

Searching for Periodic Gravitational Waves from Spinning Neutron Stars

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Outline

- 1 Introduction: What are CWs?
- 2 CW Search Methods
 - Generalities
 - Standard CW Bayes factor
 - New “Line-robust” statistic
- 3 Current status and future outlook
 - Astrophysical priors
 - Current Sensitivities
 - Future Sensitivities



Continuous GWs from Spinning Neutron Stars

Rotating neutron star:

- non-axisymmetric $\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$
- rotation rate ν

☞ GW with frequency $f = 2\nu$

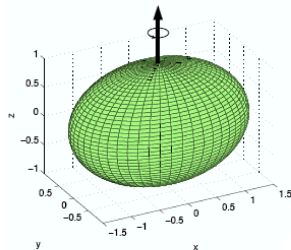
Strain-amplitude h_0 on earth:

$$h_0 = \left(\frac{16\pi^2 G}{c^4} \right) \frac{\epsilon I_{zz} \nu^2}{d}$$
$$= 4 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}} \right) \left(\frac{I_{zz}}{10^{45} \text{ g cm}^2} \right) \left(\frac{\nu}{100 \text{ Hz}} \right)^2 \left(\frac{100 \text{ pc}}{d} \right)$$

1st generation sensitivity (S5/S6): $\sqrt{S_n} \sim 2 \times 10^{-23} \text{ Hz}^{-1/2}$

☞ CW signals buried in the noise \implies need “**matched filtering**”

$$\text{SNR} \propto \frac{h_0}{\sqrt{S_n}} \sqrt{T} \quad \text{observation time } T \sim (\text{days} - \text{months})$$



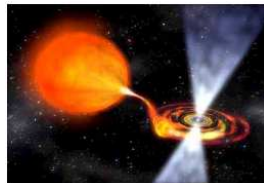
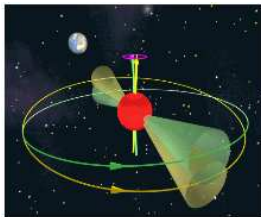
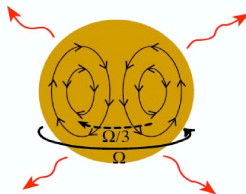
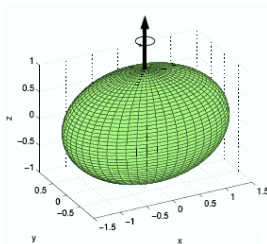
Different CW emission mechanisms

Continuous waves:

- CW lifetime $\gtrsim T_{\text{obs}}$
- quasi-monochromatic sinusoid $f \sim \mathcal{O}(\nu)$

Emission mechanisms:

- “Mountains”
($f = 2\nu$)
- Oscillations
(r-modes:
 $f \sim 4\nu/3$)
- Free precession
($f \sim \nu, 2\nu$)
- Accretion (driver)



Statistics as applied Probability Theory

Probability Theory: an extension of the framework of deductive logic to work with *incomplete information* ("Inference")

[Jaynes, Cox]

A ... logical proposition, e.g.

A = "There is a (detectable) GW signal in this data"

$A(h_0, f)$ = "The GW signal has amplitude h_0 , frequency f "

$P(A|I) \equiv$ 'plausibility' of A being true *given* I

I ... set of relevant 'knowledge' and model assumptions

$P(A|I)$ quantifies an **observer's** state of knowledge about A

☞ **not** an intrinsic property of the observed system!

(Jaynes "Mind projection fallacy")



The Three Laws

(Cox 1946, 1961, Jaynes) Requiring 3 conditions for $P(A|I)$:

(i) $P \in \mathbb{R}$, (ii) consistency, (iii) agreement with "common sense"
one can *derive* **unique** laws of probability (up to gauge):

- 1 $P(A|I) \in [0, 1]$ $\begin{cases} P(A|I) = 1 & \Leftrightarrow (A|I) \text{ certainly true} \\ P(A|I) = 0 & \Leftrightarrow (A|I) \text{ certainly false} \end{cases}$
- 2 $P(A|I) + P(\text{not } A|I) = 1$
- 3 $P(A \text{ and } B|I) = P(A|B, I) P(B|I)$

☞ $P(A|B, I) = P(B|A, I) \frac{P(A|I)}{P(B|I)}$ ("Bayes' theorem")

☞ $P(A \text{ or } B|I) = P(A|I) + P(B|I) - P(A \text{ and } B|I)$

We observe data 'x', what can we learn from it?

Formulate "question" as a proposition A and *compute* $P(A|x, I)$



Hypothesis Testing

The usual GW hypotheses

\mathcal{H}_G : data is pure Gaussian noise: $\mathbf{x}(t) = \mathbf{n}(t)$

\mathcal{H}_S : data is *signal* + GN: $\mathbf{x}(t) = \mathbf{n}(t) + \mathbf{h}(t; \mathcal{A}, \lambda)$

Data from several detectors: $\mathbf{x} = \{x^1, x^2, \dots\}$

Gaussian noise: $P(\mathbf{n}|\mathbf{S}_n) = \kappa e^{-\frac{1}{2}(\mathbf{n}|\mathbf{n})}$

Signal **amplitude** parameters $\mathcal{A} = \{h_0, \cos \iota, \psi, \phi_0\}$

CW Signal **phase** parameters $\lambda = \{\text{sky-position}, f, \dot{f}, \dots\}$

👉 Given \mathbf{x} , how can we decide between \mathcal{H}_G and \mathcal{H}_S ?



Bayes factor

Directly *compute* $P(\mathcal{H}_S|\mathbf{x}, I)$, or equivalently compute "odds":

$$O_{\text{SG}}(\mathbf{x}) \equiv \underbrace{\frac{P(\mathcal{H}_S|\mathbf{x}, I)}{P(\mathcal{H}_G|\mathbf{x}, I)}}_{\text{"Posterior odds"}} = \underbrace{\frac{P(\mathbf{x}|\mathcal{H}_S, I)}{P(\mathbf{x}|\mathcal{H}_G, I)}}_{\text{"Bayes factor"}} \times \underbrace{\frac{P(\mathcal{H}_S|I)}{P(\mathcal{H}_G|I)}}_{\text{"prior odds"}}$$

Assume *given* phase parameters λ , *unknown* \mathcal{A}

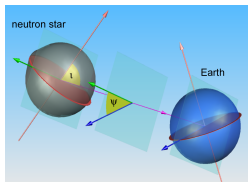
Bayes factor $B_{\text{SG}}(\mathbf{x})$ "updates" our knowledge about \mathcal{H}_S :

$$B_{\text{SG}}(\mathbf{x}) = \int \mathcal{L}(\mathbf{x}; \mathcal{A}) P(\mathcal{A}|\mathcal{H}_S, I) d^4\mathcal{A}$$

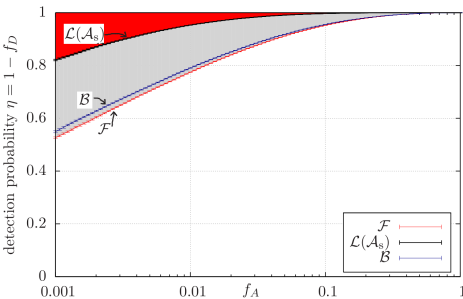
- \mathcal{A} -prior $P(\mathcal{A}|\mathcal{H}_S, I)$
- Likelihood ratio $\mathcal{L}(\mathbf{x}; \mathcal{A}) \propto \exp[-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu} \mathcal{A}^\nu + \mathcal{A}^\mu x_\mu]$



What \mathcal{A} -prior to use? Beating the \mathcal{F} -statistic ...



- simple prior: $P(\mathcal{A}^\mu | \mathcal{H}_S) = \text{const}$
 $\Rightarrow B_{\mathcal{F}}(\mathbf{x}) = \int \mathcal{L}(\mathbf{x}; \mathcal{A}) d^4 \mathcal{A}^\mu \propto e^{\mathcal{F}(\mathbf{x})}$
- *correct* prior $P(\mathcal{A} | \mathcal{H})$: isotropic NS axis
 $\Rightarrow B(\mathbf{x}) \equiv \int \mathcal{L}(\mathbf{x}; \mathcal{A}) dh_0 d\cos \iota d\psi d\phi_0$



- \mathcal{F} -statistic historically derived as $\max_{\mathcal{A}} \mathcal{L}(\mathbf{x}; \mathcal{A}) \propto e^{\mathcal{F}(\mathbf{x})}$ [JKS(1998)]
- $B(\mathbf{x})$ is *more powerful* than $\mathcal{F}(\mathbf{x})$
 R Prix, B Krishnan, CQG 26 (2009)
- $B(\mathbf{x})$ is Neyman-Pearson *optimal*
 A Searle, arXiv:0804.1161 (2008)



Can we make \mathcal{F} more robust vs "line" artifacts?

$$\text{Problem with } O_{\text{SG}}(\mathbf{x}) = \frac{P(\mathcal{H}_S|\mathbf{x})}{P(\mathcal{H}_G|\mathbf{x})} \propto e^{\mathcal{F}(\mathbf{x})}$$

Anything that resembles \mathcal{H}_S **more** than Gaussian noise \mathcal{H}_G can trigger large O_{SG} , regardless of its "goodness-of-fit" to \mathcal{H}_S !
e.g. quasi-monochromatic+stationary detector artifacts ("lines")

➡ add an *alternative* hypothesis \mathcal{H}_L to capture "lines"

"Zeroth order line": single-detector signal trigger

$\mathcal{H}_L =$ "x looks like a signal **in only one detector**"

$$\text{➡ } \mathcal{H}_L \equiv \left[\left(\mathcal{H}_S^1 \text{ and } \mathcal{H}_G^2 \right) \text{ or } \left(\mathcal{H}_G^1 \text{ and } \mathcal{H}_S^2 \right) \right]$$



Extended CW statistics

Using ‘simple’ \mathcal{F} -stat priors: $P(\mathcal{H}_L|\mathbf{x}) \propto I_1 e^{\mathcal{F}_1(x_1)} + I_2 e^{\mathcal{F}_2(x_2)}$

with prior line odds $I_D \equiv \frac{P(\mathcal{H}_L^D|I)}{P(\mathcal{H}_G^D|I)}$ in detector D

Two ways to use \mathcal{H}_L :

① line-veto statistic: $O_{\text{SL}}(\mathbf{x}) \equiv \frac{P(\mathcal{H}_S|\mathbf{x})}{P(\mathcal{H}_L|\mathbf{x})}$

e.g. for loud “candidates” with $\mathcal{F}(\mathbf{x}) > \mathcal{F}^*$

② “line-robust” detection statistic: $O_{\text{SN}}(\mathbf{x}) \equiv \frac{P(\mathcal{H}_S|\mathbf{x})}{P(\mathcal{H}_N|\mathbf{x})}$

with **extended** noise hypothesis: $\mathcal{H}_N \equiv (\mathcal{H}_G \text{ or } \mathcal{H}_L)$

(used in E@H S6Bucket, S6LV1)

[Prix, Keitel, Papa, Leaci, Siddiqi, in preparation]



Line-veto "followup" O_{SL}

$$\Rightarrow O_{SL}(\mathbf{x}) \equiv \frac{P(\mathcal{H}_S|\mathbf{x})}{P(\mathcal{H}_L|\mathbf{x})} \propto \frac{e^{\mathcal{F}(\mathbf{x})}}{l_1 e^{\mathcal{F}_1(x_1)} + l_2 e^{\mathcal{F}_2(x_2)}}$$

Special case $l_1 = l_2$: $(\mathcal{F}_{\max} \equiv \max\{\mathcal{F}_1, \mathcal{F}_2\})$

$$\ln O_{SL}(\mathbf{x}) = c_0 + [\mathcal{F}(\mathbf{x}) - \mathcal{F}_{\max}(x)] - \underbrace{\ln \left(1 + e^{(\mathcal{F}_{\min} - \mathcal{F}_{\max})} \right)}_{\in [0, \ln 2]}$$

Recover *ad-hoc* veto criterion as special case


$$\ln O_{SL}(\mathbf{x}) - c_0 \approx \mathcal{F}(\mathbf{x}) - \mathcal{F}_{\max}(x)$$

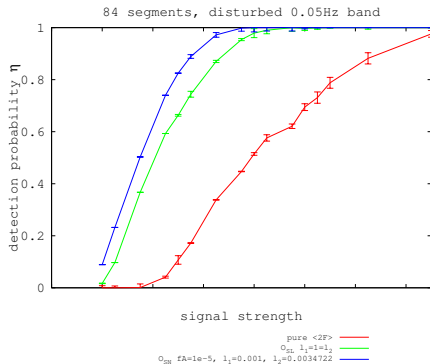
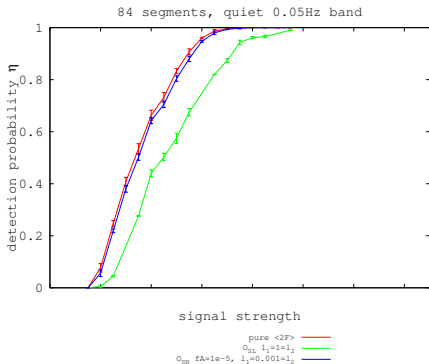
\Rightarrow veto if $\mathcal{F}_{\max}(x) > \mathcal{F}(\mathbf{x}) \iff$ special choice of threshold!



"Line-robust" detection statistic $O_{SN}(\mathbf{x})$

$$O_{SN}(\mathbf{x}) \equiv \frac{P(\mathcal{H}_S|\mathbf{x})}{P(\mathcal{H}_L|\mathbf{x}) + P(\mathcal{H}_G|\mathbf{x})} \propto \frac{e^{\mathcal{F}(\mathbf{x})}}{e^{\mathcal{F}^*} + l_1 e^{\mathcal{F}_1(x_1)} + l_2 e^{\mathcal{F}_2(x_2)}}$$

- \mathcal{F}^* is a prior constant (requires "tuning")
-  estimate prior line-odds l_D from detector data!



Neutron Star “Mountains”: What do we know?

- *Maximal possible* deformations:
 - Conventional NS crustal shear:
☞ $\epsilon \lesssim 10^{-7} - 10^{-6}$ [Ushomirsky, Cutler, Bildsten]
 - Exotic EOS: strange-quark solid cores
☞ $\epsilon \lesssim 10^{-5} - 10^{-4}$ [B. Owen]
- Models predicting *actual* deformations:
 - large toroidal field $B_t \sim 10^{15}$ Gauss \perp to rotation:
☞ $\epsilon \sim 10^{-6}$ [C. Cutler]
 - accretion along B -lines \implies “bottled” mountains
☞ $\epsilon \sim 10^{-6} - 10^{-5}$ [Melatos, Payne]
- *Minimal* deformation from magnetic field:
☞ $\epsilon \gtrsim 10^{-12} \left(\frac{B}{10^{12} \text{Gauss}} \right)^2$ [B. Haskell et al.(2008)]

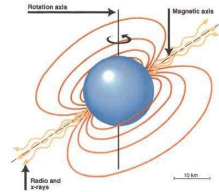
\implies Prior range: $\epsilon \in [10^{-12}, 10^{-4}]$



Spindown upper-limit for CWs from known pulsars

Rotational energy lost: $\dot{E}_{\text{rot}} \propto I_{zz} \underbrace{\nu \dot{\nu}}_{\text{observed}}$

Energy emitted in GWs: $\dot{E}_{\text{GW}} \propto \nu^6 I_{zz}^2 \epsilon^2$



Spindown upper limit: Spindown fully due to GW emission

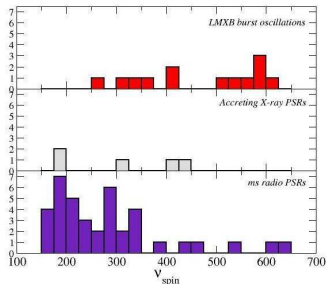
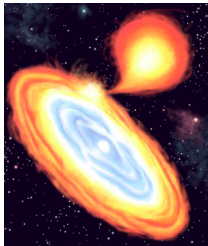
Assumed I_{zz} (from EOS) and known distance d :

\Rightarrow Upper limit on deformation ϵ : $\epsilon_{\text{sd}} \propto \sqrt{\frac{1}{I_{zz}} \frac{|\dot{\nu}|}{\nu^5}}$

\Rightarrow Upper limit on amplitude h_0 : $h_{\text{sd}} \propto \frac{1}{d} \sqrt{I_{zz} \frac{|\dot{\nu}|}{\nu}}$



Accretion



Breakup-limit $\nu_K \sim 1.5$ kHz \Rightarrow What limits the NS-spin?

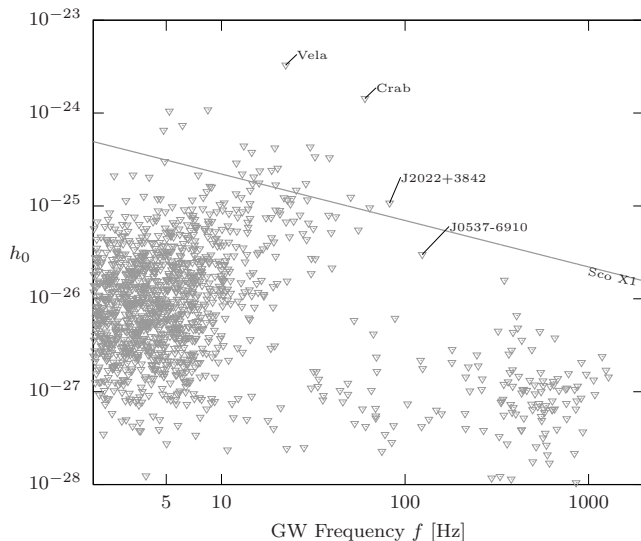
Bildsten, Wagoner: Accretion-torque = GW torque ($\propto \nu^5$)

$$h_0 \approx 5 \times 10^{-27} \left(\frac{300 \text{ Hz}}{\nu} \right)^{1/2} \left(\frac{F_x}{10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}} \right)^{1/2}$$

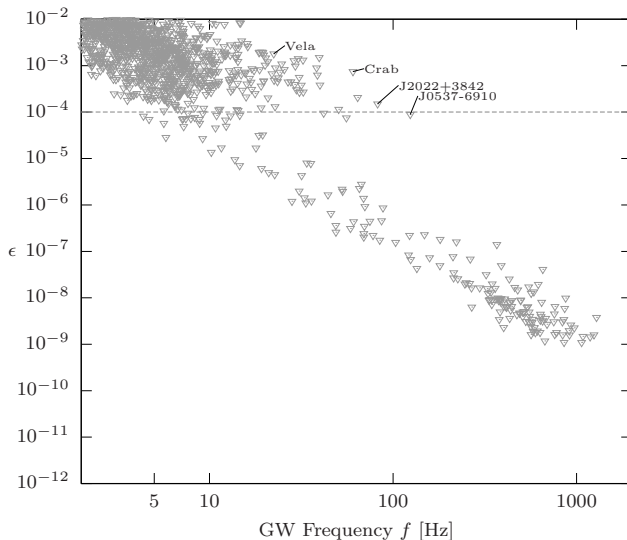
\Rightarrow Sco X-1: $h_0(f = 2\nu) \sim 3 \times 10^{-26} (540 \text{ Hz}/f)^{1/2}$



Spindown Upper Limits: h_0



Spindown and Indirect Upper Limits: ϵ



Unknown gravitar population?

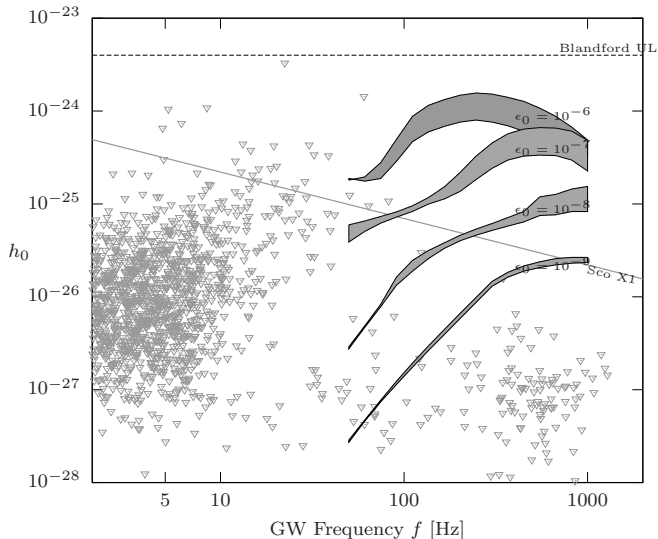
“Gravitars” \equiv {Population of unknown NSs, born spinning rapidly, spinning down purely due to GWs}

Blandford: If steady-state 2D uniform gravitar distribution in galactic disk, expected strongest signal is independent of $\{\epsilon_0, f\}$
 $h_0 \sim 4 \times 10^{-24}$ (for birth-rate $\tau_B \sim 1/30\text{y}$)

More detailed analysis by **Knispel, Allen, PRD D78 (2008)**: distribution not 2D uniform, not steady-state: h_0 depends on f and (fixed) population ϵ_0



Unknown gravitar population?



Types of CW searches

- **Targeting pulsars:** sky-position and frequency $f(t)$ known
 - ☞ 1 template, computationally cheap $\sim \mathcal{O}(\text{laptop})$
 - ☞ use optimal method (Bayes factor, “matched filtering”)
- **Directed:** sky-position known, frequency $f(t)$ unknown
- **Wide-parameter:** unknown sky-position and frequency $f(t)$
 $\text{SNR} \propto \frac{h_0}{\sqrt{S_n}} \sqrt{T}$ BUT computing cost $\mathcal{C} \propto T^p, p \gtrsim 5$
 - ☞ optimal method computationally impossible
 - **Semi-coherent** methods: break data into N_{seg} shorter segments of length T_{seg} , combine incoherently
 $\text{SNR} \propto \frac{h_0}{\sqrt{S_n}} N_{\text{seg}}^{1/4} \sqrt{T_{\text{seg}}}$, BUT **cheaper!**
NOTE: Optimal method *at fixed computing-cost* **unknown**
 - maximize available computing power by using **Einstein@Home**, clusters + GPUs



Sensitivity estimate

“Sensitivity” \equiv {weakest detectable signal amplitude h_0 }

Depends on (i) detector noise $S_n(f)$, (ii) search parameters θ :

- false-alarm p_{FA} (small) and detection $p_{\text{det}}(\sim 90\%)$
- total amount of data used T_{data}
- “size” of the parameter-space \mathbb{P}
- Computing-cost: $C_0 = \text{Computing-power} \times \text{runtime}$
- internal pipeline parameters: $N_{\text{seg}}, T_{\text{seg}}, \mu, \dots$

Define “characteristic sensitivity” $\sigma(\theta)$ of the **method** as

$$h_0(f) = \frac{\sqrt{S_n(f)}}{\sigma(\theta)}$$

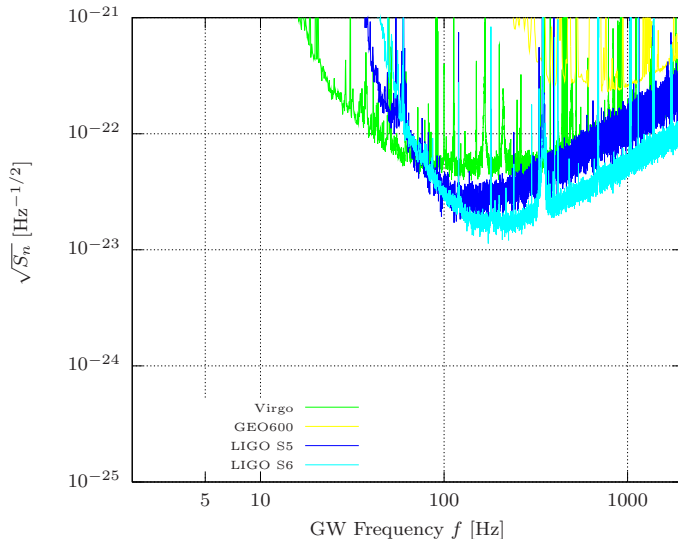


Examples of current Search Sensitivities

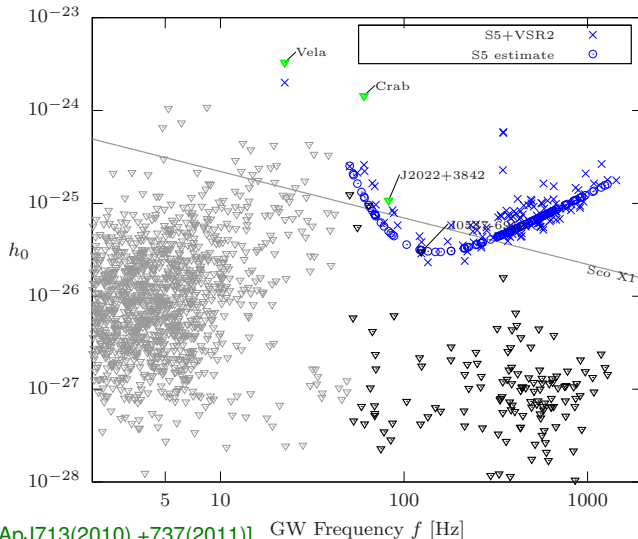
- Targeted searches (fully coherent): $h_0 = \frac{11.4}{\sqrt{T_{\text{data}}}} \sqrt{S_n}$
2 years of data from 2 detectors: $T_{\text{data}} = 2 \times 2\text{y} \approx 10^8\text{s}$
☞ $\sigma \sim 1000 \text{ Hz}^{-1/2}$
- Directed semi-coherent (e.g. Galactic-center, Cas-A,...)
☞ $\sigma \sim 70 \text{ Hz}^{-1/2}$ ($N_{\text{seg}} = 630, T_{\text{seg}} = 2 \times 11.5\text{h}, \mu \sim 0.17$)
- All-sky searches for *isolated* NSs ($C_0(E@H) \sim 10^{21}$ flop)
☞ $\sigma \sim 30 \text{ Hz}^{-1/2}$ ($N_{\text{seg}} = 121, T_{\text{seg}} = 2 \times 25\text{h}, \mu \sim 0.6$)
[K. Wette, PRD85 (2012), Prix&Wette LIGO-T1200272]
- TwoSpect: First all-sky *binary* search
☞ $\sigma \lesssim 10 \text{ Hz}^{-1/2}$ [E. Goetz, GWPAW12 talk]
(huge parameter space, search ongoing)



Current Sensitivities: noise PSD $S_n(f)$



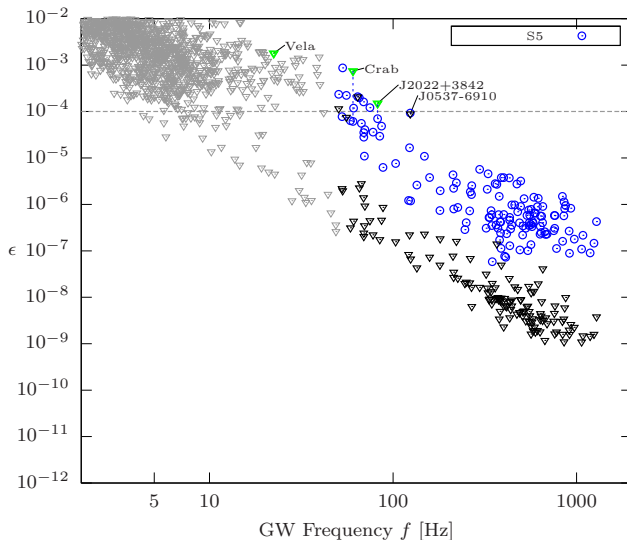
Current Sensitivity: Targeted searches ($\sigma \approx \frac{1000}{\sqrt{\text{Hz}}}$)



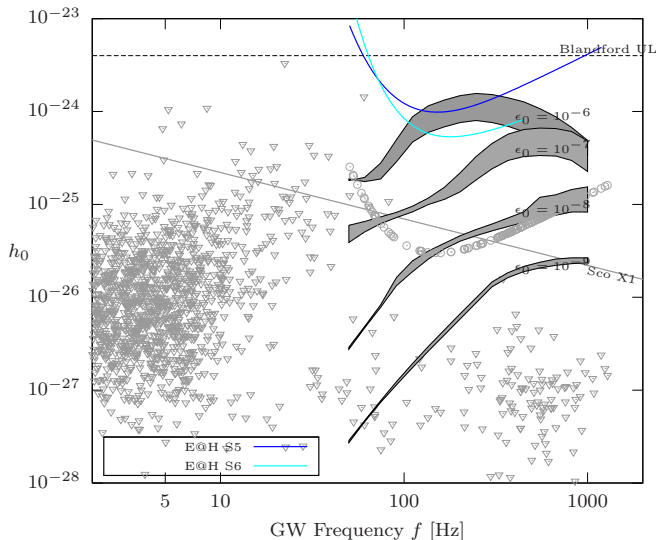
[LSC+Virgo,ApJ713(2010),+737(2011)] GW Frequency f [Hz]



Current Sensitivities: Targeted searches ($\sigma \approx \frac{1000}{\sqrt{Hz}}$)



Current Sensitivities: Einstein@Home ($\sigma \approx \frac{30}{\sqrt{\text{Hz}}}$)



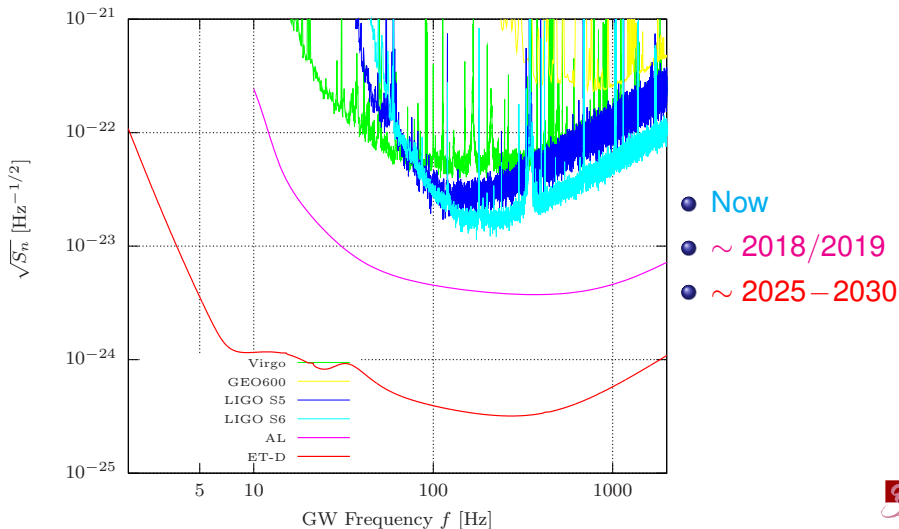
What future sensitivity improvements can we expect?

Generally, sensitivity gains can come from 3 factors:

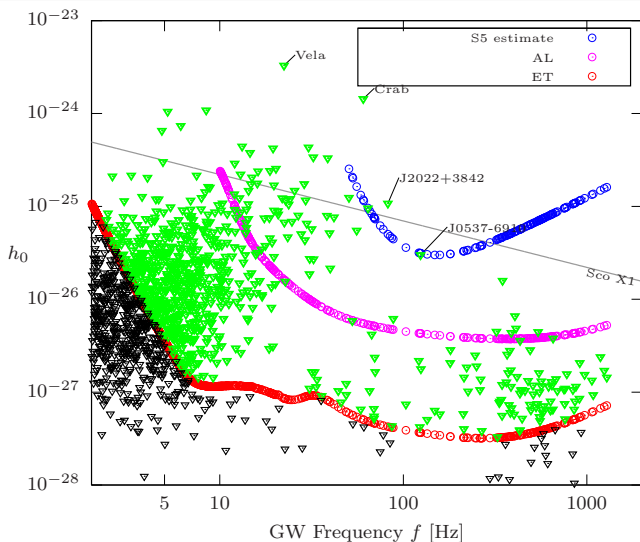
- 1 better (more sensitive) detectors $\sqrt{S_n}$
- 2 more computing power (Moore's law)
- 3 better *search methods*



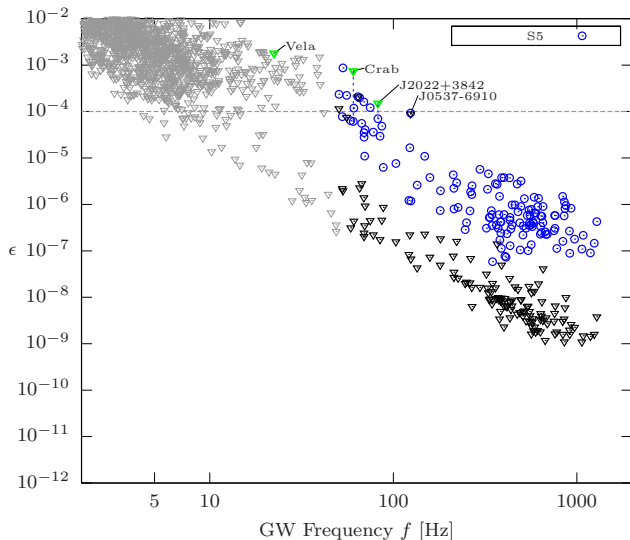
1. How much can we gain from future detectors?



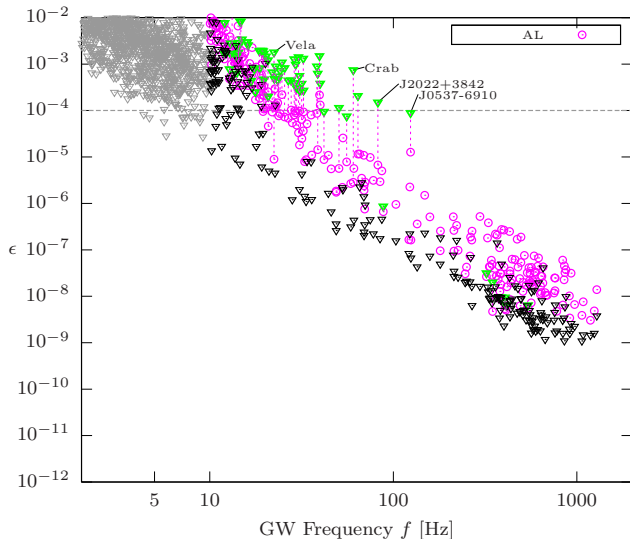
Future sensitivity: Targeted Searches h_0 ($\sigma \approx \frac{1000}{\sqrt{\text{Hz}}}$)



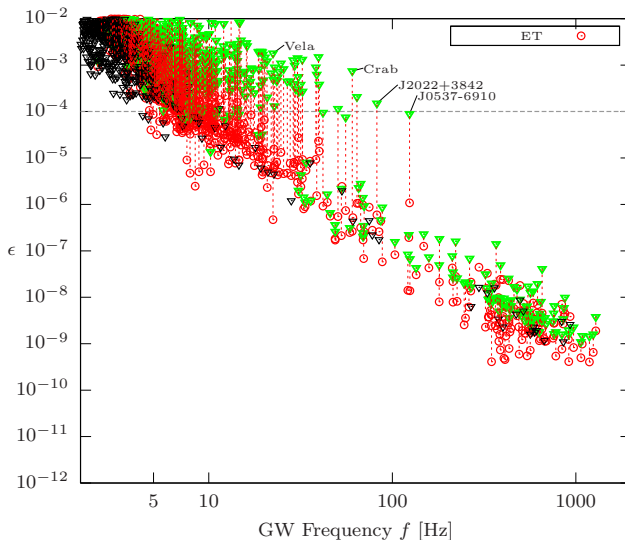
Future sensitivity: Targeted Searches ϵ ($\sigma \approx \frac{1000}{\sqrt{\text{Hz}}}$)



Future sensitivity: Targeted Searches ϵ ($\sigma \approx \frac{1000}{\sqrt{\text{Hz}}}$)



Future sensitivity: Targeted Searches ϵ ($\sigma \approx \frac{1000}{\sqrt{\text{Hz}}}$)



2. How much can we gain from Moore's law?

“Computing power doubles every ~ 2 years”

- 2nd generation: Advanced LIGO+Virgo, KAGRA, ...
 $\sim 2018/2019$ $\Rightarrow \sim 3$ doublings $\Rightarrow C_0[\text{AL}] \sim 8 \times C_0$
- 3rd generation, e.g. “Einstein Telescope” (ET):
 $\sim 2025 - 2030$ $\Rightarrow \sim 8$ doublings $\Rightarrow C_0[\text{ET}] \sim 256 \times C_0$

How does h_0 sensitivity scale with C_0 ?

- Targeted searches: no gain
- Wide parameter-space searches: [Prix, Shaltev, PRD85 (2012)]

$$h_0 \sim [C_0^{-1/16}, C_0^{-1/8}] \overset{\text{here}}{\approx} C_0^{-1/10}$$

Sensitivity increase due to Moore's law (e.g. for E@H)

$\sigma[\text{AL}] \sim +25\%$ in Advanced-detector (AL) era

$\sigma[\text{ET}] \sim +75\%$ in Einstein Telescope (ET) era



3. How much can we gain from *improved methods*?

Wide parameter-space searches are computationally limited,
optimal search method unknown.

How much improvement do we expect?

- *tuning* of semi-coherent method (StackSlide) can yield **+25%** wrt recent E@H searches [Prix,Shaltev,PRD85 (2012)]
- Coherent *follow-up* can yield up to **+80%** improvement, unclear if computing cost affordable [✉ Shaltev, PhD thesis]

Combined: Future all-sky sensitivities (e.g. E@H)

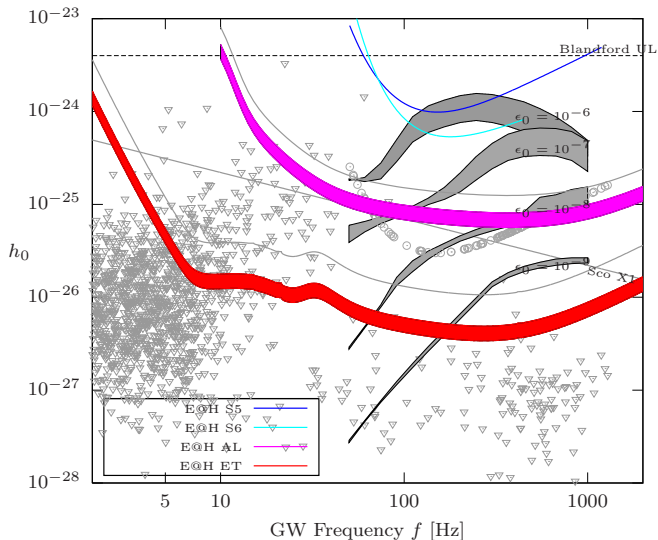
$$\sigma[S5, S6] \sim 30 \text{ Hz}^{-1/2}$$

$$\sigma[AL] \sim [47, 67] \text{ Hz}^{-1/2}$$

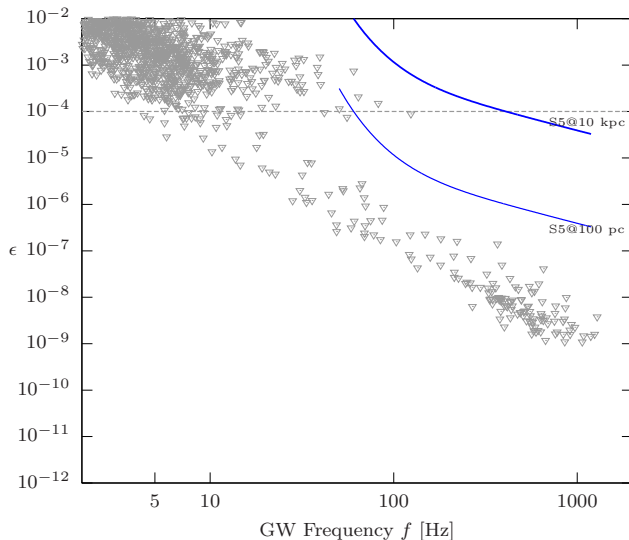
$$\sigma[ET] \sim [65, 94] \text{ Hz}^{-1/2}$$



Future sensitivity of All-Sky Searches h_0



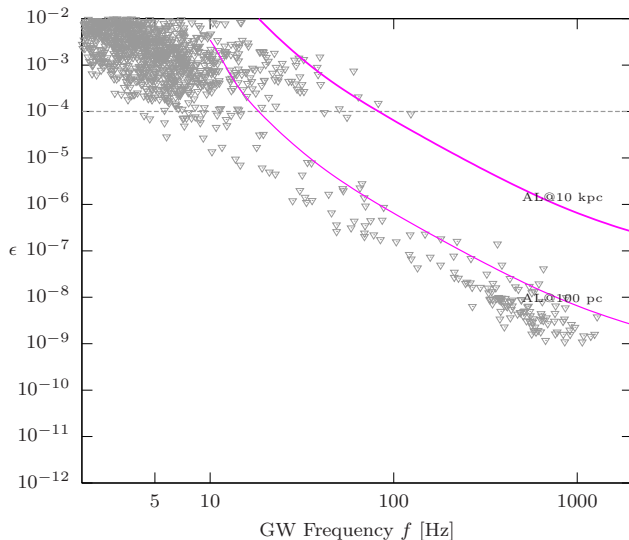
Future sensitivity: Wide parameter-space ϵ



$$\sigma[S5] = 30\text{Hz}^{-1/2}$$



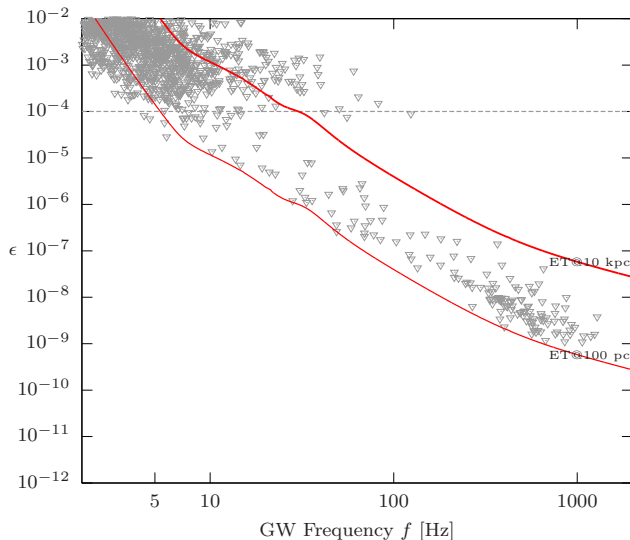
Future sensitivity: Wide parameter-space ϵ



$$\sigma[\text{AL}] = 67\text{Hz}^{-1/2}$$



Future sensitivity: Wide parameter-space ϵ



$$\sigma[\text{ET}] = 94\text{Hz}^{-1/2}$$

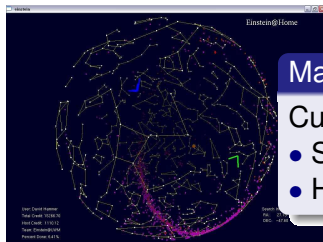


Conclusions

- No guaranteed future CW detections, but . . .
- . . . entering increasingly interesting territory!
- Future observations will definitely be **informative** (one way or the other), cutting substantially into the prior ranges
- Astrophysical conclusions will depend on exact nature of (non-)detection and assumed astrophysical models
- Lots of work remaining to improve our wide-parameter search *methods* (eg “Line-Veto”, Hierarchical, ...)
- Expand our searches to new *categories*: e.g. “transient CWs” (lifetime \sim days) from NS glitches?



You can help by running Einstein@Home!



Maximize available computing power

Cut parameter-space λ in small pieces $\Delta\lambda$

- Send workunits $\Delta\lambda$ to participating hosts
- Hosts return finished work and request next

- Public distributed computing project, launched Feb. 2005
- Currently $\sim 100,000$ participants, ~ 1 PFlop/s (24x7)
- All-sky search for GWs from unknown neutron stars
- Analyzed LIGO data from S3, S4, S5, S6
- March 2009: also search for binary radio pulsars in Arecibo+Parkes data 🖱️ First E@H discovery [Science 2010]
- Aug 2011: also search for γ -ray pulsars in Fermi-LAT data

