General Relativity

Project student lectures – 2013

Frank Ohme

Cardiff University

07/10/2013

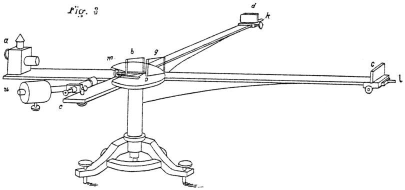
Schedule

Topic	Lecturer	Date
General Relativity	Frank Ohme	07/10
Gravitational Waves (GWs)	Frank Ohme	10/10
GW detectors	Tom Adams	14/10
GW Bursts	Tom Adams	17/10
Compact Binary Coalescences	Erin Macdonald	21/10
Multi-Messenger Astronomy	Erin Macdonald	24/10
Numerical Relativity I	Patricia Schmidt	28/10
Numerical Relativity II	Patricia Schmidt	31/10

Special Relativity

Maxwell's equations describing electrodynamics show that *light* travels at a finite velocity

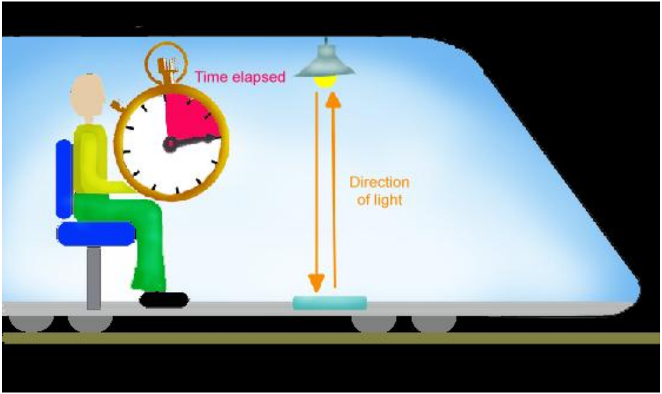
- Reference frame/medium for light ("Aether")?
- Michelson & Morley showed that there is no preferred reference frame.



Einstein's special relativity

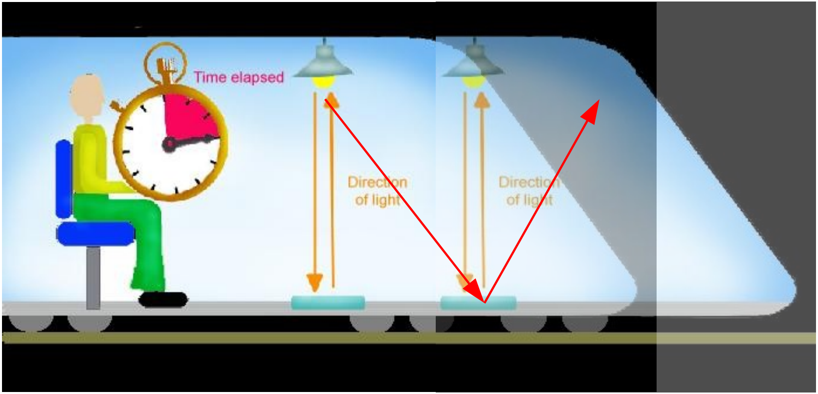
- Space and time unified in *spacetime*
- Speed of light is **constant, same for all observers**

Conclusions from the constant speed of light



- From inside: time measured by light beam going up and down, no way of telling whether the train is moving at constant speed

Conclusions from the constant speed of light



- From outside: light travels a longer path but with the *same* speed ⇒ takes longer

The passenger could, of course, tell that the train is moving if

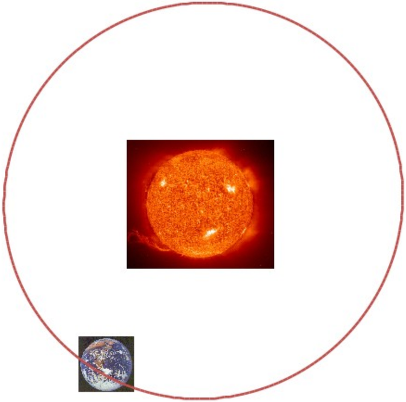
. . .

The passenger could, of course, tell that the train is moving if . . .

the train is **accelerating/decelerating**!

How does gravity fit in?

- Gravity does not fit within Special Relativity
- Gravity acts instantaneously

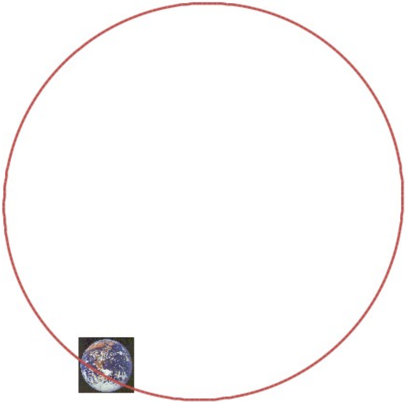


How does gravity fit in?

- Gravity does not fit within Special Relativity
- Gravity acts instantaneously

Gedankenexperiment:

- Take the sun away.

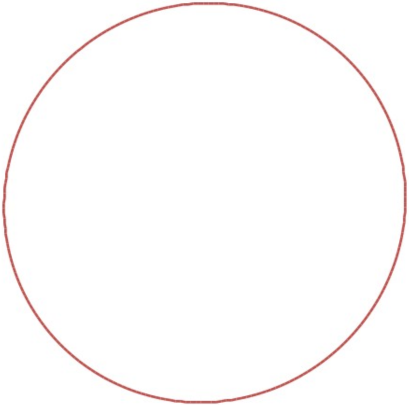


How does gravity fit in?

- Gravity does not fit within Special Relativity
- Gravity acts instantaneously

Gedankenexperiment:

- Take the sun away.
- The earth immediately flies off.

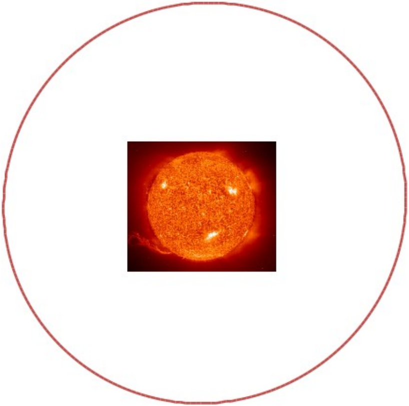


How does gravity fit in?

- Gravity does not fit within Special Relativity
- Gravity acts instantaneously

Gedankenexperiment:

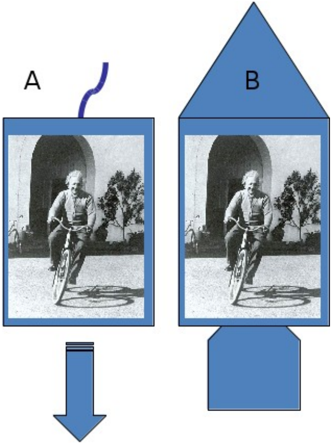
- Take the sun away.
- The earth immediately flies off.
- However, sun still visible for 8 mins.



Einstein's thoughts

There is no way to tell the difference between

1. A freely falling elevator and
2. zero gravitational field in deep space,



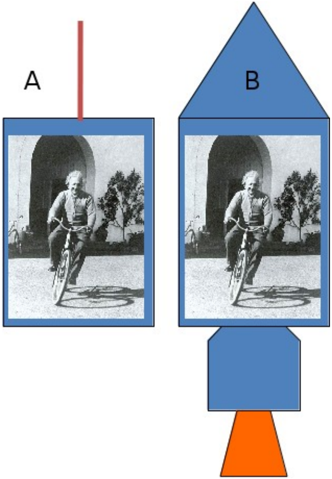
Einstein's thoughts

There is no way to tell the difference between

1. A freely falling elevator and
2. zero gravitational field in deep space,

or

1. the attraction of the earth's field and
2. an accelerated spaceship.



Einstein's thoughts

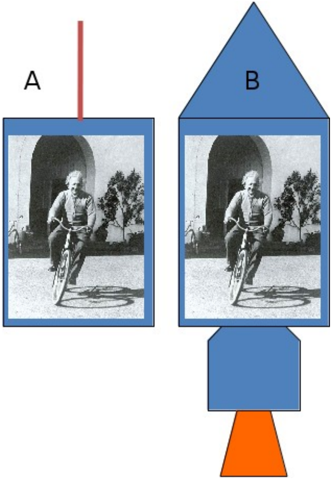
There is no way to tell the difference between

1. A freely falling elevator and
2. zero gravitational field in deep space,

or

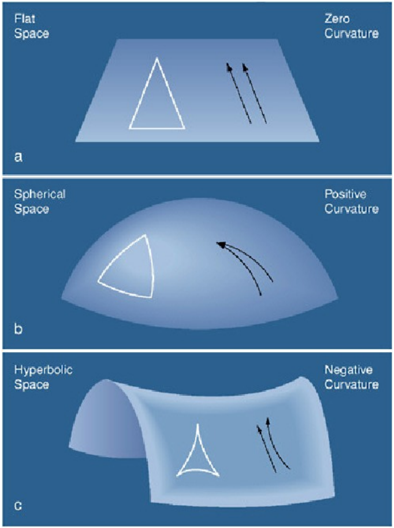
1. the attraction of the earth's field and
2. an accelerated spaceship.

Reason:
$\vec{F} = m\vec{a}$ and $\vec{F} = m\vec{g}$ contain the *same* mass.



GR is geometry!

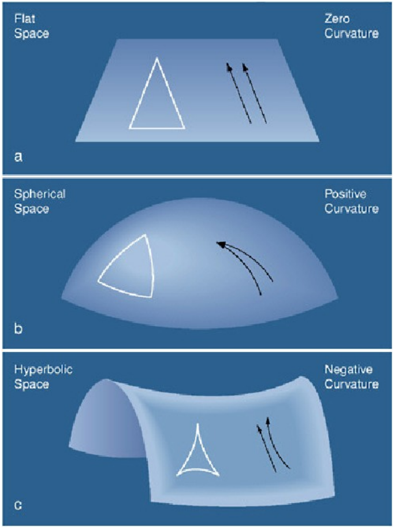
- *Gravity* is seen as a warping in space and time
- Caused by the mass and energy in the universe



GR is geometry!

- *Gravity* is seen as a warping in space and time
- Caused by the mass and energy in the universe

But how can we see curvature in a 4-dimensional spacetime without having more dimensions?

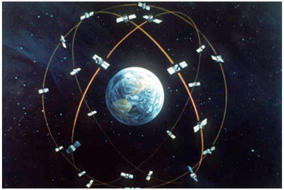


Home exercise I

Clocks in GPS satellites

Assume time is measured by laser beams just as in the canonical example of Special Relativity. According to Einstein's Equivalence Principle, being in a gravitational field is no different from travelling in an accelerating rocket. Therefore, clocks are ticking at different rates depending on their location in the gravitational field.

Are the clocks in GPS satellites ticking faster or slower compared to the surface of the earth?



Mathematical interlude

Distance

- Consider two points separated by
$$d\vec{x} = (dx, dy, dz)$$
- Their distance in flat space reads
$$ds^2 = dx^2 + dy^2 + dz^2$$

Metric

- Slightly more generalized $(dx, dy, dz) = (dx^1, dx^2, dx^3)$
$$ds^2 = g_{ab}\, dx^a\, dx^b$$
(summed over indices a and b)
- g_{ab} is the metric of the space
(in flat space: simply 3×3 identity matrix)

Generalize the concept

Flat spacetime

- Space and time $(dt = dx^0)$ unified

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$
$$= \eta_{ab}\, dx^a\, dx^b$$

- η_{ab} is the Minkowski metric
  - $ds^2 > 0$ spacelike separation, no information can be passed between both points
  - $ds^2 = 0$ null separation, (only) light can travel from one point to the other
  - $ds^2 < 0$ timelike separation, particles can go from one point to the other

Generalize the concept

General (curved) spacetime

- In general,

$$ds^2 = g_{ab}\, dx^a\, dx^b \ ,$$

  where metric g_{ab} can differ from Minkowski, but has to be symmetric and obey a particular "signature"
- There is no unique or special coordinate system, and metric has different form in every system

$\Rightarrow$ Neither coordinates nor metric mean very much by themselves

- Distance of (closely) neighbouring points are invariant

Home exercise II

Metric transformations

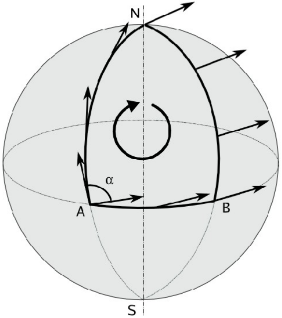
The flat space metric in Cartesian coordinates is the 3×3 identity matrix. How does the same metric look in cylindrical and spherical coordinates?

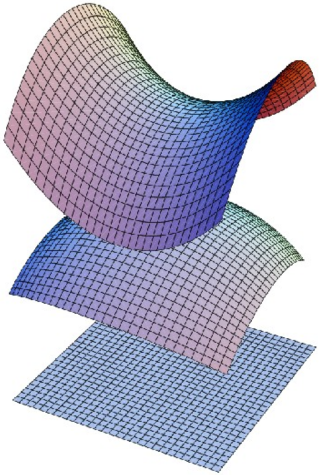
Can you think of a coordinate system in which the flat space metric is *not* diagonal?

Curvature

Riemann tensor

- (Intrinsic) curvature is defined by the Riemann tensor
  $$(\nabla_c \nabla_d - \nabla_d \nabla_c) v^a = R^a{}_{bcd} v^b$$
- ∇ is the covariant derivative



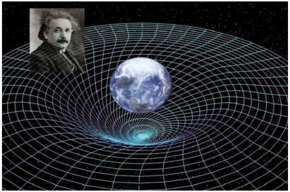


Einstein's Equations

Condensed form

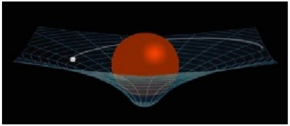
- Matter and energy warp spacetime

$$\underbrace{R_{ab} - \frac{1}{2} R\, g_{ab}}_{\textit{curvature}} = \underbrace{8\pi T_{ab}}_{\textit{energy/momentum}}$$



Geodesics

- Free particles move along *geodesics*, the equivalent of a straight line in curved spacetime
- Geodesics are the shortest paths between two points.



Geodesic deviation

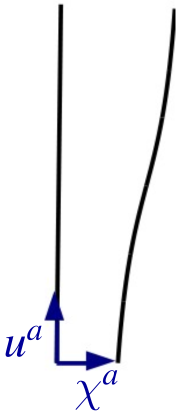
The effect of gravity

- Gravity can be detected by monitoring the distance between two freely falling particles (on geodesics)
- Geodesic deviation describes how separation evolves

$$\nabla_u \nabla_u \chi^a = R^a{}_{bcd}\, u^b u^c \chi^d$$

⇒ The "acceleration" of the separation is determined by curvature

⇒ Gravity acts as a tidal force



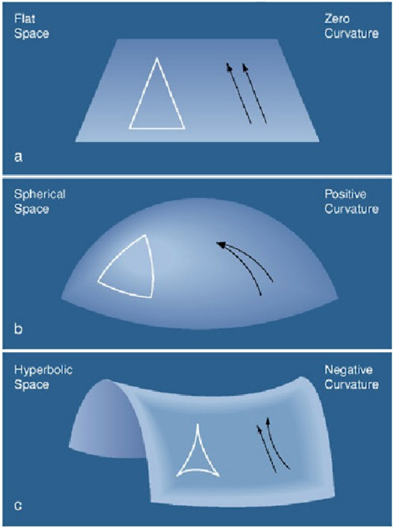
Geodesic deviation

Examples

→ Flat space, parallel lines stay parallel

→ Positive curvature, initially parallel lines will intersect

→ Negative curvature, initially parallel lines diverge



Observable Consequences

Examples

- Precession of Mercury
- Bending of the light by the sun
- “Inspiral” of binary pulsars