpha\mu i} \mathcal{M}^{\alpha\beta} R_{\beta\nu j} \\ =& \begin{pmatrix} m_{ij}^1 & m_{ij}^3 & 0 & m_{ij}^4 \\ m_{ij}^3 & m_{ij}^2 & -m_{ij}^4 & 0 \\ 0 & -m_{ij}^4 & m_{ij}^1 & m_{ij}^3 \\ m_{ij}^4 & 0 & m_{ij}^3 & m_{ij}^2 \end{pmatrix}_{\mu\nu}\end{aligned}$$

- $g_{ij}^{\mathcal{F}}$ still depends on $\{\mathcal{A}^\mu(D, \iota, \psi, \phi_0)\}$

[7] Drew Keppel. *PRD* 86 (2012), p. 123010.

Average Metric

- How do we remove the amplitude dependence?
  - average the extremal values of mismatch

$$\bar{g}^{\mathcal{F}}_{ij} = \frac{Bm^1_{ij} + Am^2_{ij} - 2Cm^3_{ij}}{2\left(AB - C^2\right)}$$

  - marginalize over physical parameters[8]

$$\langle g^{\mathcal{F}}_{ij} \rangle = \int\int\int\int \frac{\mathcal{A}^\alpha \mathcal{G}_{\alpha\beta ij} \mathcal{A}^\beta}{\mathcal{A}^\alpha \mathcal{M}_{\alpha\beta} \mathcal{A}^\beta} P(D)P(\iota)P(\psi)P(\phi_0)\,\mathrm{d}D\,\mathrm{d}\iota\,\mathrm{d}\psi\,\mathrm{d}\phi_0$$
$$\approx \frac{m^1_{ij} + m^2_{ij}}{A + B}$$

(valid for both face-on and randomly oriented signals)

[8] Drew Keppel. *in preparation* (2013).

Metric Separation

- Four-dimensional metric depends on sky parameters and mass parameters

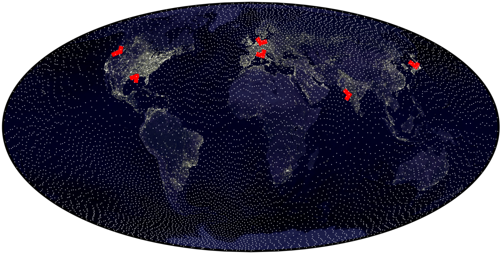
$\langle g_{ij}^{\mathcal{F}} \rangle$	α	δ	$\mathcal{M}_c$	η
α	1	−0.87	−0.010	−0.083
δ	−0.87	1	0.015	0.078
$\mathcal{M}_c$	−0.010	0.015	1	−0.87
η	−0.083	0.078	−0.87	1

- Metric separable into 2 two-dimensional metrics[9]

[9] Drew Keppel. *in preparation* (2013).

Sky Metric

- Sky metric mostly dependent on detector separation and bandwidth



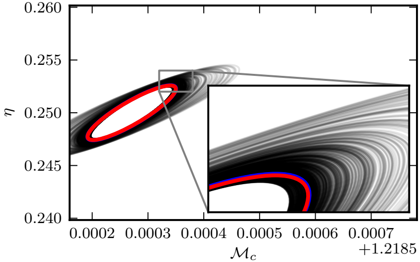
- Need $N \approx 10^3$–10^4 templates to cover sky

Mass Metric

- Marginalization of SNR over amplitude and sky parameters

$$\langle \rho^2 \rangle \propto \sum_Y \langle (h^Y | h^Y) \rangle \propto \int \left\langle \left| \tilde{h}(f) \right|^2 \right\rangle \sum_Y \frac{1}{S_n^Y(f)} \, \mathrm{d}f$$

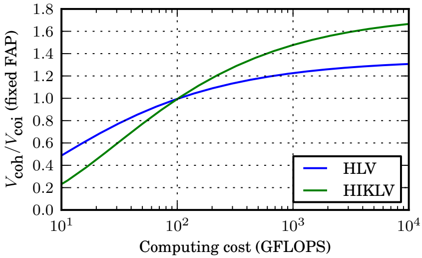
- Motivates synthetic detector with harmonic sum PSD
- This mass metric agrees with sky-marginalized $\langle g_{ij}^{\mathcal{F}} \rangle$[10]



[10] Drew Keppel. *in preparation* (2013).

Computational Cost?

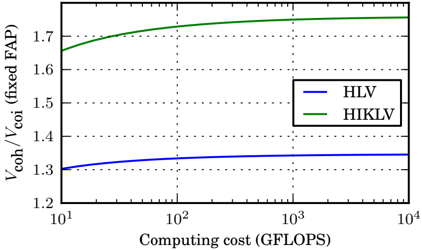
- Computational cost greatly increased compared to coincident search.[11]
  - $\gtrsim$ 20 additional operations per output SNR sample per mass template per sky direction
- Reduction in minimal match can overwhelm gains from fewer DoF



[11] Tito Dal Canton and Drew Keppel. *in preparation* (2013).

Hierarchical Search

- Hierarchical coherent SNR construction reduces cost of coherent search[12]
  - Coherent SNR computation fraction $= 10^{-3}$



[12] Tito Dal Canton and Drew Keppel. *in preparation* (2013).

Summary

- Computational costs considerations are important in designing GW searches
- Optimization can involve non-intuitive choices