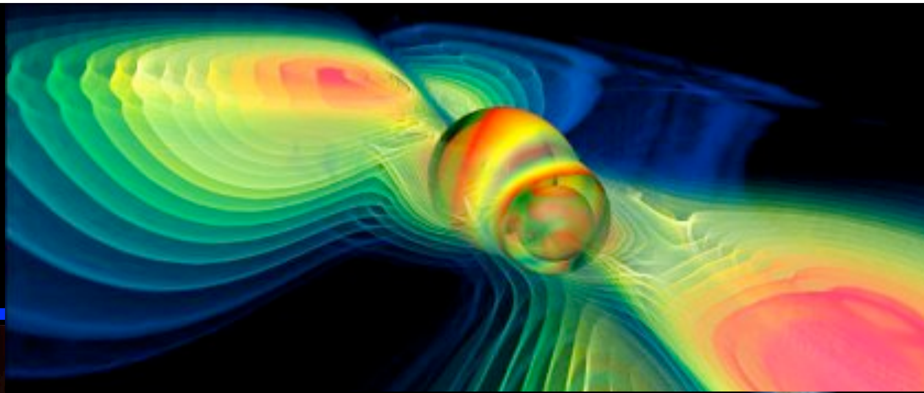
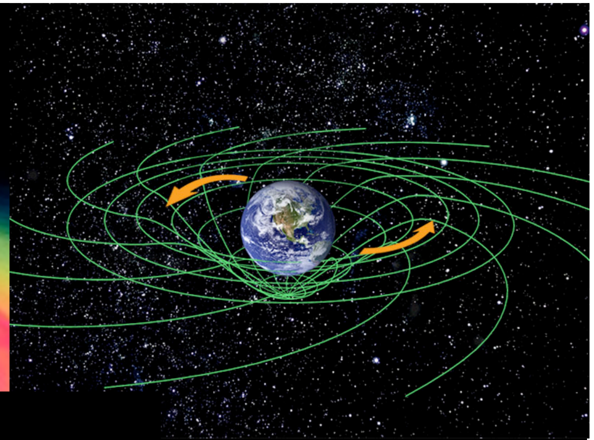
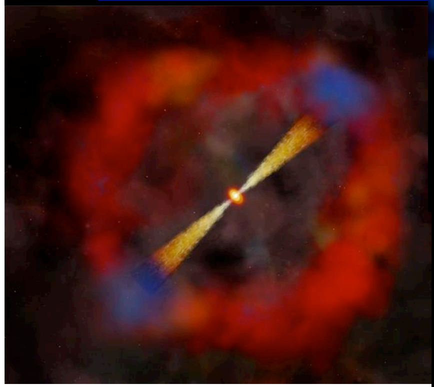
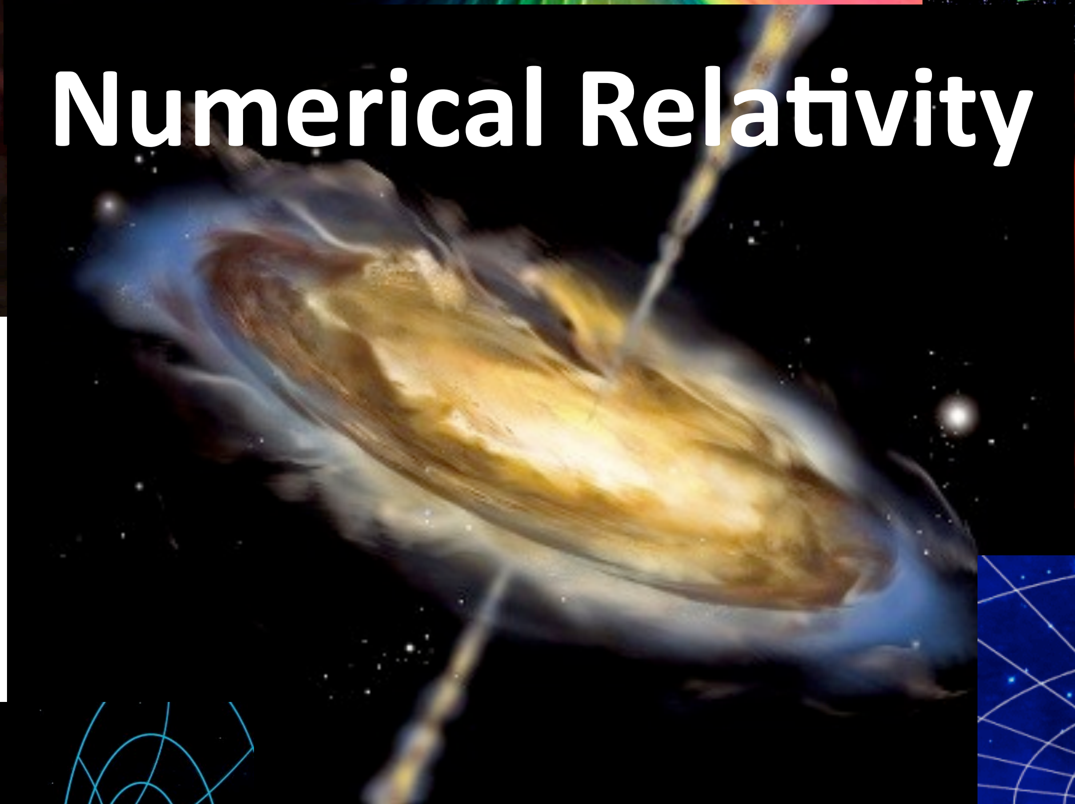
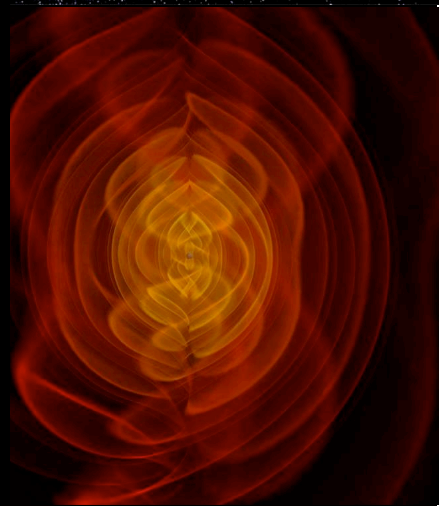


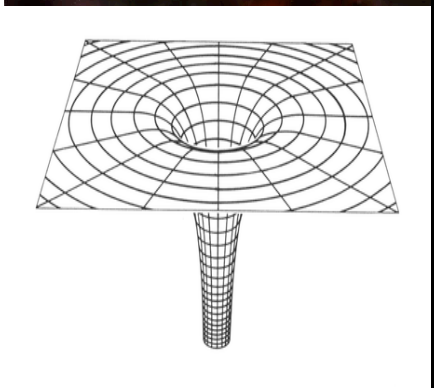


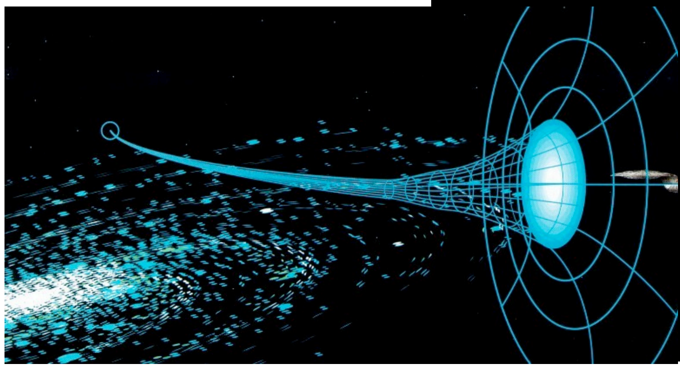


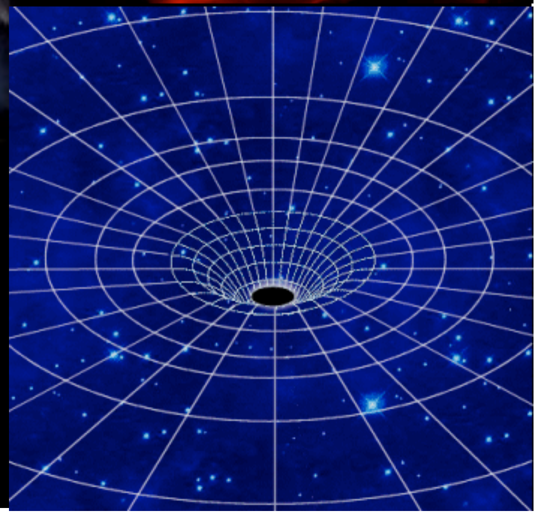
Numerical Relativity











Project Lectures 2013

Patricia Schmidt

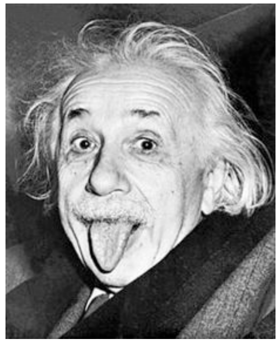
Outline

- General Relativity (GR) refresher
- Exact solutions
- (A few remarks about) Black Holes
- Numerical Relativity
- Gravitational Waves & Astrophysics

General Relativity in a nutshell

- Gravity is represented as a geometric property (curvature) of a 4D manifold (spacetime) (M,**g**)
- Matter/energy curves flat spacetime
  - Coupling between the geometry, **g,** and the energy content, **T,** described by **Einstein field equations**:

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi T_{ab}$$



- Mathematically elegant but:
  - 10 coupled non-linear second-order PDEs

Exact solutions of the field equations

- Only very few exact solutions are known:
  - ***Minkowski***: flat, empty → Special Relativity
  - ***Schwarzschild*** (1916): unique spherical symmetric vacuum solution
  - ***Kerr*** (1963): axisymmetric vacuum solution
  - Friedmann-Lemâitre-Robertson-Walker
  - ....

The Schwarzschild solution

- Karl Schwarzschild 1916
- Solution *outside* any static spherical symmetric mass distribution (sun, etc.), i.e. *vacuum solution*
- *Solution:*

$$ds^2 = -\left(1 - \frac{2M}{R}\right)dt^2 + \left(1 - \frac{2M}{R}\right)^{-1} dR^2 + R^2 d\Omega^2$$

  - M is the mass of the body
  - for M=0 we recover Minkowski space
  - Time-independent (static)
  - Spherically symmetric

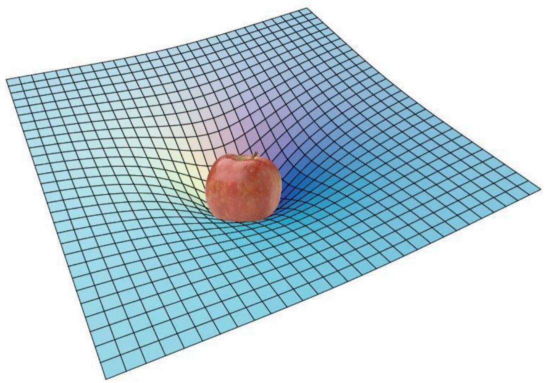
The Schwarzschild solution

- Far from the body, we can expand the metric as:

$$ds^2 \approx -\left(1 - \frac{2M}{R}\right)dt^2 + \left(1 + \frac{2M}{R}\right)dR^2 + R^2 d\Omega^2$$

  – Schwarzschild spacetime is ***asymptotically flat*** → we recover Minkowski space

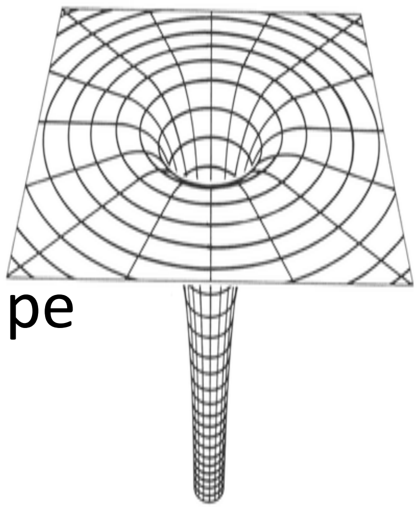
- Close to the mass distribution, strange things happen ....



The Schwarzschild solution

- At R=2M, the metric diverges (g_{tt}=0 and g_{rr}→ ∞)
  - Coordinate singularity
- BUT: At R=0, the metric diverges as well → real *physical singularity* !
- R=2M is still "special": *Schwarzschild radius:* R_S≈3km for the sun
  - R>2M: light rays can escape
  - R=2M: light rays stay there: event horizon
  - R<2M: light rays fall back and cannot escape

  → **Black Hole**

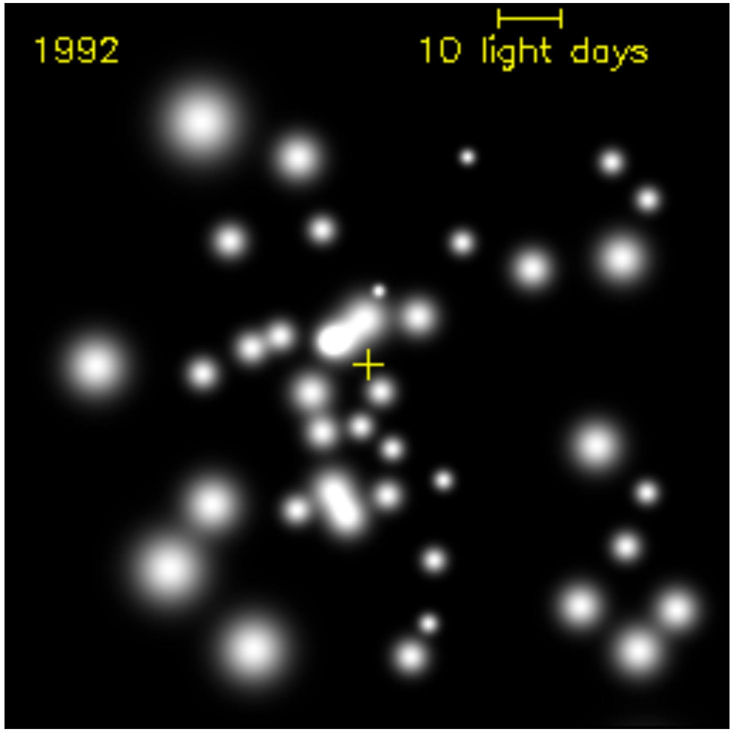


Black Holes

- Physical spacetime singularity
- May be formed when matter undergoes gravitational collapse
- Compact regions in spacetime
- Different types: Schwarzschild, Kerr, Reissner-Nordström, Kerr-Newman

Astrophysical Black Holes

- Expect most BH to be Kerr, i.e. *spinning*
- Evidence for super-massive BHs in the centres of galaxies
- *Binary black holes* (BBH) assumed to form in globular clusters and old galaxies
- BBHs, colliding BHs are promising sources of gravitational waves



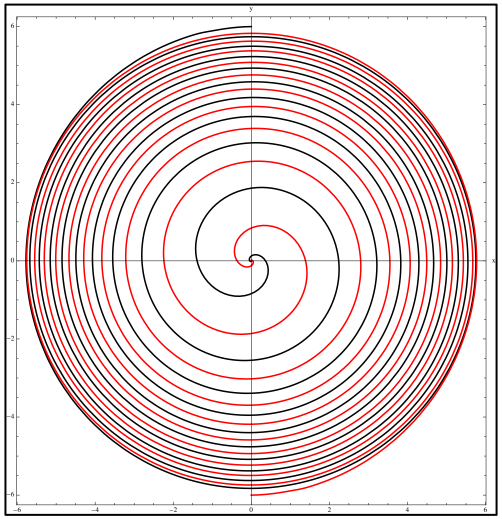
www.eso.org

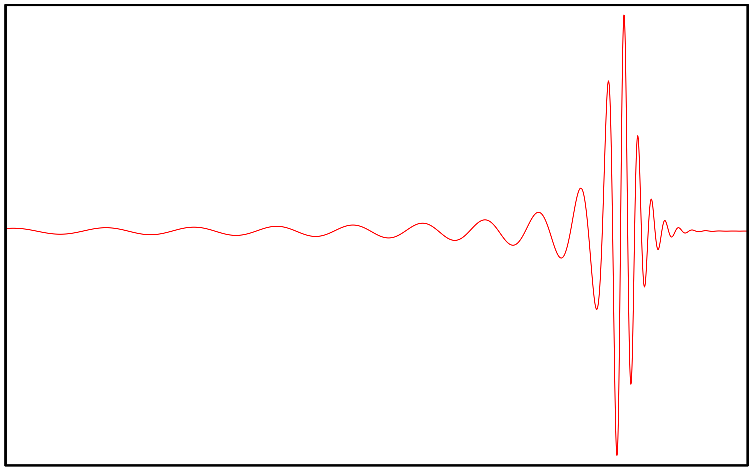
Numerical Relativity – Why ?!

- Analytic solutions of the field equations only known in a handful of cases
- NO analytic solution for the binary black hole problem ! (and many other astrophysical spacetimes)
- We need to solve the Einstein equations on a supercomputer to solve these problems in full General Relativity and to extract the desired information, e.g. gravitational waves

Numerical Relativity

- Successful numerical evolution of binary black hole spacetimes possible since 2005 (Pretorius, PRL 95(2005) 121101)
- Allows the *accurate* modelling of the complete inspiral-merger-ringdown signal





Astrophysics with NR

- What astrophysics can we do with numerical relativity ?
  - Gravitational waves
  - Final mass and spin of the remnant BH (necessary for understanding BH population in the universe)
  - Gravitational recoil

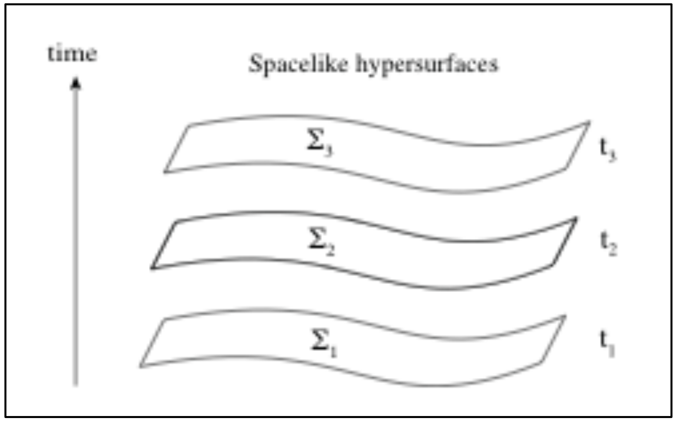
Numerical Relativity - Basics

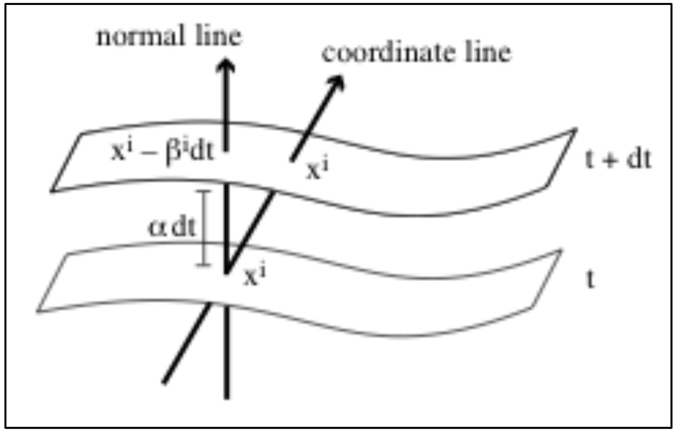
- 4D spacetime mathematically very elegant but very inconvenient for numerical techniques

- Need to reformulate the Einstein equations as initial value problem (IVP):
  - There exists a (mathematically) well-posed IVP for GR (Y. Choquet-Bruhat, 1950s)

Numerical Relativity - Basics

- 3+1 decomposition → constrained IVP
  - ***6 evolution equations***
  - ***4 constraint equations***
  - ***Gauge conditions*** for the coordinate evolution
- Variables: (γ_{ij}, K_{ij}), (α, β^i)
- Produce "meaningful" initial data for a configuration





M. Alcubierre: Introduction to 3+1 numerical relativity

The 3+1 decomposition (vacuum)

- Choose a basis → decomposed metric:

$$ds^2 = \left(-\alpha^2 + \beta_i\beta^i\right)dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j$$

- Data on one time slice

$$\left\{\gamma_{ij}, K_{ij} \propto \partial_t \gamma_{ij}\right\}$$

must satisfy the constraints

$$R + K^2 - K_{ij}K^{ij} = 0$$
$$D_j K^{ij} - \gamma^{ij} D_j K = 0$$

ADM equations

- Given (α, β^i) go to next slice with evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$
$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha\left(R_{ij} - 2K_{ik}K_j{}^k + KK_{ij}\right) + \beta^k D_k K_{ij} + K_{ik} D_j \beta^k + K_{kj} D_i \beta^k$$

Coordinates - Gauge

- The physics is the same in all coordinates but if you solve on a computer, coordinates matter:
  - Different coordinates have different numerical properties
  - Some coordinates allow to obtain an accurate solution more easily
  - Not all coordinates allow a stable numerical evolution → you might not get a solution at all !

Initial Data

- Have to specify gravitational fields (**γ**,**K**) on some initial spatial slice Σ_0 that satisfy the constraint equations
- Constraints determine 4 out of 12 independent components → 8 undetermined !
- 4 of these 8 are related to coordinate choices (freely specifiable variables)
- 4 represent the dynamical DOF of a gravitational field in GR → 2 polarisations of gravitational waves
- Difficult task for general spacetimes !

Evolution Schemes

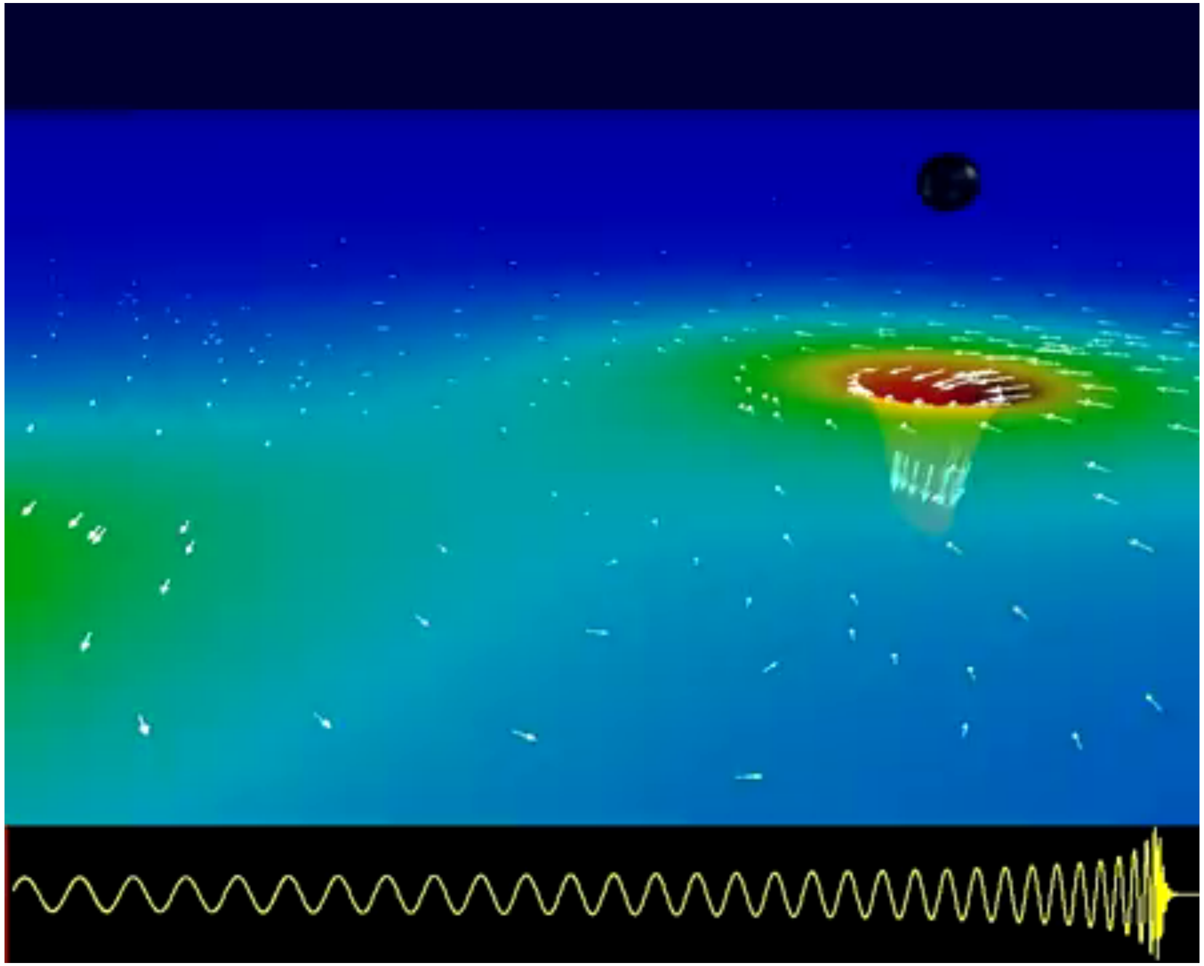
- Free vs. constrained evolution
  - Free: solve constraint equations initially and evolve with evolution equations
    - Bianchi identities assure that evolved data satisfy constraints at later times (exact statement)
    - Monitor constraint violation !
  - Constrained: solve constraint equations at each time step t for a subset of variables and evolve the remaining ones with the evolution equations

Implementation & Alternatives

- ADM formulation is numerically unstable
  - Reformulation for numerical implementation needed: BSSN (Baumgarte, Shapiro, Shibata and Nakamura)

- Alternatives to the 3+1 approach:
  - Characteristic evolution (light cones)
  - Conformal evolution (hyperboloidal slices)
  - Full 4D spacetime evolution (adequate choice of coordinates to expand Einstein eqn.)

Numerical Relativity



Visualisation by the SXS Collaboration, www.black-holes.org

Summary

- Only very few exact solutions to GR known
  - Schwarzschild, Kerr, FRWL, …
- Use approximate solving methods → Numerical Relativity
- 3+1 approach: separate space and time to formulate initial value problem
- Construct physical initial data and make appropriate gauge choices

A brief history of BBH simulations

- 1960s: First attempts to simulate BHs on a computer
- 1960 and 1970s: 3+1 formulation, some gauge conditions
- 1980s and 1990s: initial data for multiple BHs
- 1990s: resources to run full 3D simulations
  - But still no long-term evolutions of binary mergers !
- 2005: Breakthrough !

The Breakthrough

- Frans Pretorius (2005):
  - Generalised harmonic slicing
  - Moving excision
  - Compactified spatial domain to simplify boundary conditions
  - “constraint damping”
- UT-Brownsville and NASA-Goddard (2005):
  - Standard BSSN with puncture initial data
  - 1+log and Γ-driver gauge conditions
  - Modified treatment of the conformal factor
  - “moving” punctures

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Progress & Results since 2005

- Progress:
  - Simulations became longer and covered more orbits
  - Simulations became more accurate
    - Higher order finite differencing, pseudo-spectral methods
    - Radiation extraction at larger distances
    - Greater code efficiency: from mesh-refinement to multipatch
- Physics & Astrophysics:
  - Parameter (mass, spin) of final remnant BH
  - High energy BH collisions
  - Gravitational waveforms
  - Gravitational recoil
  - Formation of accretion discs …

Gravitational Waves

- Why are we looking for gravitational waves from BBH ?
  - Amongst the most promising sources for LIGO/Virgo
  - Exploring the dark side of the universe: in a matter-free system, NO EM counterparts
  - GWs are like the DNA of binary systems: revealing the physical properties (mass, spin) of the objects
- How are we looking for gravitational waves from BBH ?

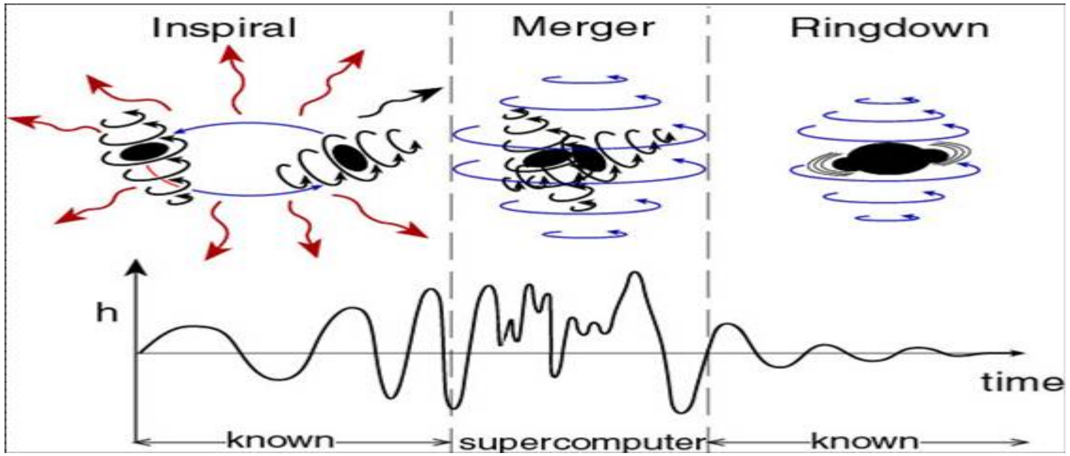
  Accurate waveform templates for matched filtering technique & astrophysics !
  - Large BBH parameter space to cover
  - *Phenomenological description* of generic systems required

Gravitational Waves

- How do we extract the gravitational wave signal in a numerical relativity simulation ?
- Numerical simulation uses specified coordinates but we want the GW signal in a gauge-invariant way !
  - ***Moncrief formalism***
    - Decompose metric in spherical harmonics and combine in a gauge-invariant way
  - ***Newman-Penrose formalism***
    - Project Weyl tensor on a null tetrad $\rightarrow$ Weyl scalars
    - 2 scalars can be interpreted as ingoing and outgoing gravitational radiation

BBH & Gravitational Waves

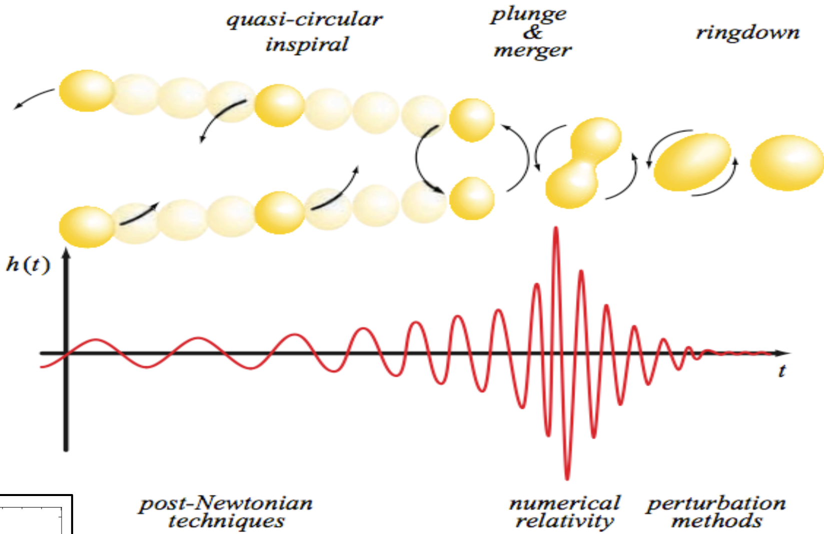
- Binary motion in three stages:
  - Inspiral: post-Newtonian approximation technique
  - Merger: ???
  - Ringdown: perturbation theory (LRR Kokkotas, Schmidt)
- M between $25M_{\odot}$ and 100 $M_{\odot} \rightarrow$ merger and ringdown “in band“
- We need to solve the full Einstein equations numerically to obtain the merger waveform !



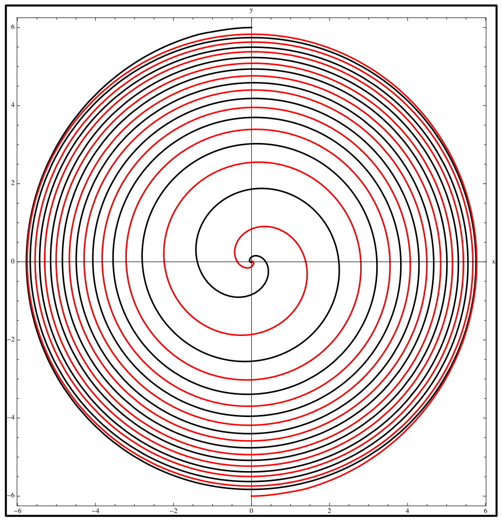
Impression by Kip Thorne

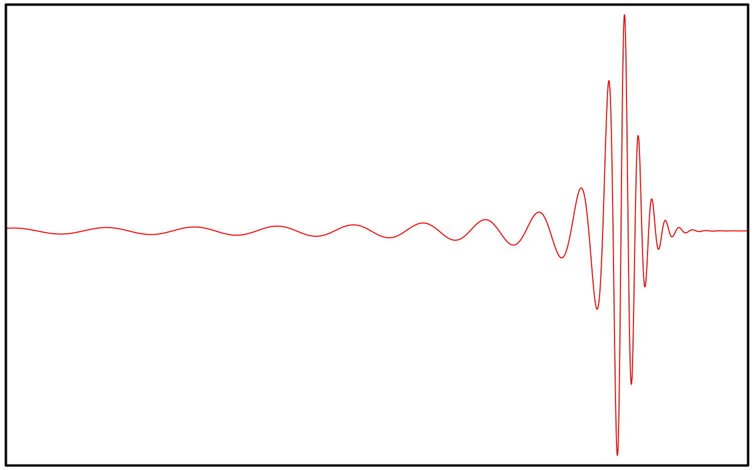
Gravitational Waves: after 2005

- NR:



From Baumgarte &Shapiro: Numerical Relativity





Gravitational Waves

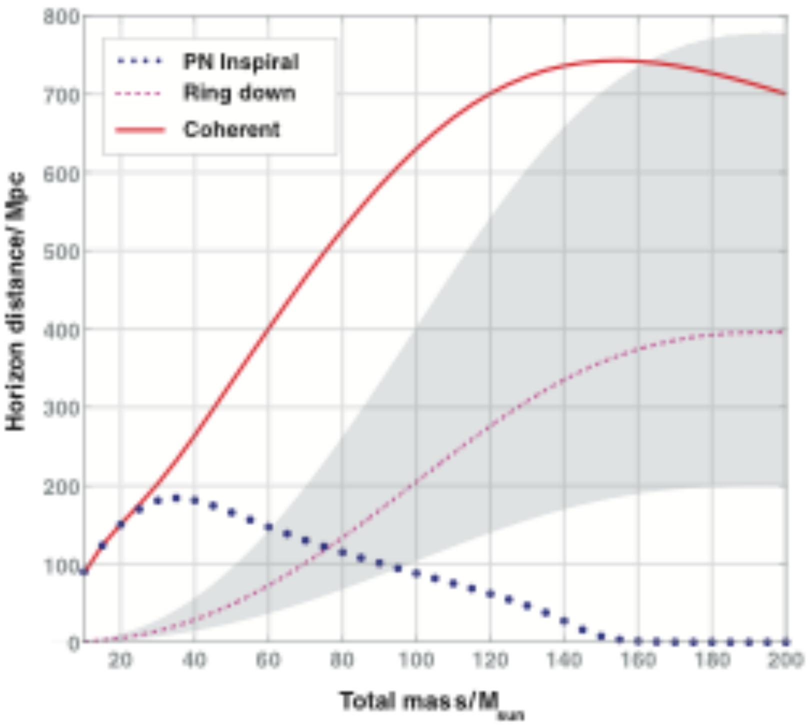
- This complete picture allows us to model the full signal emitted by binary configurations
  - Different approaches:
    - Semi-analytic phenomenological description of the signal
    - **E**ffective **O**ne **B**ody approach (EOB)

→Combine analytical and numerical information

- Improves
    - Detectability
    - Parameter estimation
    - Astrophysical understanding !

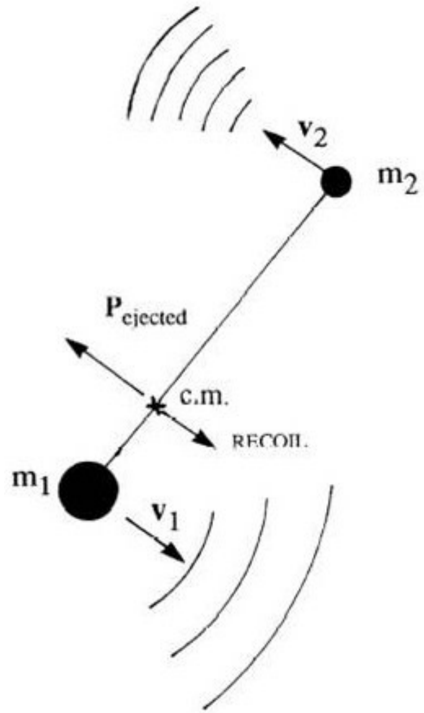
Gravitational Waves

- Complete inspiral-merger-ringdown waveforms enhance detectability
- Until now, accurate IMR description of:
  - Non-spinning binaries
  - Binaries with (anti-)aligned spins
- A lot left to do:
  - Eccentric, precessing binaries
  - Neutron star – BH binaries
  - Supernovae
  - …



Gravitational Recoil

- Gravitational recoil:
  - GW carry energy, linear and angular momentum
  - Asymmetric GW emission combined with inspiral → net momentum loss → the system will spiral outwards !
  - After merger the emission switches off and the final BH recoils
  - Kick velocities up to 4000 km/s !
  - Could explain extra-galactic BHs and has influence on formation models



Astrophysical implications

- Most BH-binaries will be inside galaxies
- If the recoil velocity is large enough → final BH can escape its host galaxy
- Highest galactic escape velocity is ~2000km/s (giant elliptic galaxy)
- If recoil large enough → effect on galactic BH populations
- Also creates a population of "wandering" extra-galactic BHs

Gravitational Recoil: calculations

- Effect first discussed in the 1960s.
- Gravitational collapse calculations (1970s): 20-300 km/s
- Fitchett (1983):
  - Considered circular orbits of Newtonian point particles
  - Max. kick velocity: 100s or 1000s of km/s
  - Max. kick velocity for a mass ratio of ~2.6.
- 1992-2006: Post-Newtonian calculations
  - Expansion of the Einstein equations on powers of v/c
  - Estimates are generally lower than Fitchett's
  - "best" PN estimate: max. between 50 and 500km/s
  - Large uncertainty and crucial merger and ringdown phase not included !

Approximate methods → full calculation needs NR !

Kicks and Superkicks

- When NR simulations were possible …
  - Unequal mass nonspinning binaries:
    - Max. recoil of 175±11km/s
    - Max. recoil is at mass ratio 2.78
    - BH ejection seems to be very unlikely …
  - Spinning binaries:
    - Aligned spins: recoil up to 500km/s !
    - Spins in the orbital plane (equal & opposite): recoil out of the plane with a velocity up to 4000km/s !

Current research

- Investigation of improved numerical methods to improve efficiency and accuracy of numerical simulations
- Construction of initial data for binaries with high spins
- Achieving simulations of high mass ratios
- Waveform production to allow waveform modelling
  - Generic spins → precession, eccentricity
- Studying the interaction with matter, electric and magnetic fields:
  - Neutron star binaries, neutron star – black hole binaries:
    - Testing the equation of state of neutron star matter
    - Formation of accretion discs and jets
- Cosmology