Gravitational Wave Detectors

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Postgrad Lectures 2013

Outline

- Interferometer basics
  - Recap of GW basics
  - Detecting gravitational waves
  - Interferometers
  - GW detectors
- Noise sources
  - Seismic noise
  - Optimal readout noise
  - Thermal noise
- Transient noise

Who am I?

- Thomas Adams
- 4th year PhD student working with Patrick
  - Searches for gravitational wave bursts
  - Characterisation of the GEO 600 detector
- Spent half my PhD on site at the British - German GEO 600 detector near Hannover

Recap of gravitational waves

- Distance between two points:

$$ds^2 = \left(dx_1^2 + dx_2^2 + dx_3^2\right)$$

- Can rewrite this in General Relativity as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- Where *g* is the metric (identity for flat space) and *dx* is the displacement vector
- Includes new term for displacement in time

Recap of gravitational waves

- Gravitational waves are perturbation of the space-time metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

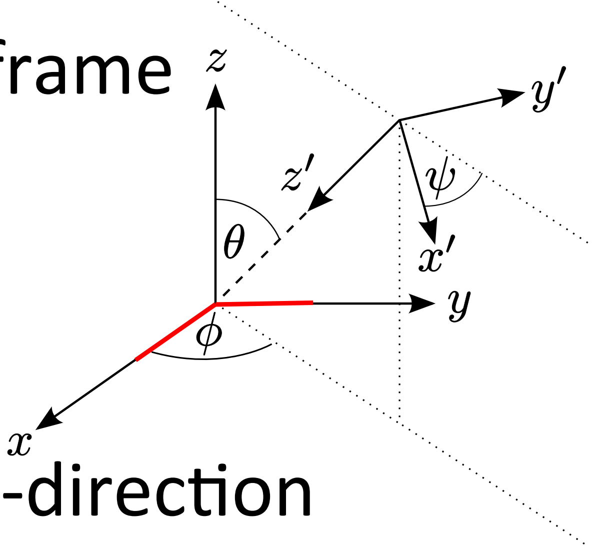
- In the simplest form, waves travelling in z-direction take this form:

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Detecting gravitational waves

- Rotation gives simple form in lab frame

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xx} & h_{xy} & h_{xz} \\ 0 & h_{yx} & h_{yy} & h_{yz} \\ 0 & h_{zx} & h_{zy} & h_{zz} \end{pmatrix}$$



- Consider light beam travelling in x-direction (down the lab) between two test masses, and GW travelling in z-direction (vertically down). Distance between them changes:

$$\begin{aligned} ds^2 = g_{\mu\nu}dx^\mu dx^\nu &= \left(\eta^{\mu\nu} + h_{\mu\nu}\right) dx^\mu dx^\nu \\ &= -c^2dt^2 + dx^2\left(1 + h_{xx}^{TT}\right) \end{aligned}$$

Detecting gravitational waves

- Consider the following experiment
  - Shine a laser down the lab (in the x-direction)
  - Time how long it takes to hit the test mass
- For a laser beam:

$$ds^2 = 0$$

- so the distance to the end of the lab changes

$$L_x = \int_0^L c\,dt = \int_0^L \sqrt{1 + h_{xx}} \simeq \left(1 + \frac{1}{2}h_{xx}\right) L$$

- We can measure the light travel time change to detect and study gravitational waves directly

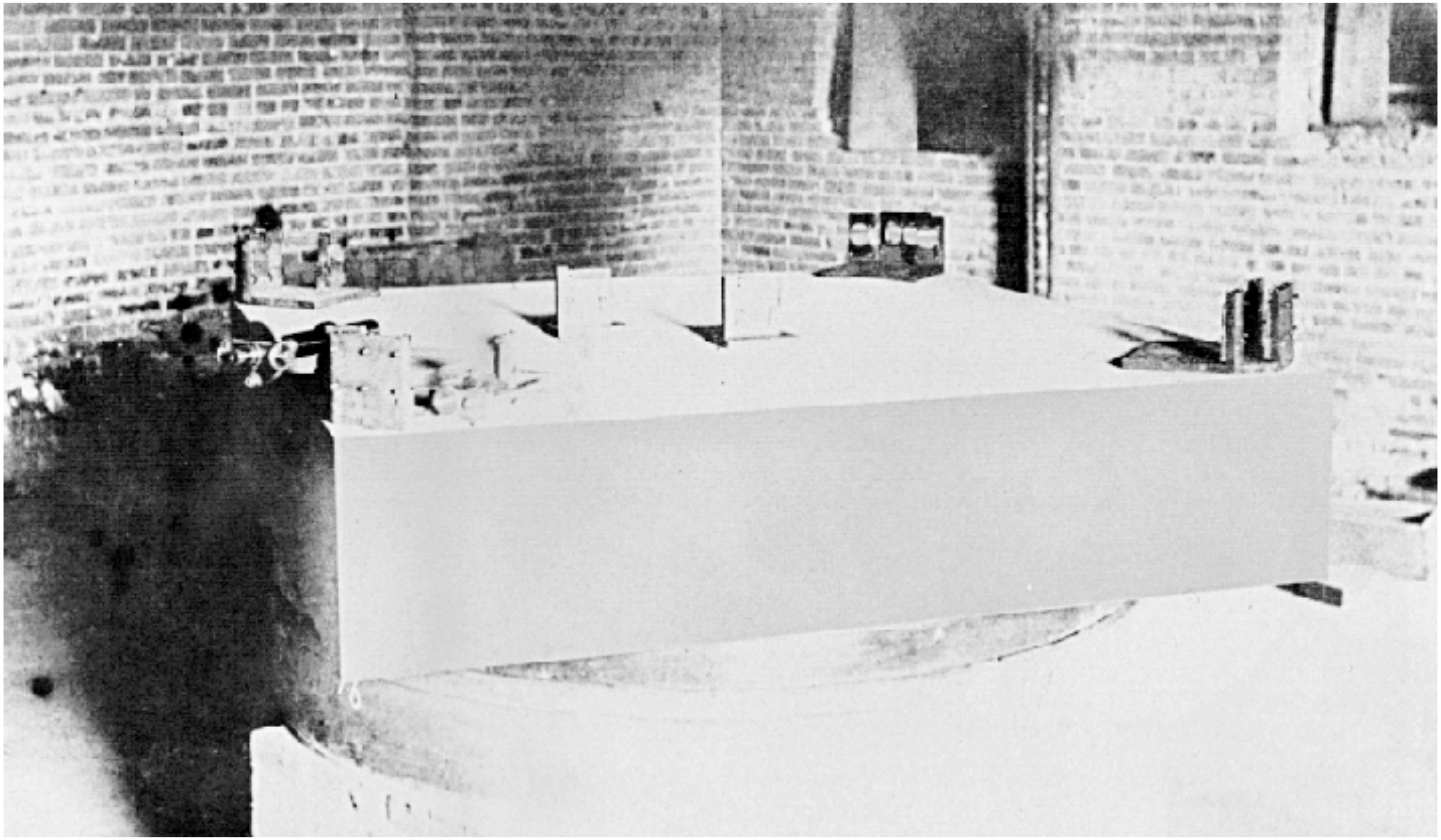
Detecting gravitational waves

- If we do this in both the x- and y-directions we can measure gravitational wave strain

$$h(t) = \frac{\Delta L}{L} = \frac{L_x - L_y}{L} = \frac{1}{2}(h_{xx} - h_{yy})$$

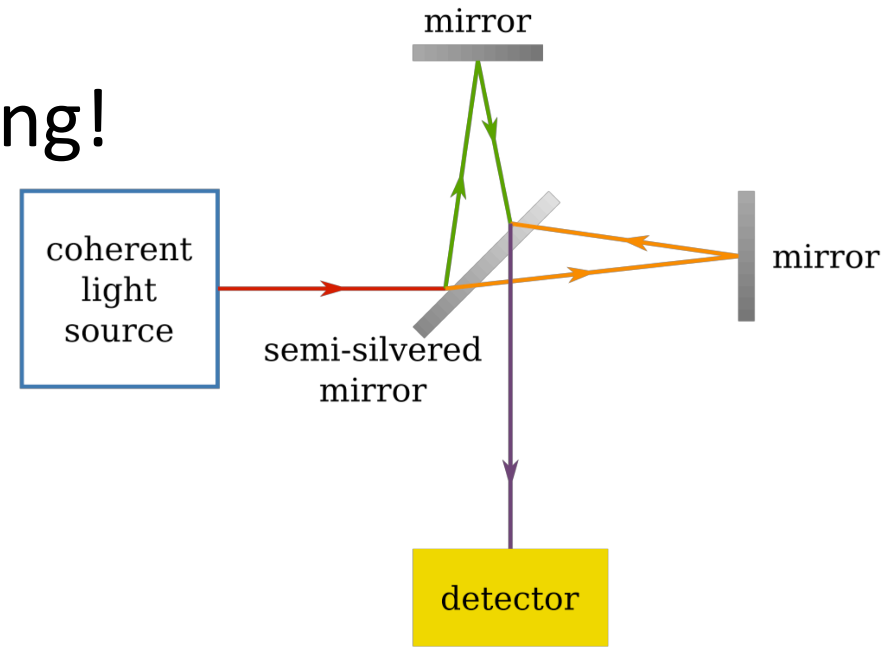
- Conveniently, such experiments have been around for more than a hundred years

Interferometers



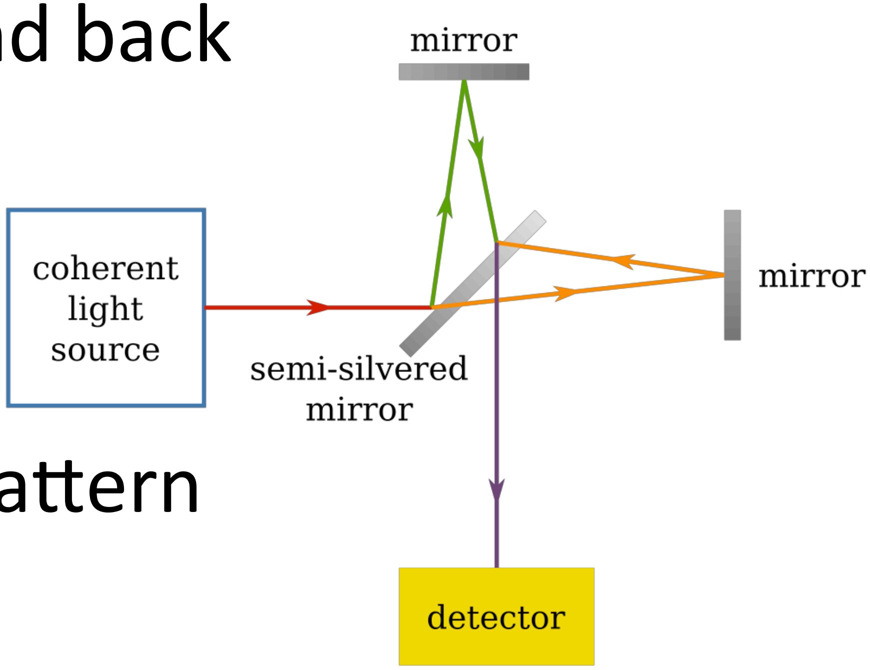
Interferometers

- Michelson-Morley designed this experiment to detect the Æther in 1887
- Proved themselves wrong!



Interferometers

- Shine laser light onto partially reflecting mirror – "beam splitter"
- Send down two arms and back
- Recombine at beam splitter and exit at output photo detector
- Measure interference pattern to accuracy of λ/20



GW detectors

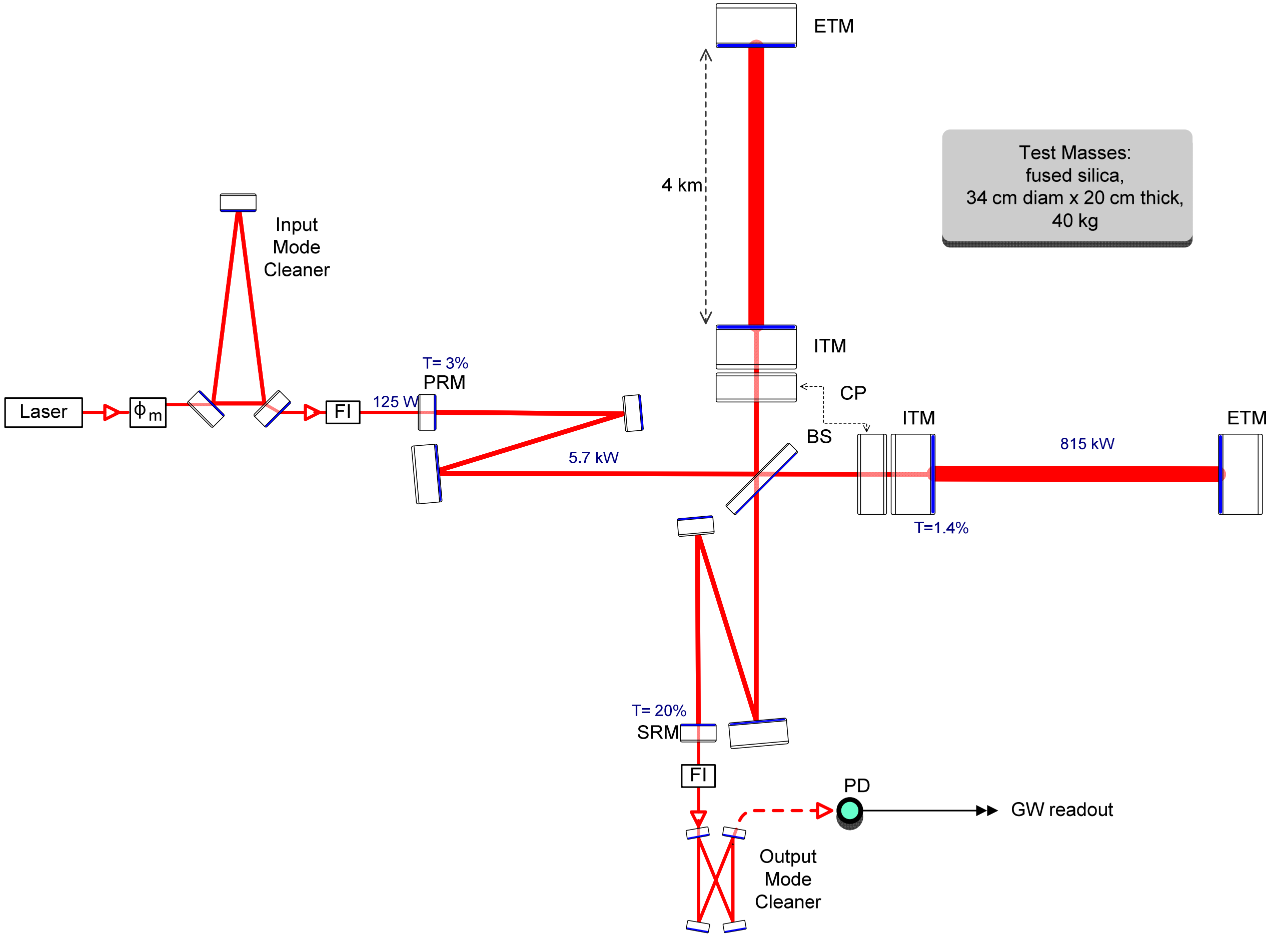

American LIGO at Hanford


American LIGO at Livingston


British-German GEO

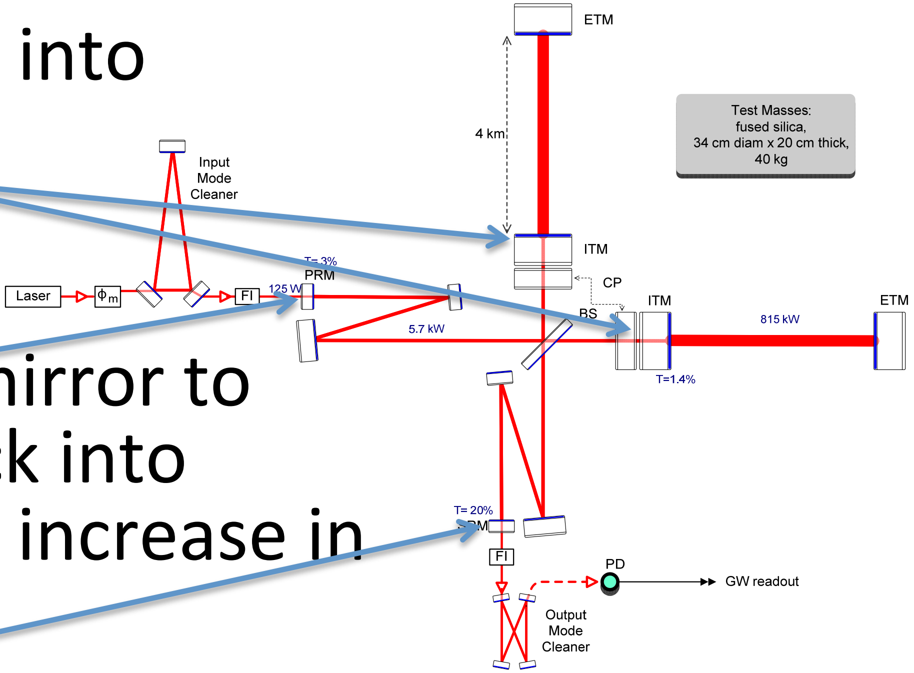
French-Italian Virgo near PISA

GW detectors



http://www.ligo.caltech.edu/docs/M/M060056-08/M060056-08.pdf

GW detectors



- Added mirrors turn arms into Fabry-Perot cavities. Effective increase in arm length of 30x-50x.
- Added power recycling mirror to return reflected light back into interferometer. Effective increase in power by 20x.
- Added signal recycling mirror to return output light (signal) into interferometer. Effective increase in signal time in interferometer.

GW detectors

- Have global network of large scale detectors
- Laser Interferometer Gravitational-Wave Observatory (LIGO):
  - Two detectors at Hanford, WA
    - ‘H1’ – 4 kilometer Fabry-Perot arm cavities
    - ‘H2’ – 2 km FP cavities
  - One detector at Livingston, LA
    - ‘L1’ – 4 km FP cavities
- Virgo at Pisa, Italy, with 3 km FP cavities
- GEO600 at Hannover, Germany with 600m folded arms
  - No FP cavities, arms fold back on themselves

GW detector sensitivity

- Optimum sensitivity is achieved when light takes ½ period of GW to traverse the arm
- For a GW at 200Hz, this corresponds to arm length of 500km…

- Fabry-Perot cavities store light in arms for ~50 round trips, LIGO with FP has effective arm length 200km.

GW detector sensitivity

- Consider a GW travelling in z-direction (i.e. landing on top of detector).
- We have $h_{xx} = -h_{yy} = h$, and recall for a single trip down the x-arm

$$c\Delta t = \left(1 + \frac{1}{2}h_{xx}\right) L$$

- So, for round trip, time delay between signals in two arms is:

$$\Delta t = \frac{2}{c}\left(\left(1 + \frac{1}{2}h_{xx}\right) L - \left(1 - \frac{1}{2}h_{yy}\right) L\right)$$

GW detector sensitivity

- Solving that gives a time-delay

  $$\Delta t = \frac{2h(t)L}{c}$$

  or a phase shift of

  $$\Delta\phi = \frac{4\pi h(t)L}{\lambda}$$

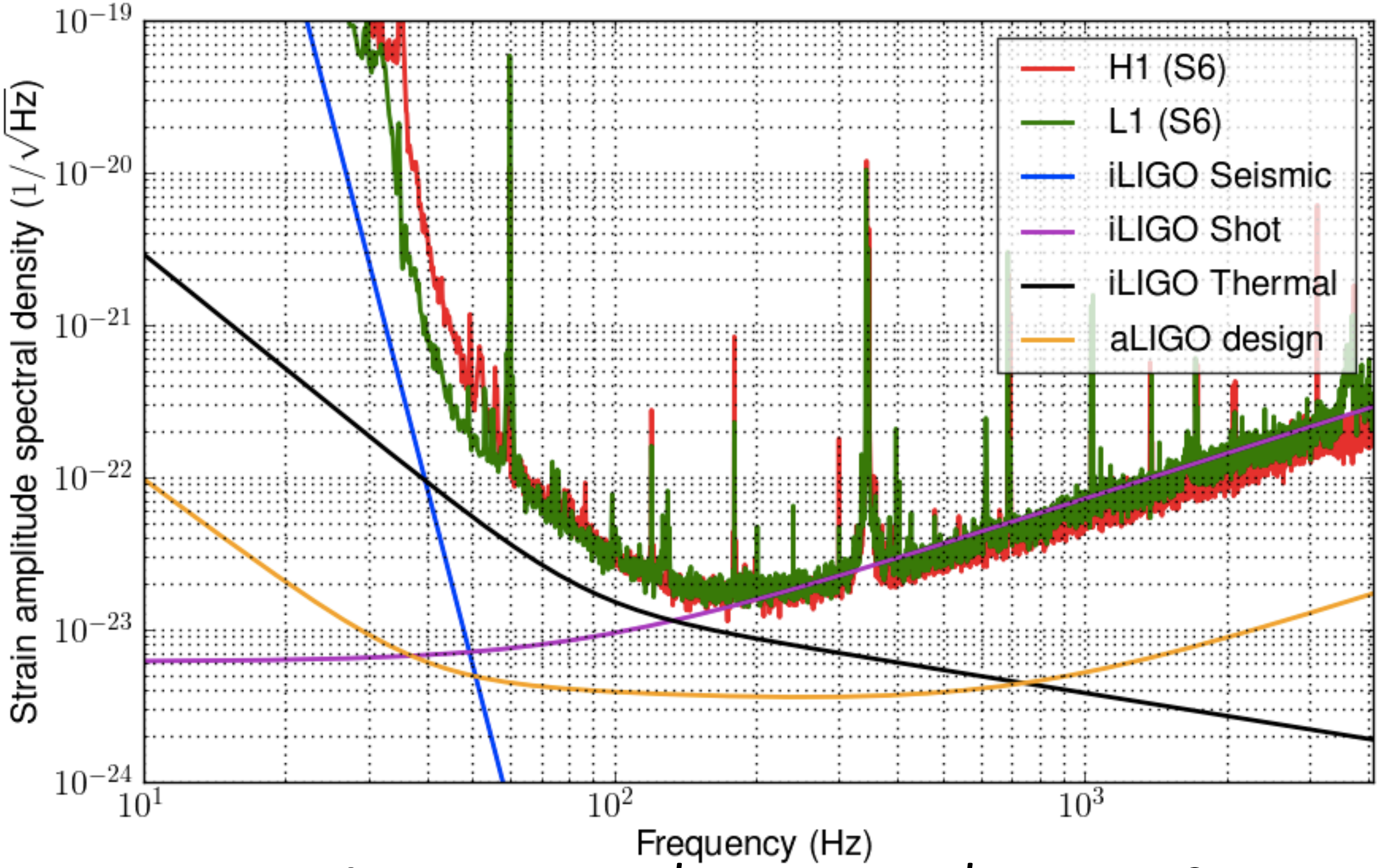
- If we plug in realistic numbers, $\lambda = 1064$nm, $L = 4$km, $h = 10^{-21}$ we get:

$$\Delta L = 2 \times 10^{-18}\text{m}, \quad \Delta\varphi = 2 \times 10^{-9}$$

GW detector sensitivity

- Patrick Sutton’s reality check:
  - Sensitivity $\Delta L = 2 \times 10^{-18}$m
    $\Delta\varphi = 2 \times 10^{-9}$
- These are:
  - 10^{10} times smaller than 1 MM fringe
  - 10^{12} times smaller than laser wavelength
  - 10^{9} times smaller than atoms in mirrors whose position we try to measure
  - 10^{12} times smaller than seismic motion shaking mirrors

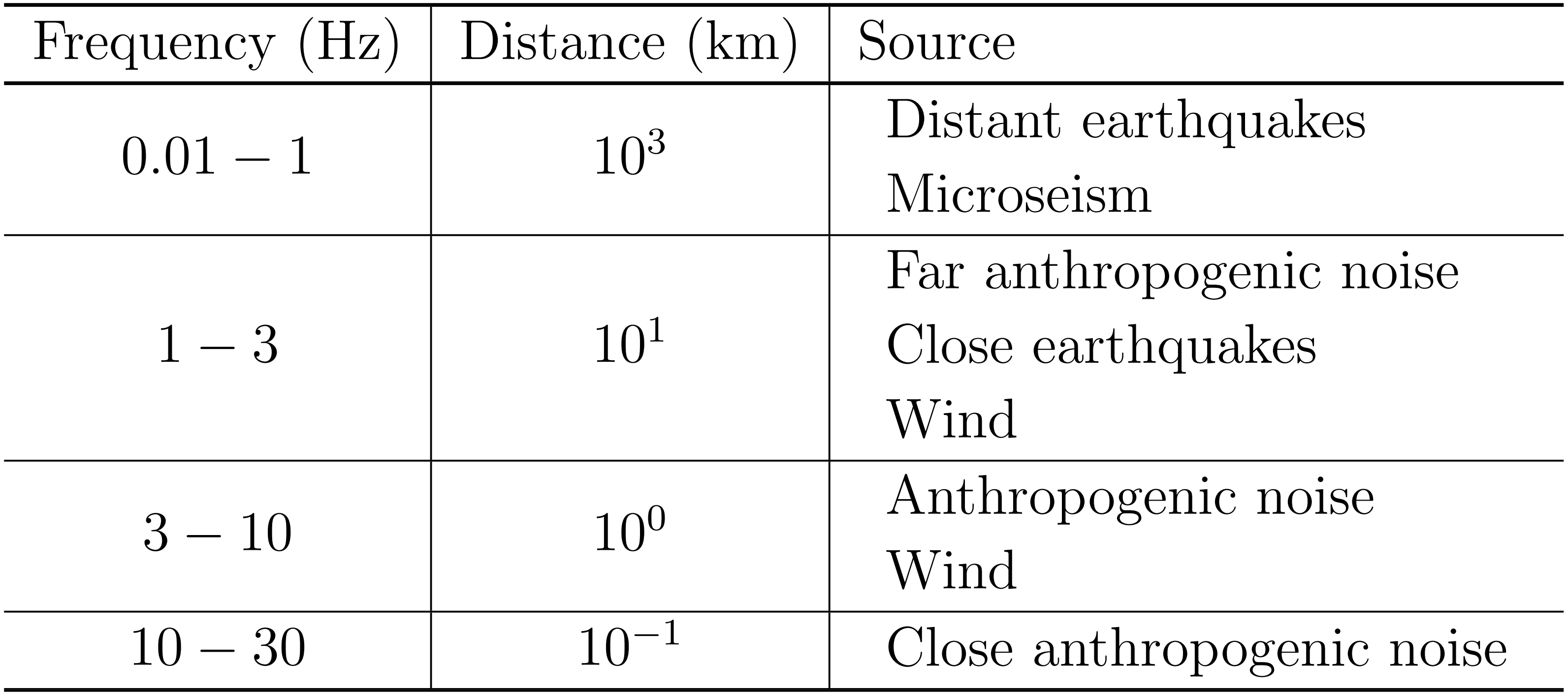
GW detector sensitivity



- It works! …but it cost \$250M + \$250M for aLIGO

Seismic noise

- Shaking of ground due to earthquakes, weather, human activity

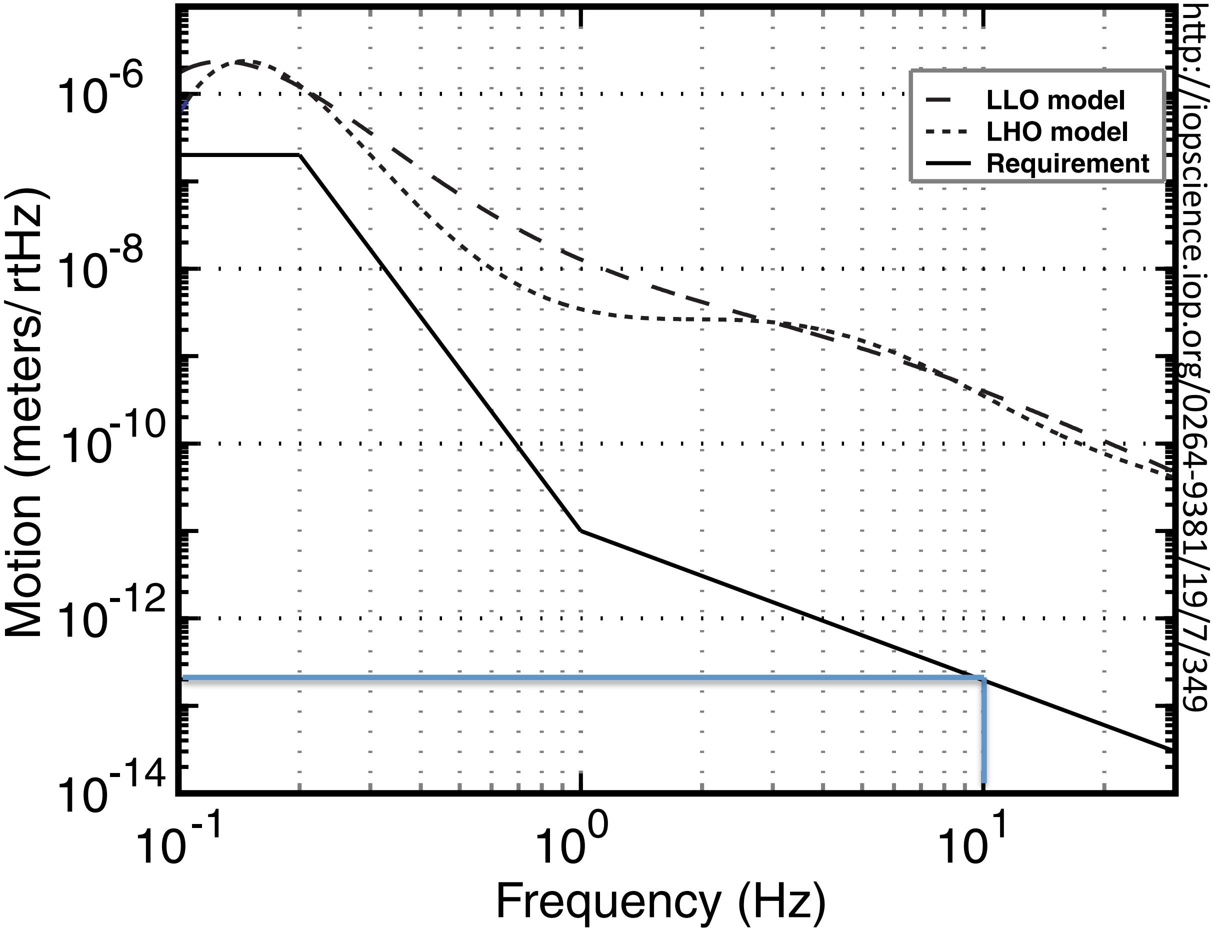


Frequency (Hz)	Distance (km)	Source
$0.01 - 1$	10^3	Distant earthquakes Microseism
$1 - 3$	10^1	Far anthropogenic noise Close earthquakes Wind
$3 - 10$	10^0	Anthropogenic noise Wind
$10 - 30$	10^{-1}	Close anthropogenic noise

Table 1. Description of the main seismic frequency bands and their sources

http://arxiv.org/abs/1108.0312

Seismic noise

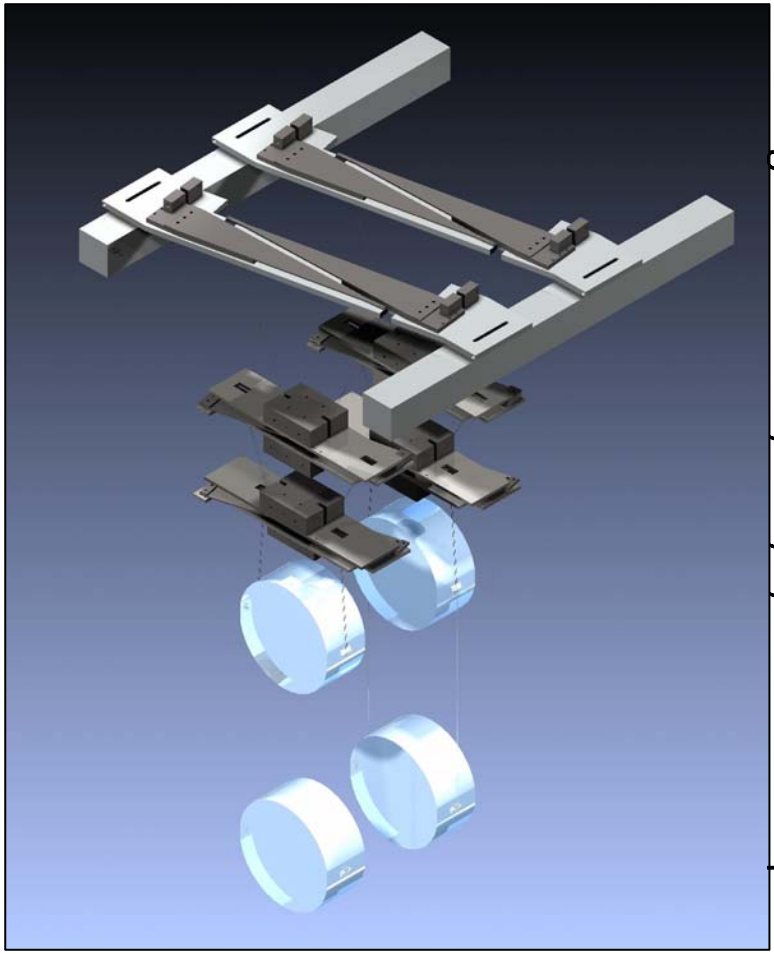


http://iopscience.iop.org/0264-9381/19/7/349

At 10 Hz, seismic motion is several orders of magnitude above GW spectrum

Passive seismic isolation

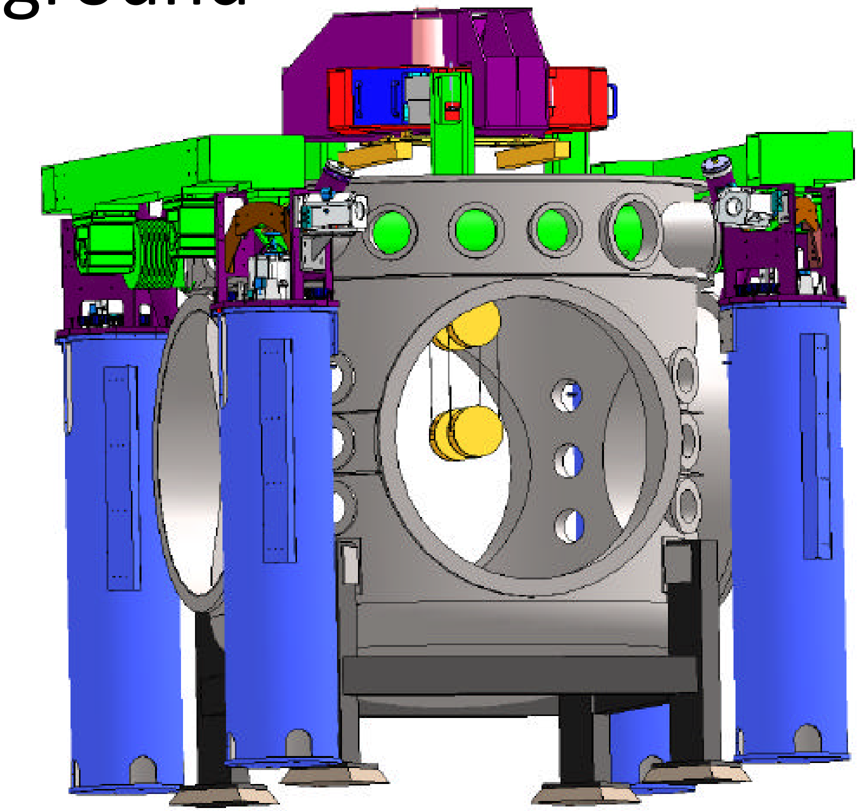
- Need to supress motion by factor of 10^8
- Mirrors suspended as quadruple pendula
- 40 kg silica test masses
- Lab floor on separate slab



www.ligo.caltech.edu/docs/G/G040046-00.pdf

Active seismic isolation

- Use sensors to detect ground motion and correct accordingly
- Hydraulic actuators reduce factor of 10 at low frequency (<2 Hz)
- EM actuators give factor of 30 at 10Hz



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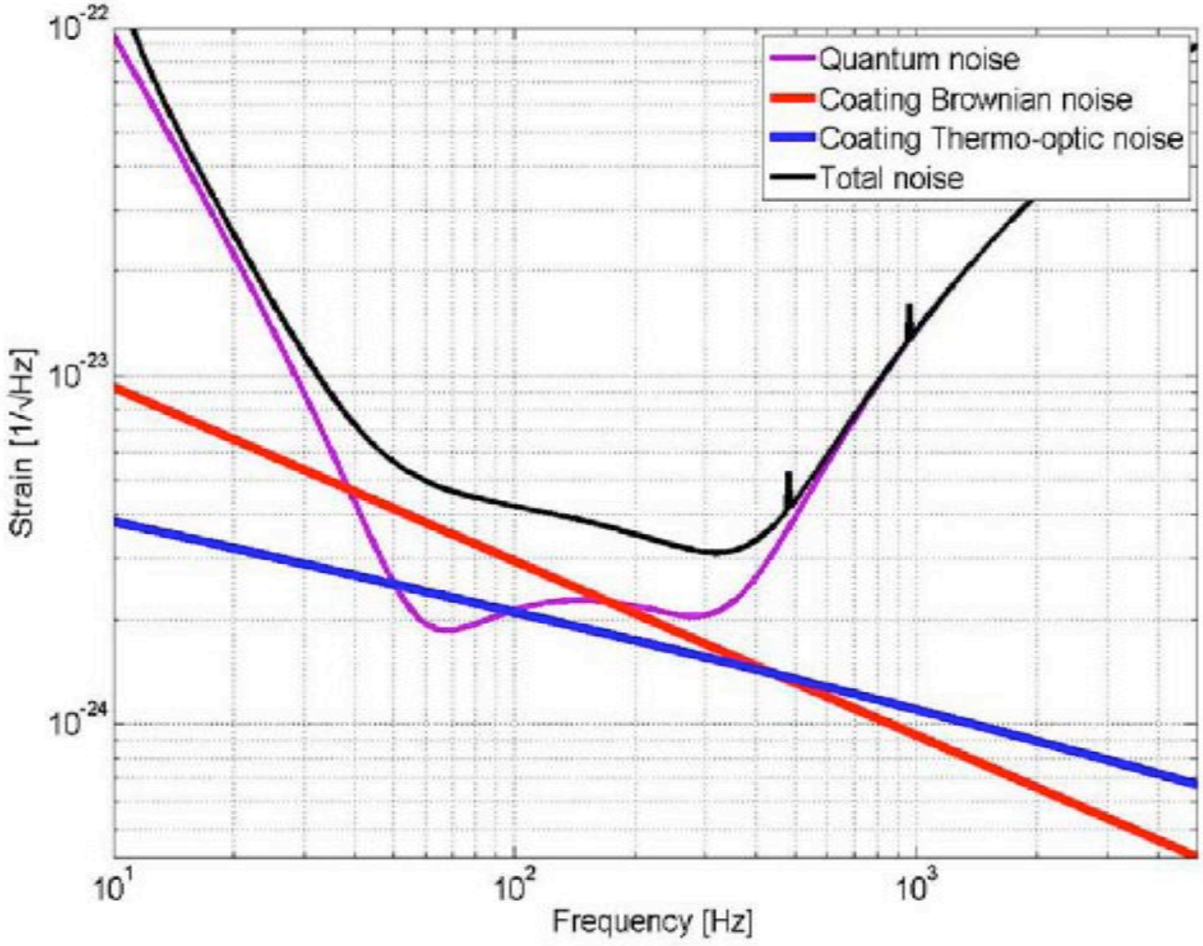
Thermal noise



www.ligo.caltech.edu/LIGO_web/seminars/pdf/villar.pdf

Thermal noise

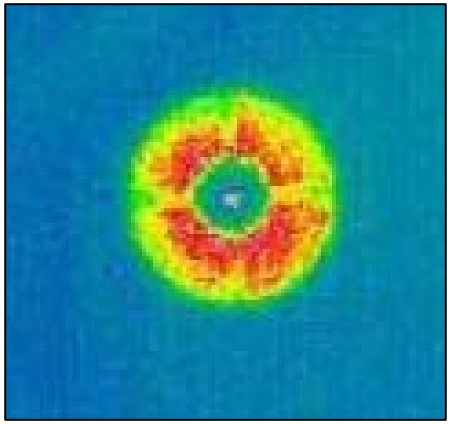
- All detector parts are subject to thermal noise:
  - Brownian motion (random motion of atoms in the mirrors)
  - Vibrational modes (resonances) of the suspension wires
  - Dissipation from friction in the wires
- Recall equipartition theorem: each degree of freedom contributes energy of ½k_BT. For 40 kg mass suspended from 1m wire:

$$\Delta L_{\mathrm{rms}} = \sqrt{\frac{k_B T l}{m g}} \simeq 3 \times 10^{-12}\,\mathrm{m}$$

- 10^7 times larger than motion due to GW

Thermal noise

- Further thermal noise due to heat transfer from laser to mirrors
- Mitigate thermal noise with:
  - – Fused silica wires – less friction, less noise
  - – Better connection to mirror
  - – Better coatings for mirrors
  - – Heavier mirrors
  - – Thermal compensation



Quantum noise – shot noise

- Quantum noise is introduced by the fluctuating number of photons arriving at the output port
- The uncertainty in the phase of a laser beam due to quantization of light into photons is called shot noise.
- Dominant noise source at high frequency
- If error in photon number if σ_N, and error in phase is σ_ϕ HUP:

$$\sigma_N \sigma_\phi = 1$$

Quantum noise – shot noise

- Number of photons per second:

$$N_\gamma = \frac{P}{E} = \frac{P\,\lambda}{hc} = 50\frac{200\mathrm{W}\;1064\mathrm{nm}}{hc} \simeq 6\times 10^{22}\,\mathrm{s}^{-1}$$

- So error (assuming Poisson statistics):

$$\sigma_N = \sqrt{N} = 2.5\times 10^{11}$$

- And so the phase error is:

$$\sigma_\phi = \frac{1}{\sigma_N} = 4\times 10^{-12}\mathrm{rad}$$

- This is equivalent to an arm length change of:

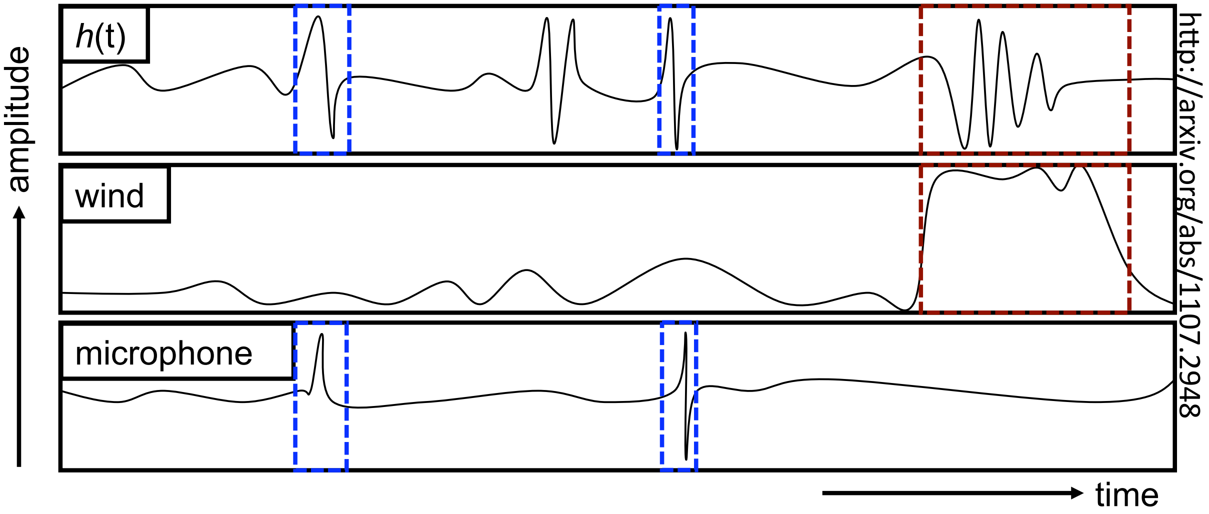
$$\Delta L = \frac{1064\,\mathrm{nm}}{2\pi}4\times 10^{-12} = 6\times 10^{-19}\,\mathrm{m}$$

Quantum noise – radiation pressure

- Quanta in laser beam carry energy and momentum, which pass to mirrors upon transmission/reflection
- Higher power laser gives better sensitivity in areas (less shot noise), but higher radiation pressure noise
- Limiting noise source in 10-50 Hz band.

Transient noise

- Limiting sources are stationary (time and freq)
  - form baseline for sensitivity
- Many searches for short-duration GW events
- Also short-duration noise events that mask/ mimic GWs



http://arxiv.org/abs/1107.2948

Summary

- Differential arm motion can detect GW!
- Large-scale interferometers have been built for this purpose
  - Can detect differential motion to 10^{-18} m!
- Subject to noise sources
  - Seismic noise due to ground motion (low frequency)
  - Thermal noise in suspension and mirrors (mid frequency)
  - Radiation pressure noise (mid) and shot noise (high) due to laser
- Subject to transient events that can mimic GW