Twist and Shout

Generic black-hole-binary waveform models

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work with
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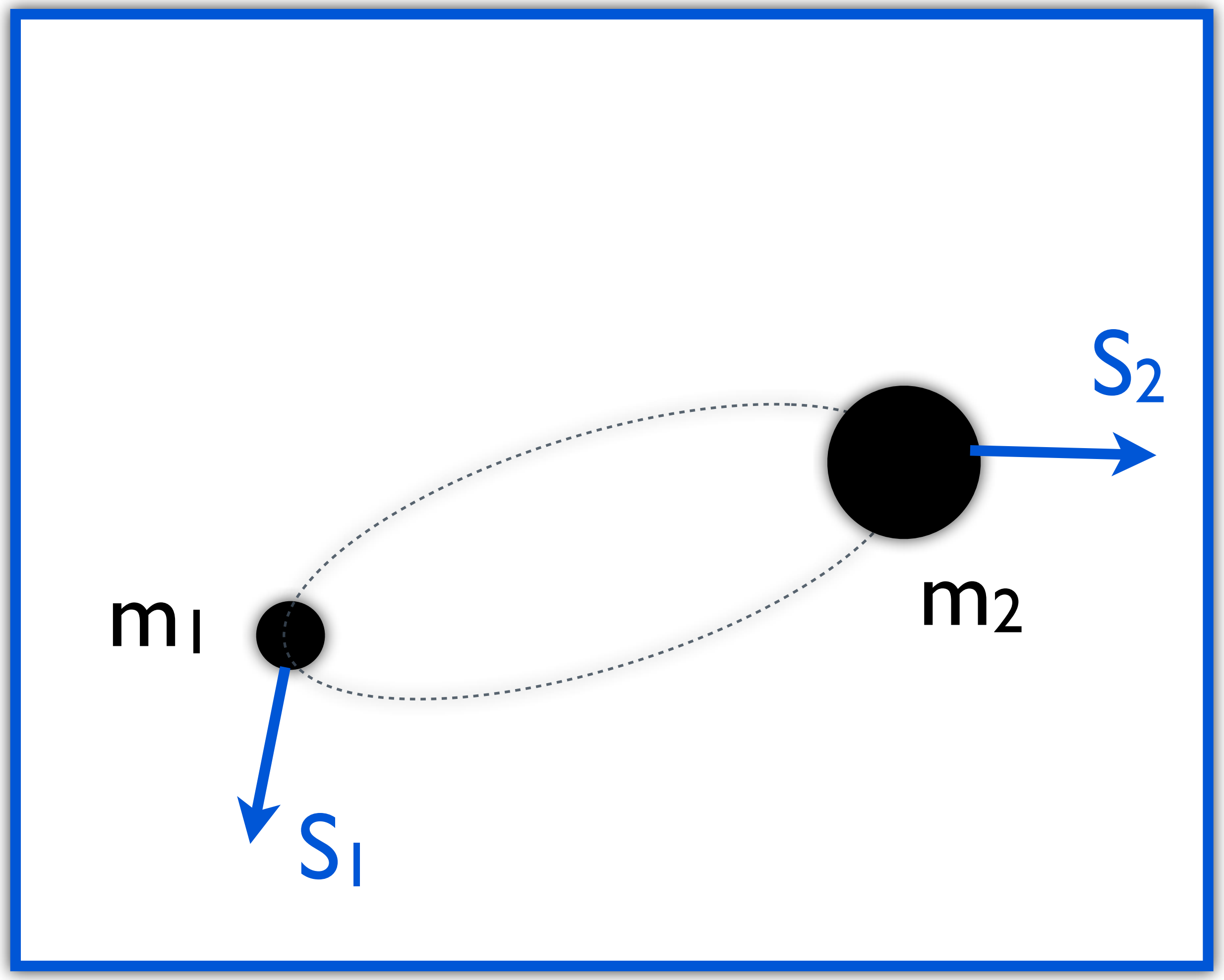
Motivation to produce waveform models:

The future of gravitational wave astronomy depends on them!

Consider the dynamics
and
gravitational waveforms
of black-hole binaries

Masses: m_1, m_2

Spins: $\mathbf{S}_1, \mathbf{S}_2$



useful combinations:

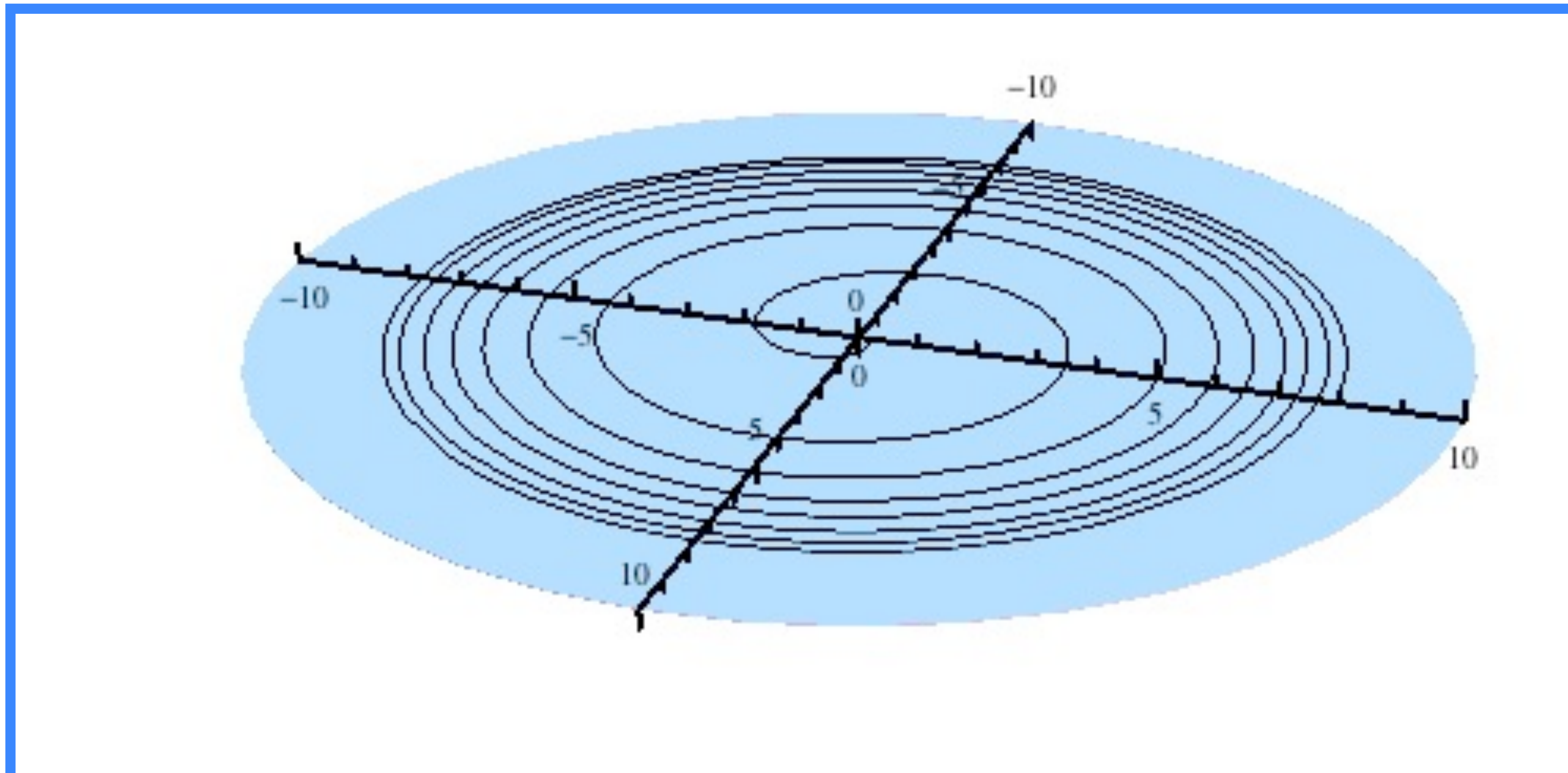
$$M = m_1 + m_2$$

$$q = m_2 / m_1$$

$$\eta = m_1 m_2 / M^2$$

$$\chi = S/m^2$$

Nonspinning black holes

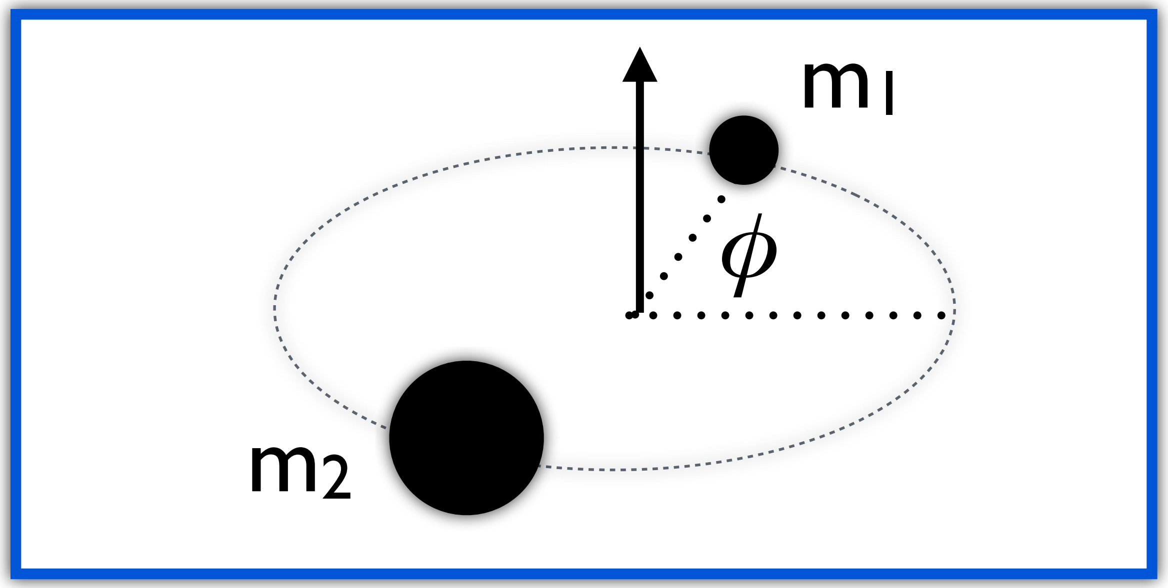


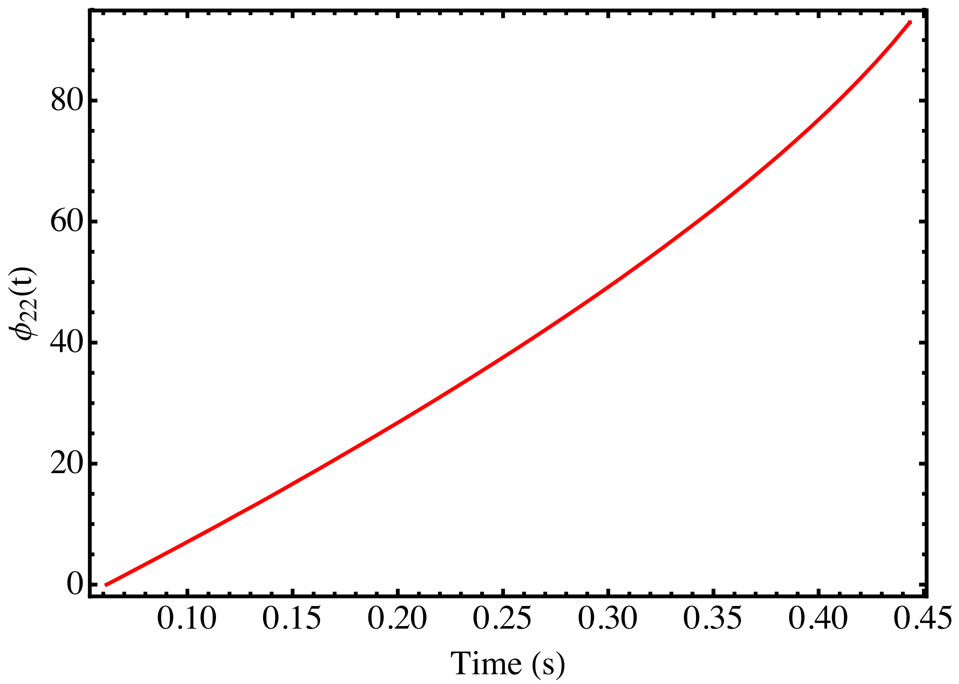
(Mass ratio 1:4, $\eta = 0.16$)

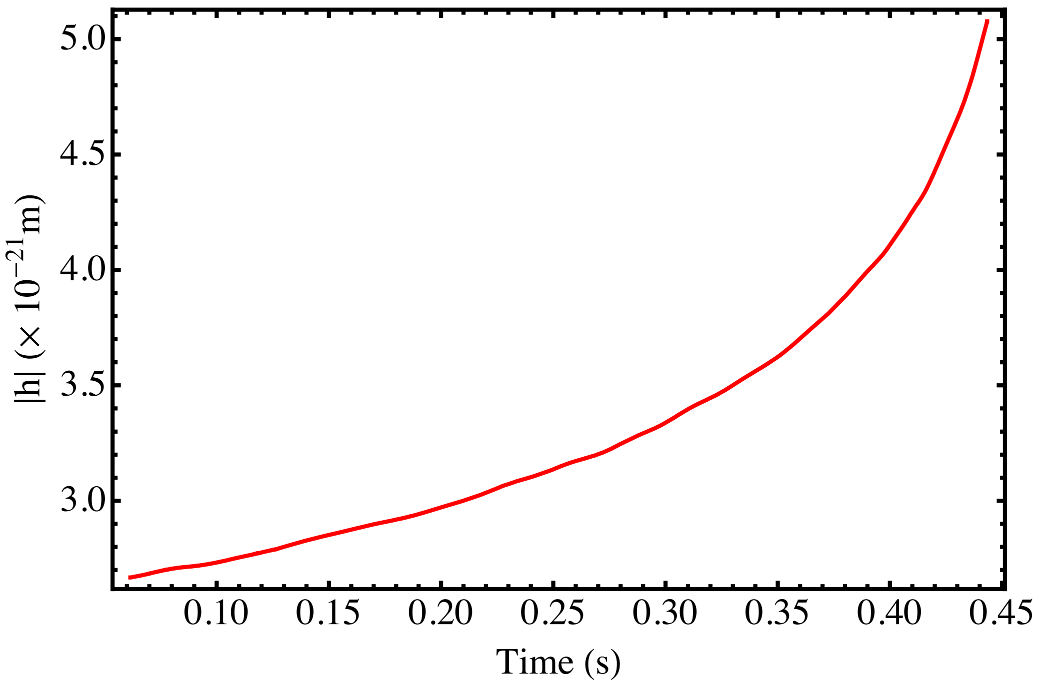
Gravitational wave signal: quadrupole approximation

$$\omega = \dot{\phi}$$

$$h_{22} = A(t)e^{-2i\phi(t)}$$







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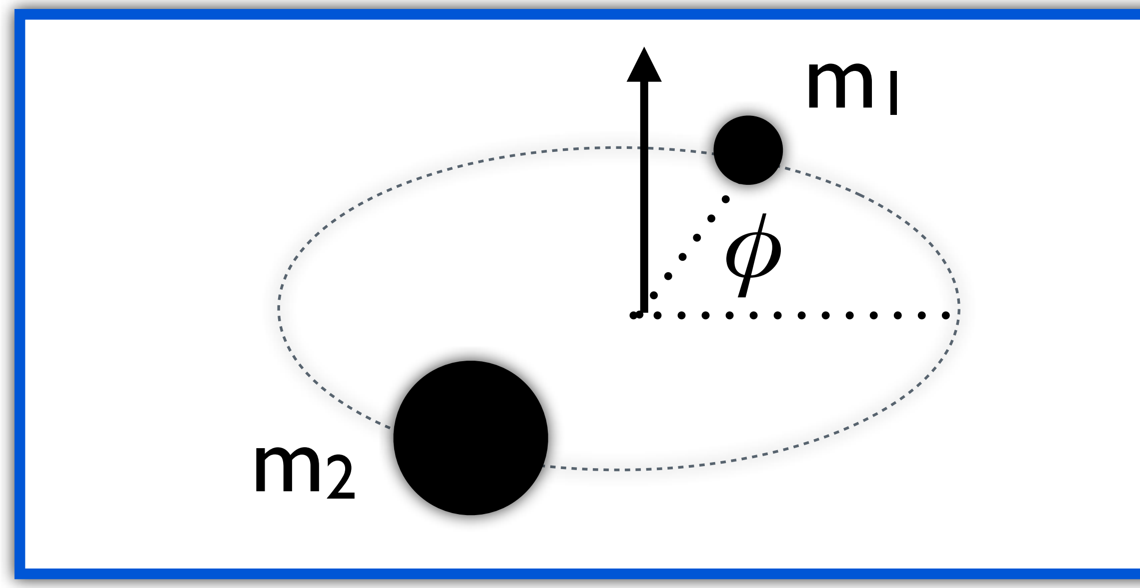


$$h(t;\theta,\phi) = h_{22}(t)\,{}^{-2}Y_{2,2}(\theta,\phi) + h^*_{22}(t)\,{}^{-2}Y_{2,-2}(\theta,\phi)$$

h_+

Optimally oriented (face on)

Edge on

Signal shape is independent of orientation

Leading-order PN GW phasing:

$$\Psi_0(f) = \frac{3}{128\eta(\pi M f)^{5/3}}$$

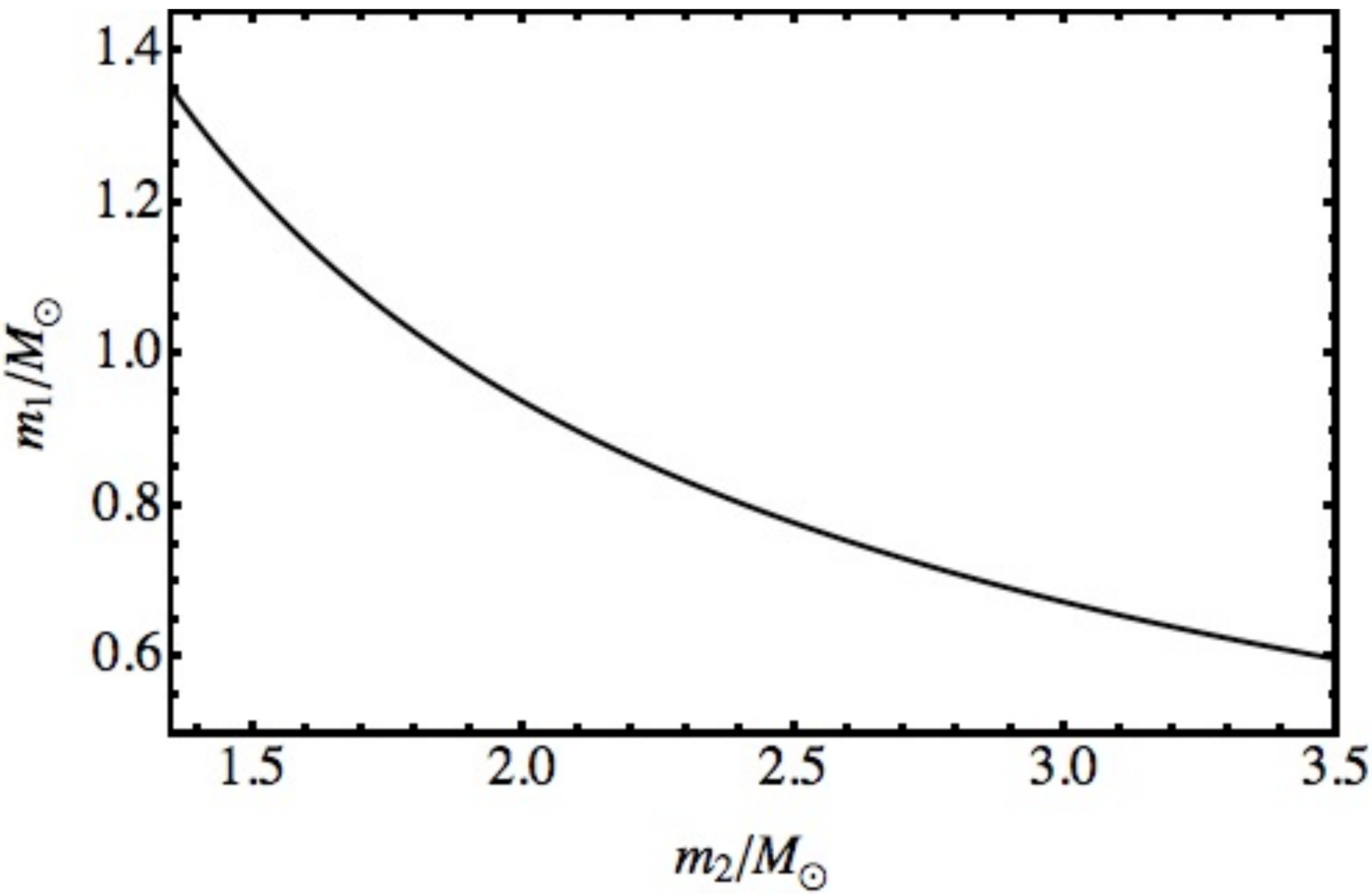
symmetric mass ratio

Total mass

Depends on combination of total mass and mass ratio:

$$\mathcal{M} = M\eta^{3/5}$$

Mass measurements



Next PN order:

$$\Psi(f) = \Psi_0 \left(1 + v^2 \left[\frac{3715}{756} + \frac{55\eta}{9}\right]\right)$$

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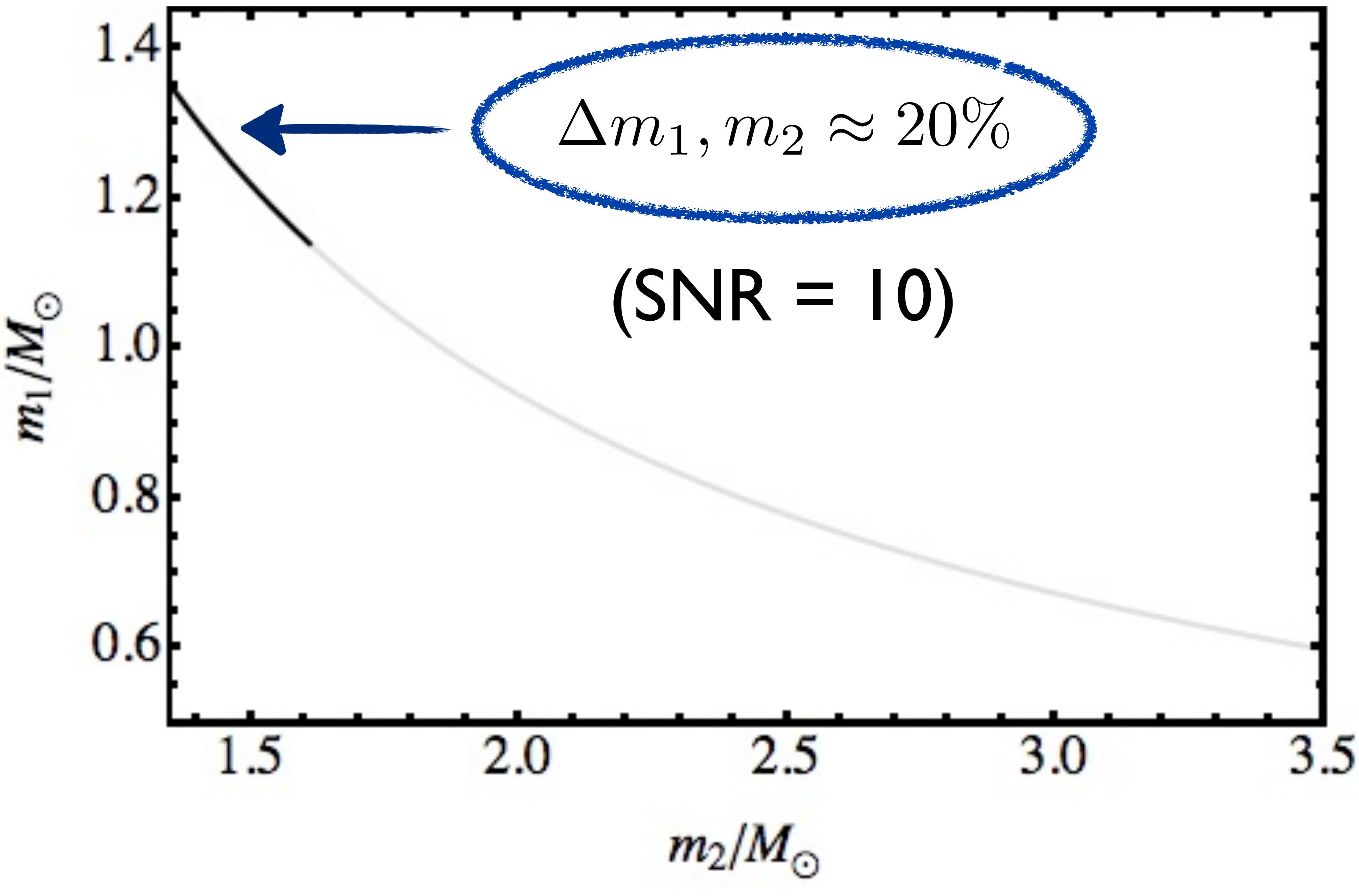
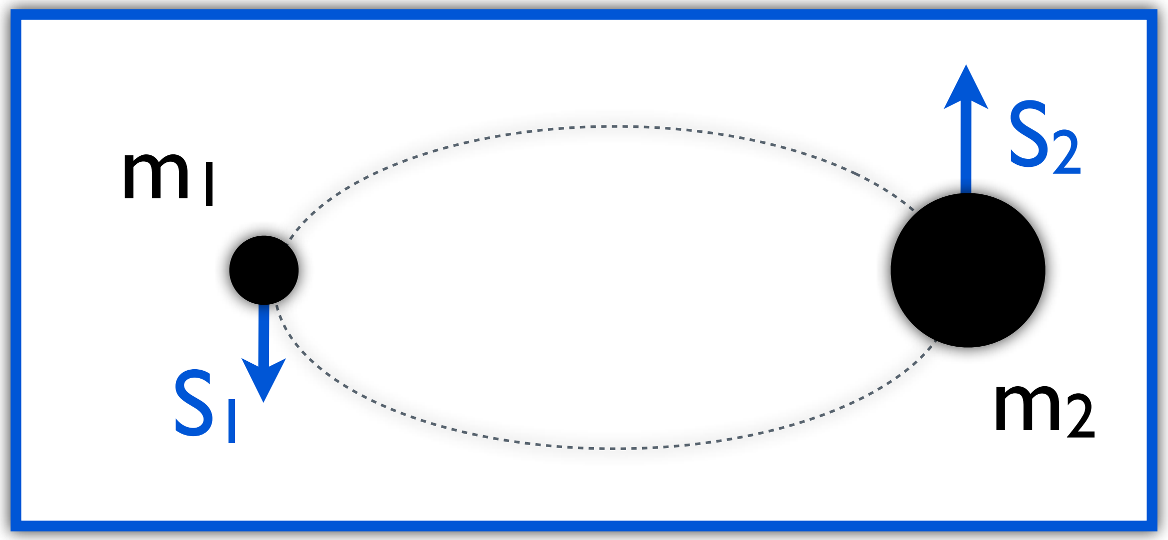
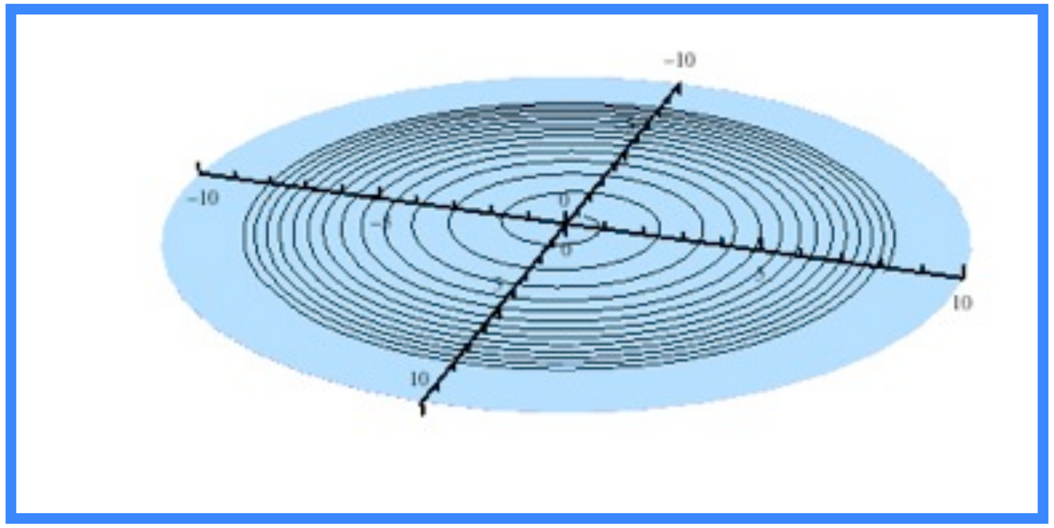


Aligned spins





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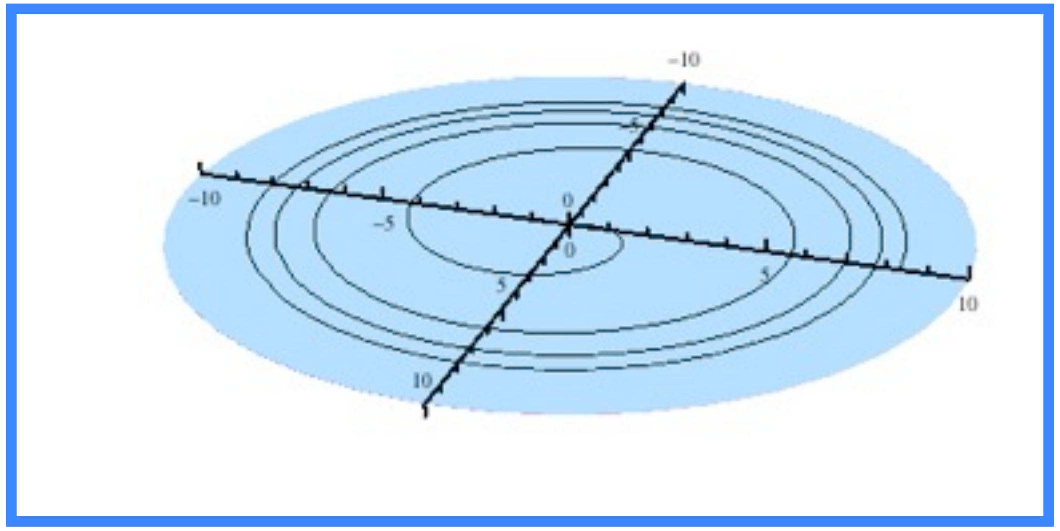

↓↓

Yet another order introduces spin, χ: Uh-oh!

$$\Psi(f) = \Psi_0 \left(1 + v^2 \left[\frac{3715}{756} + \frac{55\eta}{9}\right] + v^3 \left[\frac{113}{3}\chi - 16\pi\right]\right)$$

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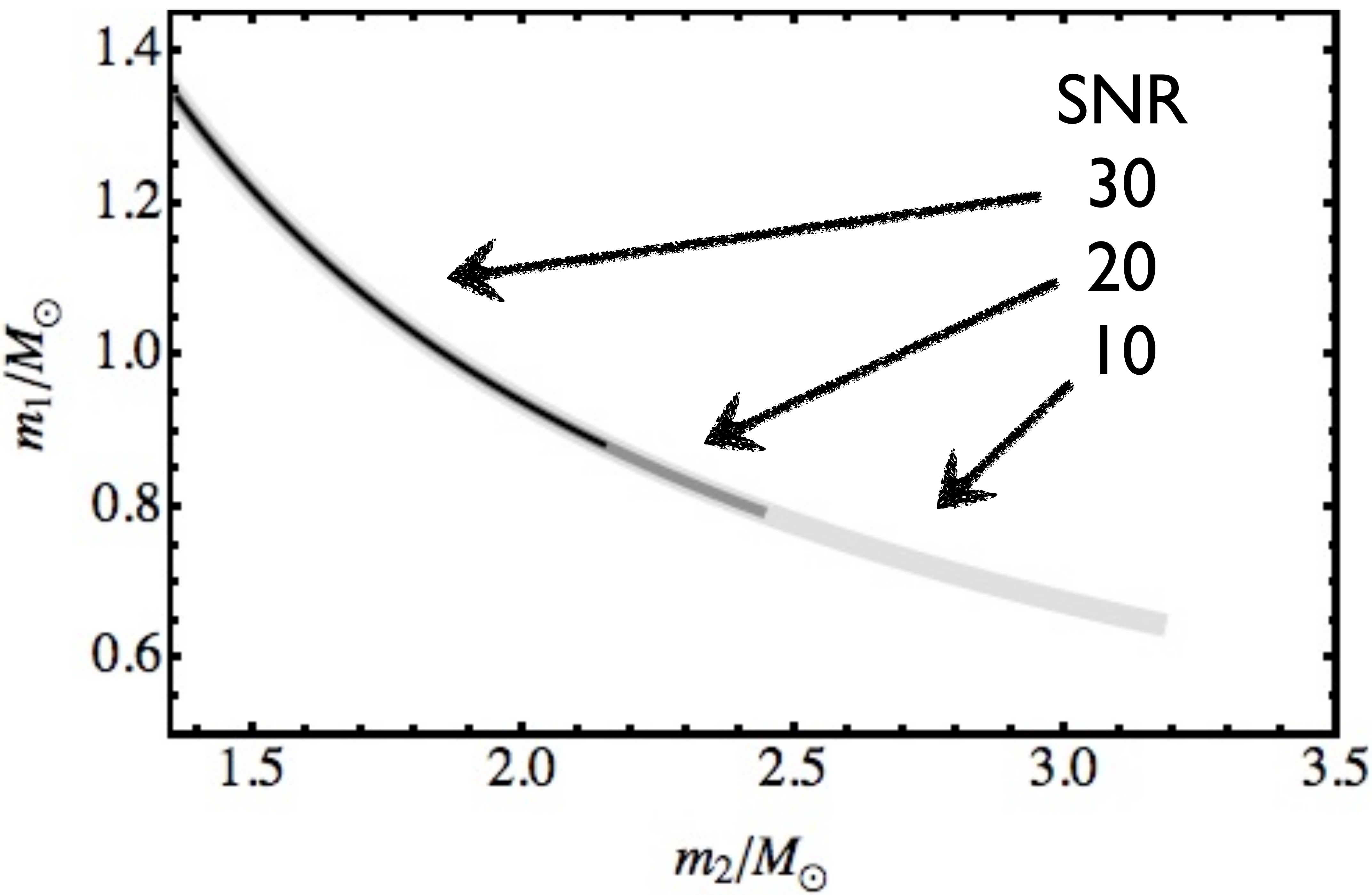
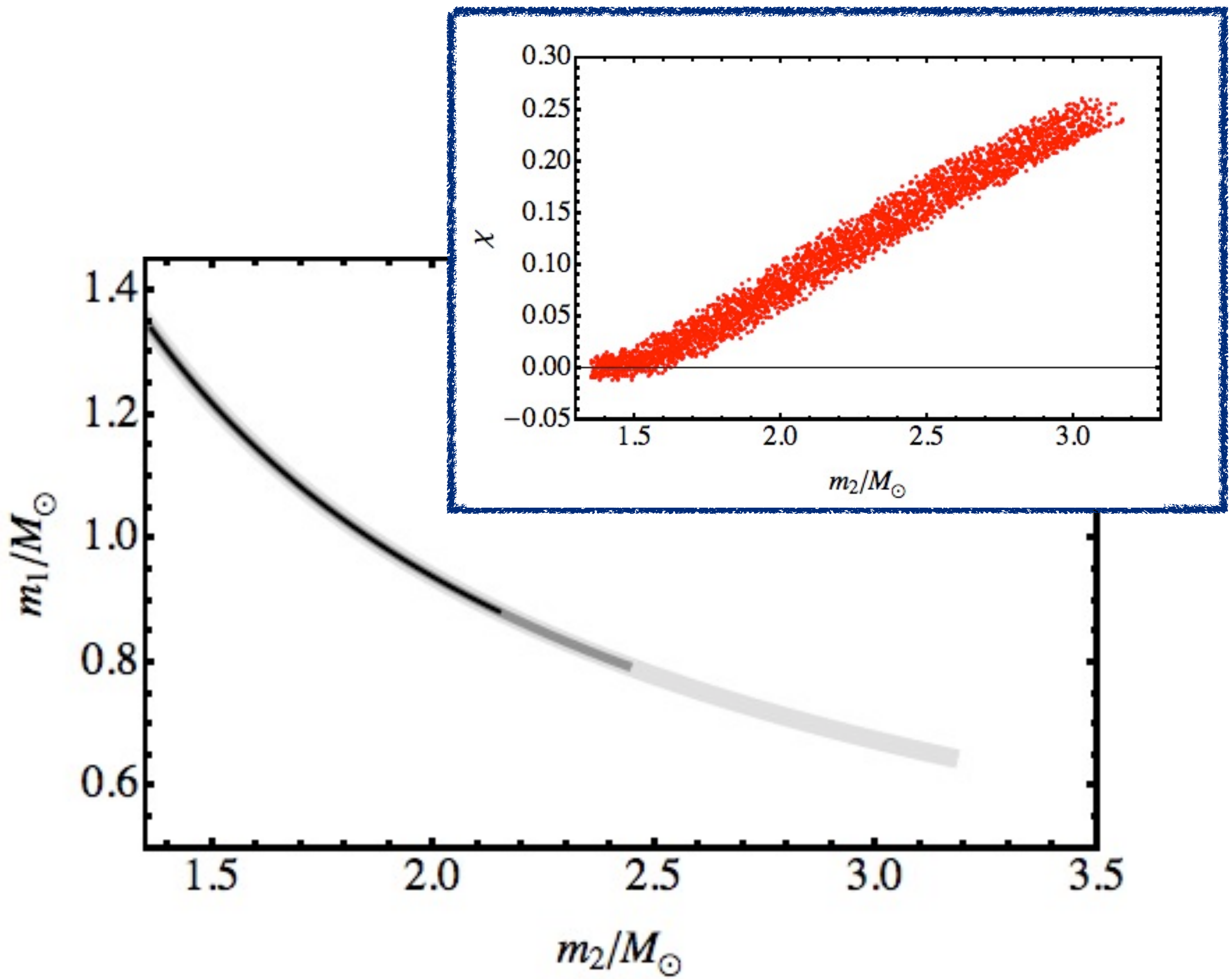
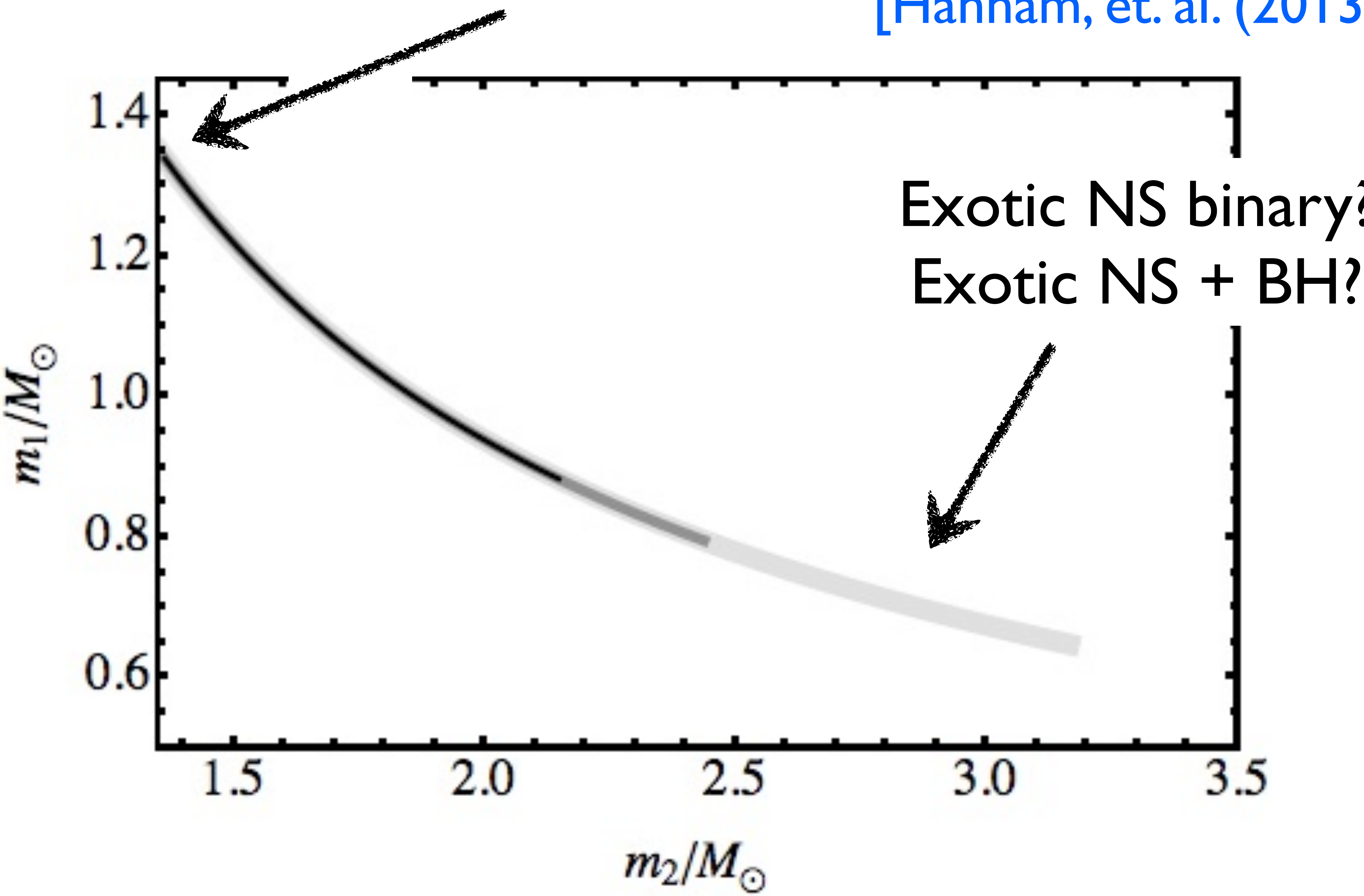




1.4
1.2
1.0
0.8
0.6
$m_1/M_\odot$
1.5
2.0
2.5
3.0
3.5
$m_2/M_\odot$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
χ
1.5
2.0
2.5
3.0
$m_2/M_\odot$

Standard 1.35-1.35 $M_\odot$ NS binary

[Hannam, et. al. (2013)]



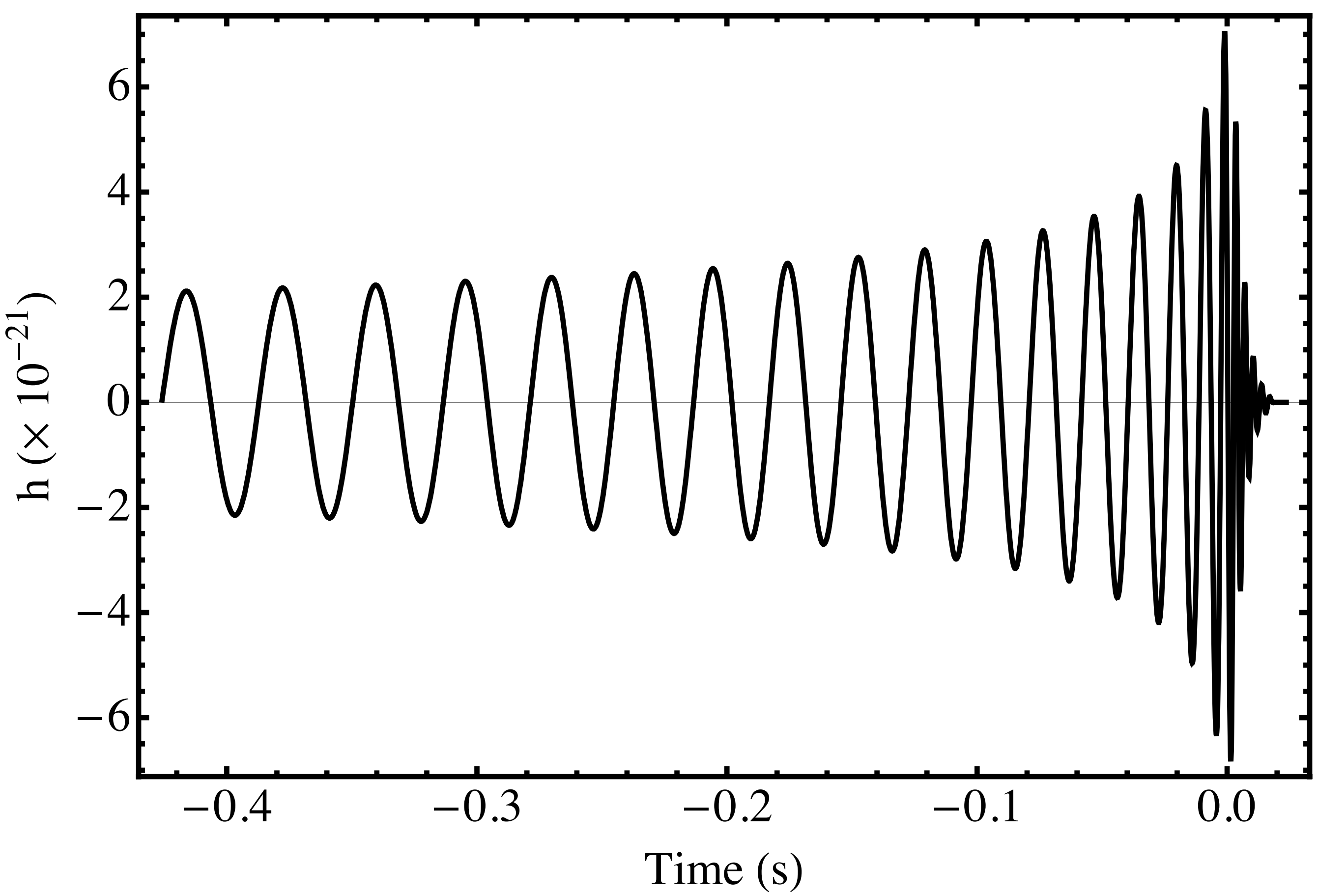
Effective total spin

The PN phasing is dominated by a weighted sum of the two spins:

$$\chi_{\rm eff} = \frac{m_1\chi_1 + m_2\chi_2}{M} - \frac{38\eta(\chi_1+\chi_2)}{113}$$

[Poisson and Will (1995), Ajith (2011)]

Merger and ringdown



Optimally oriented
50 $M_\odot$
equal-mass
nonspinning binary

Ringdown waveform determined by final mass and spin

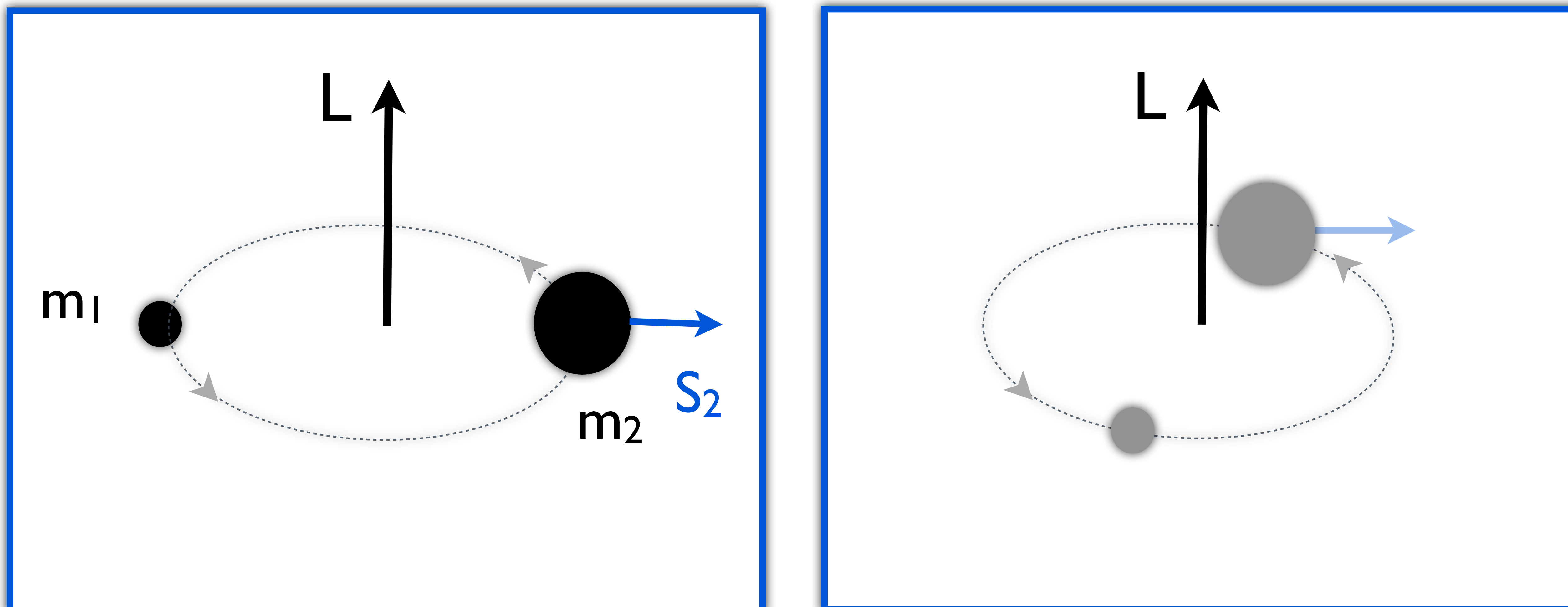
Inspiral-merger-ringdown model

$$h_{22}(f) = A(f)e^{i\Psi(f)}$$

- Inspiral: TaylorF2.
- Merger-ringdown:
  - power series in f, fit to NR data
  - final spin from formulas in literature

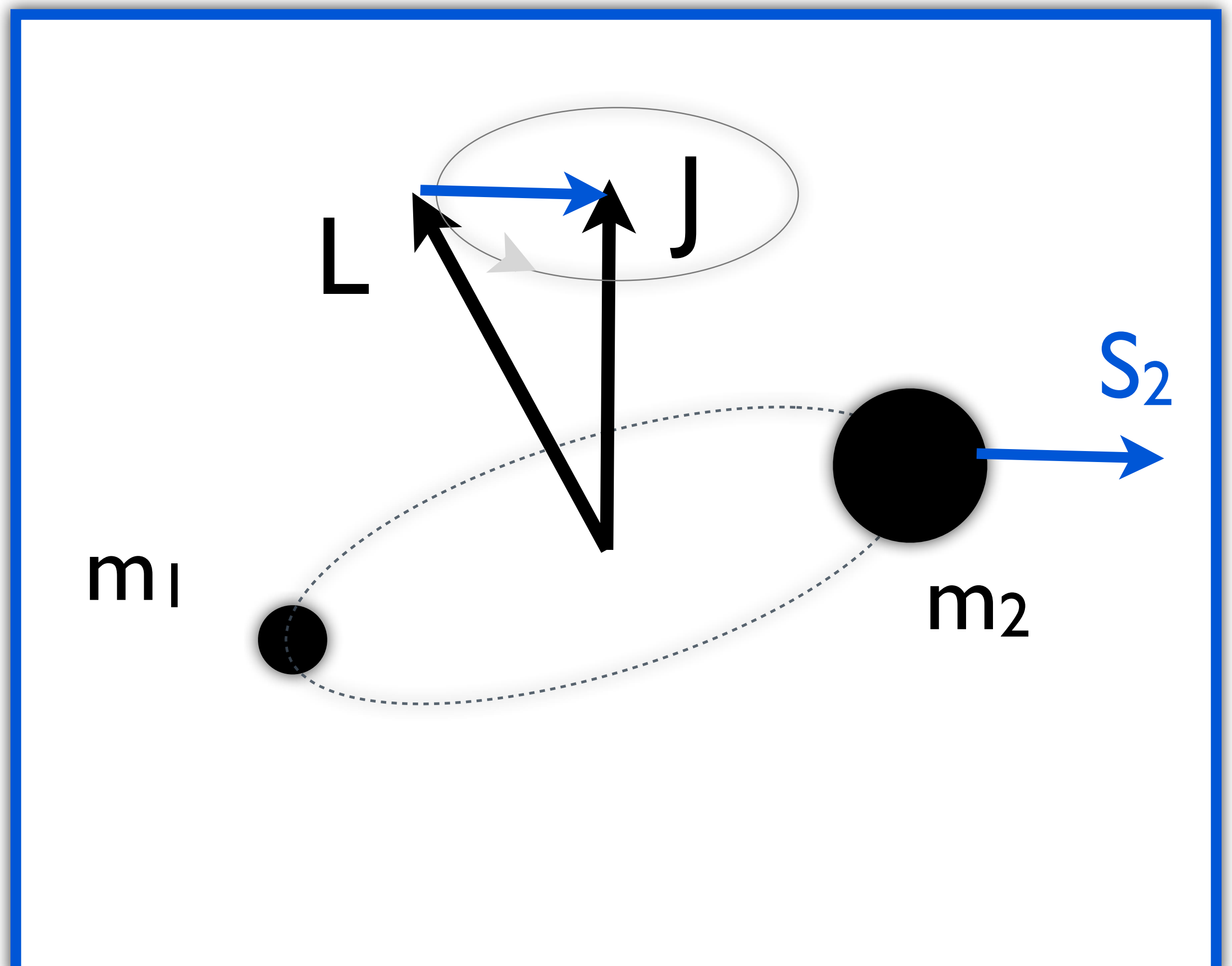
[Ajith, et. al., (2009), Santamaria, et. al., (2010)]

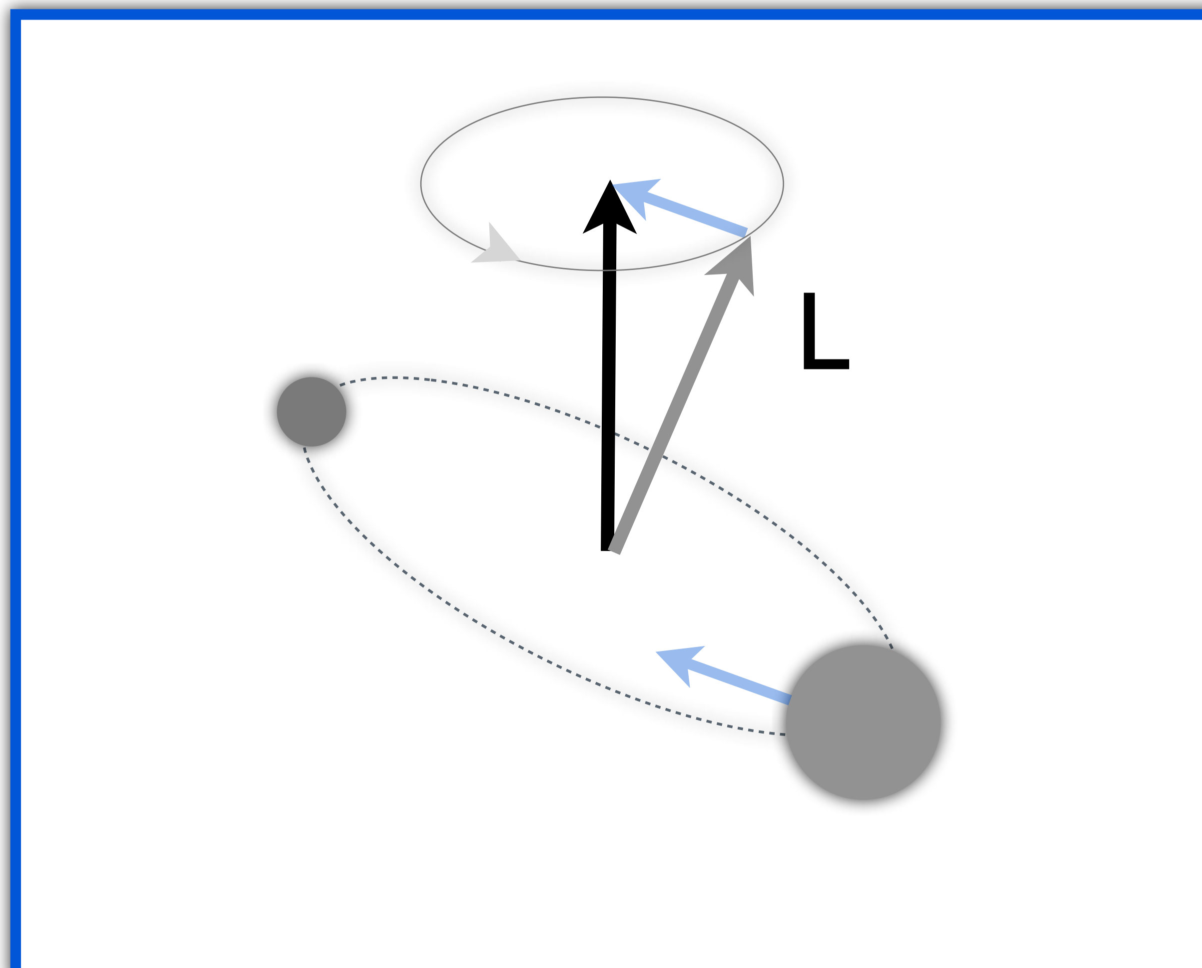
Orbital precession



Newtonian gravity:
L, S_1, S_2 remain fixed

Orbital precession

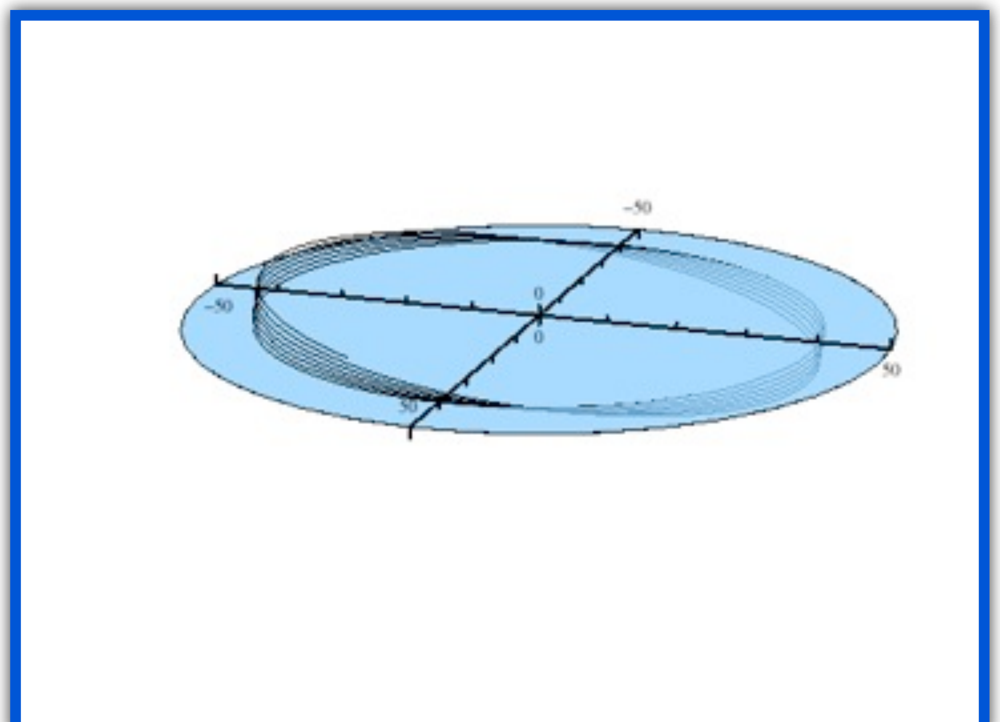




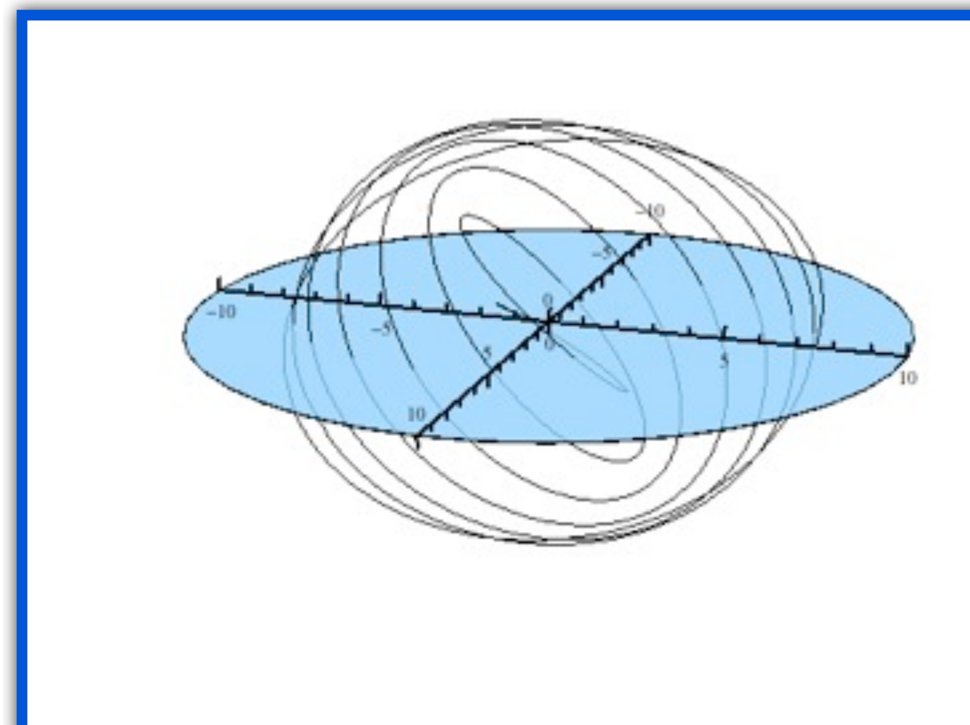
General relativity

(L, S_1, S_2) precess around J

Precessional dynamics

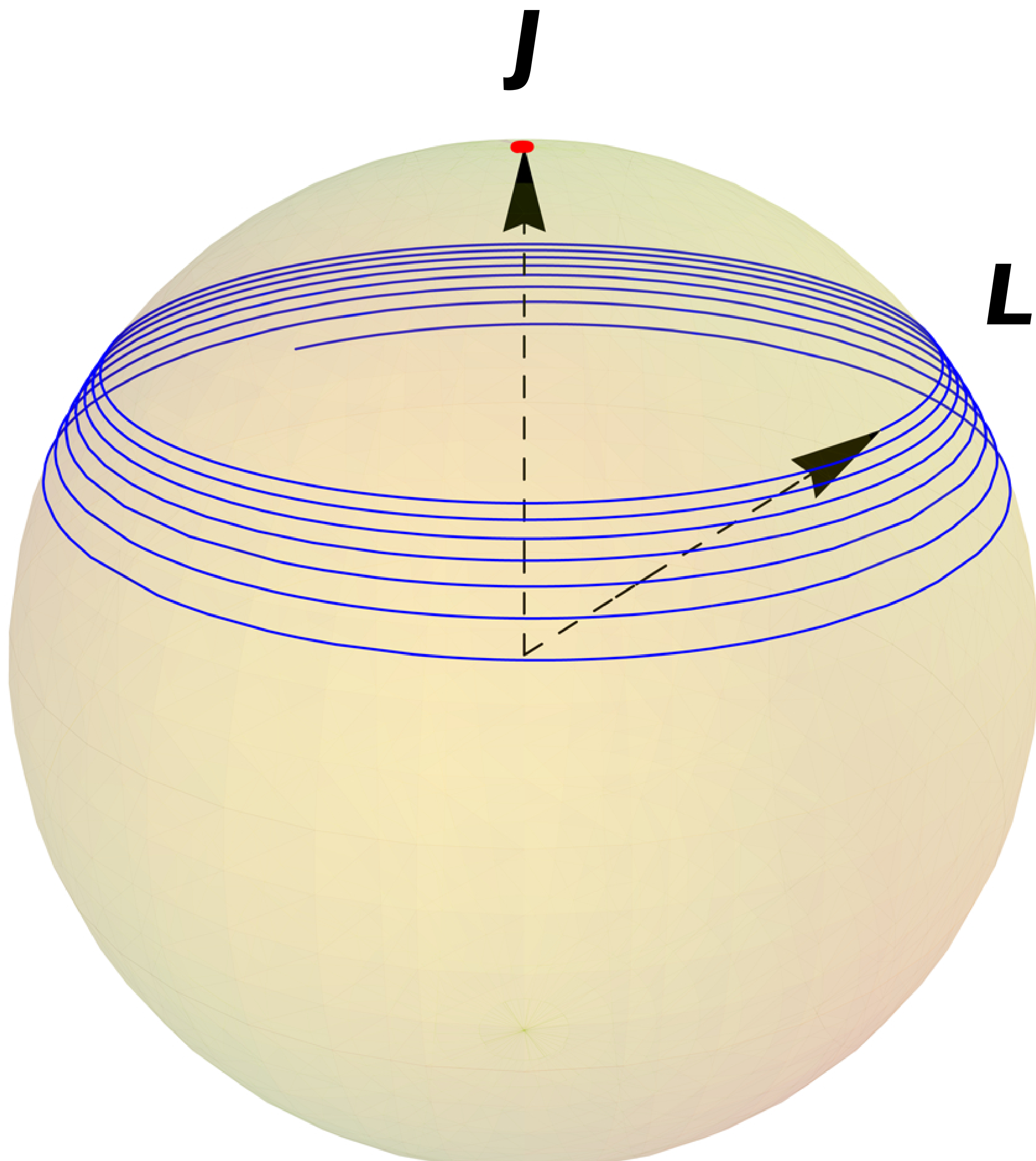


Large separation



Merger

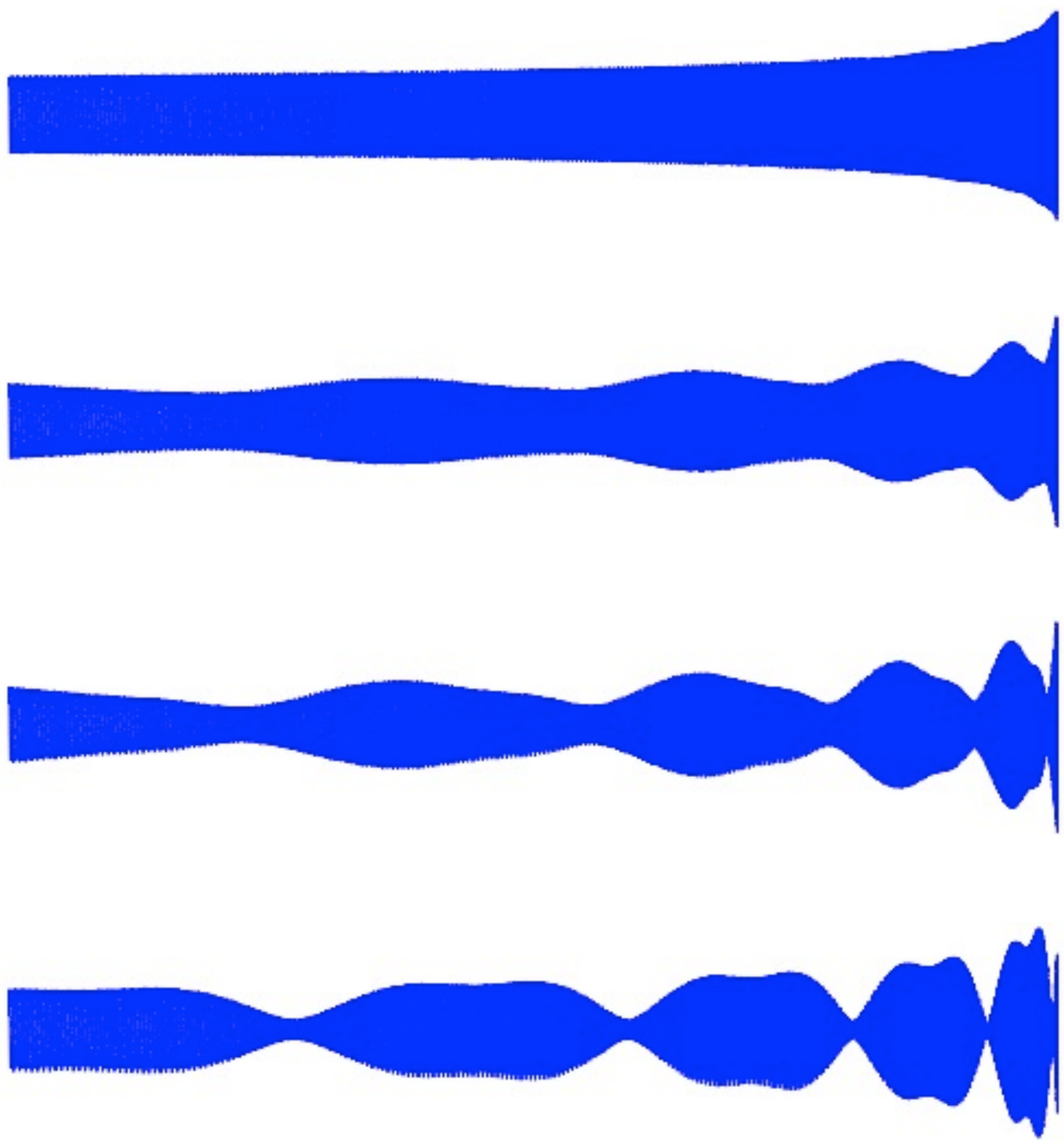
Example:
q=3, $|\mathbf{S}_2|$ = 0.75
(in plane)



q=3, $|\mathbf{S}_2|$ = 0.75 (in plane)

Observer aligned with $\boldsymbol{J}$

q=3, $|\mathbf{S}_2|$ = 0.75 (in plane)



Observer aligned with $\boldsymbol{J}$

Observer inclined $\pi/6$ to $\boldsymbol{J}$

Observer inclined $\pi/3$ to $\boldsymbol{J}$

Observer inclined $\pi/2$ to $\boldsymbol{J}$

How do we model
these complicated
waveform features?

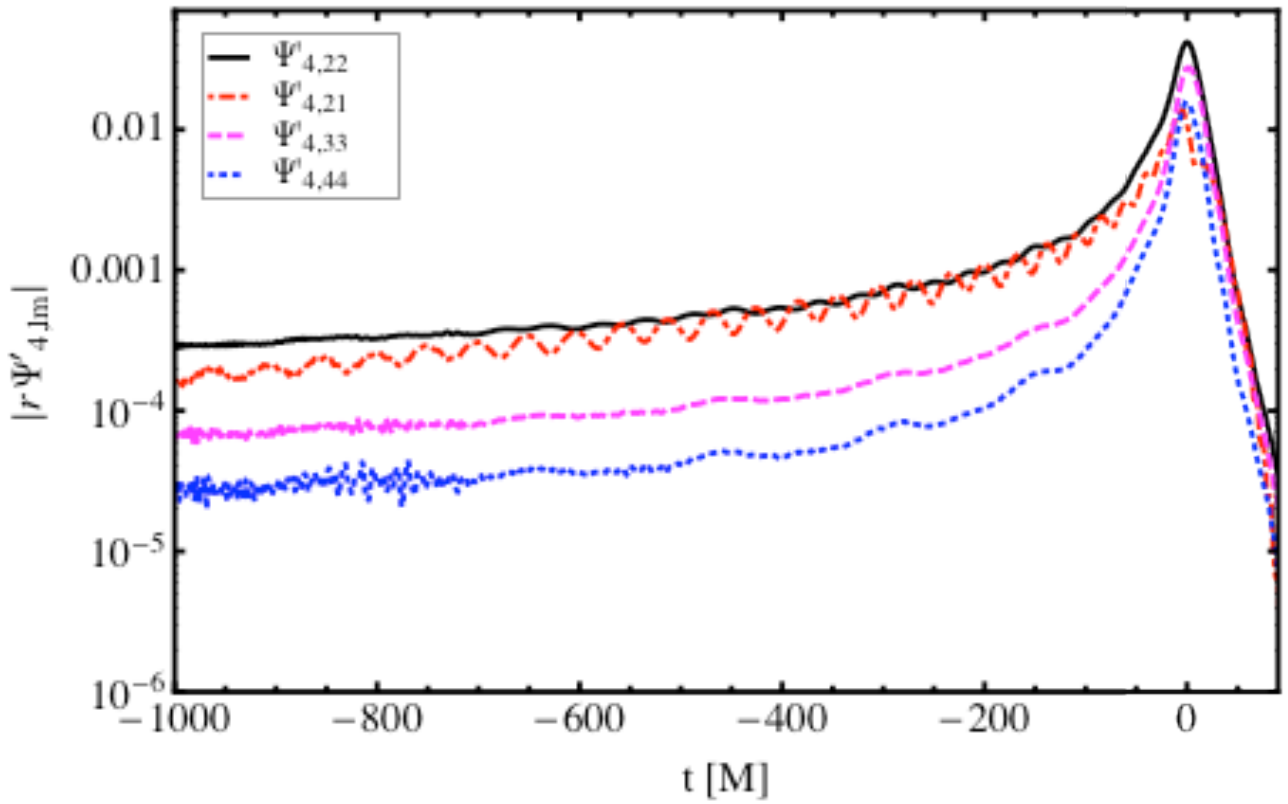
How do we cover
a seven-dimensional
parameter space
with NR simulations?

Untangling precession

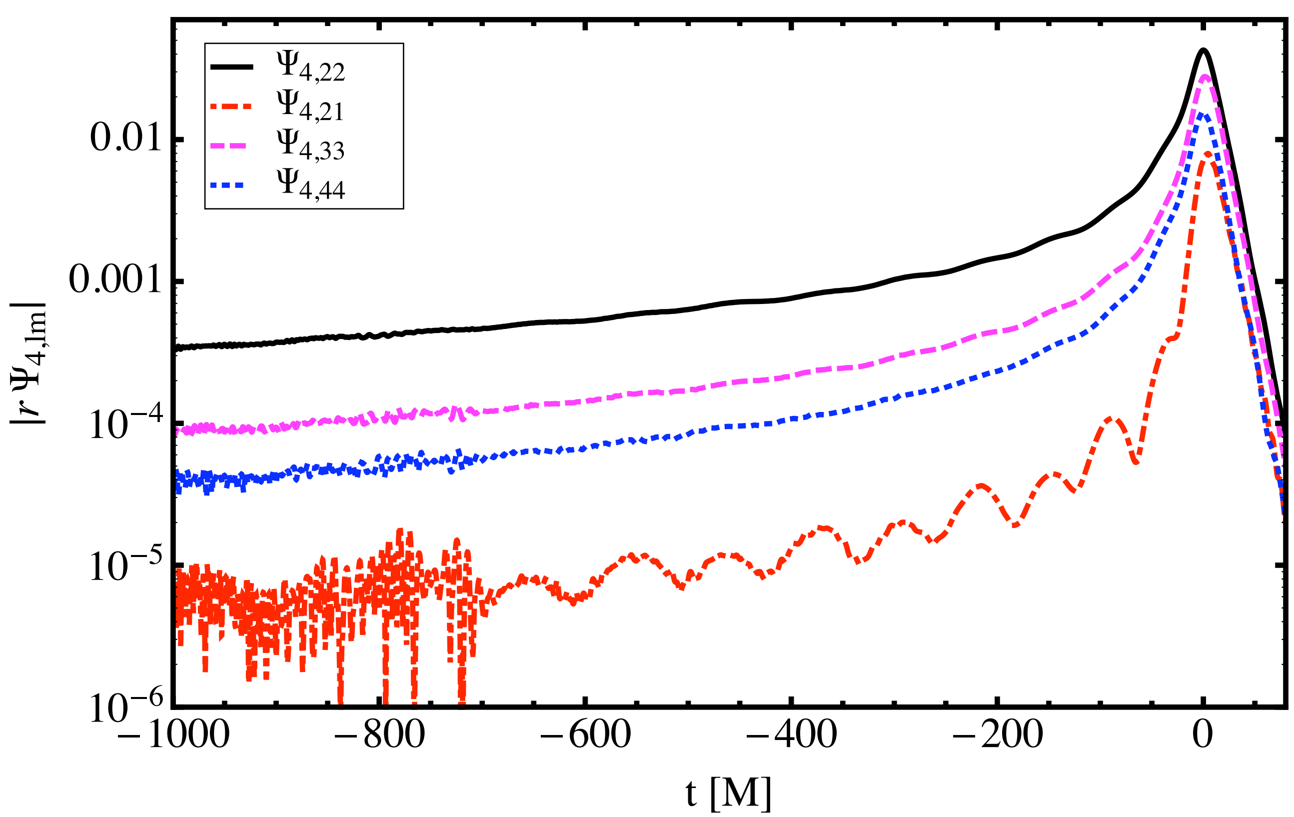
The waveforms are much simpler if viewed from a "co-precessing" frame

(Remain face-on to the binary)

[Schmidt, et. al. (2010), Boyle, et. al. (2011)]

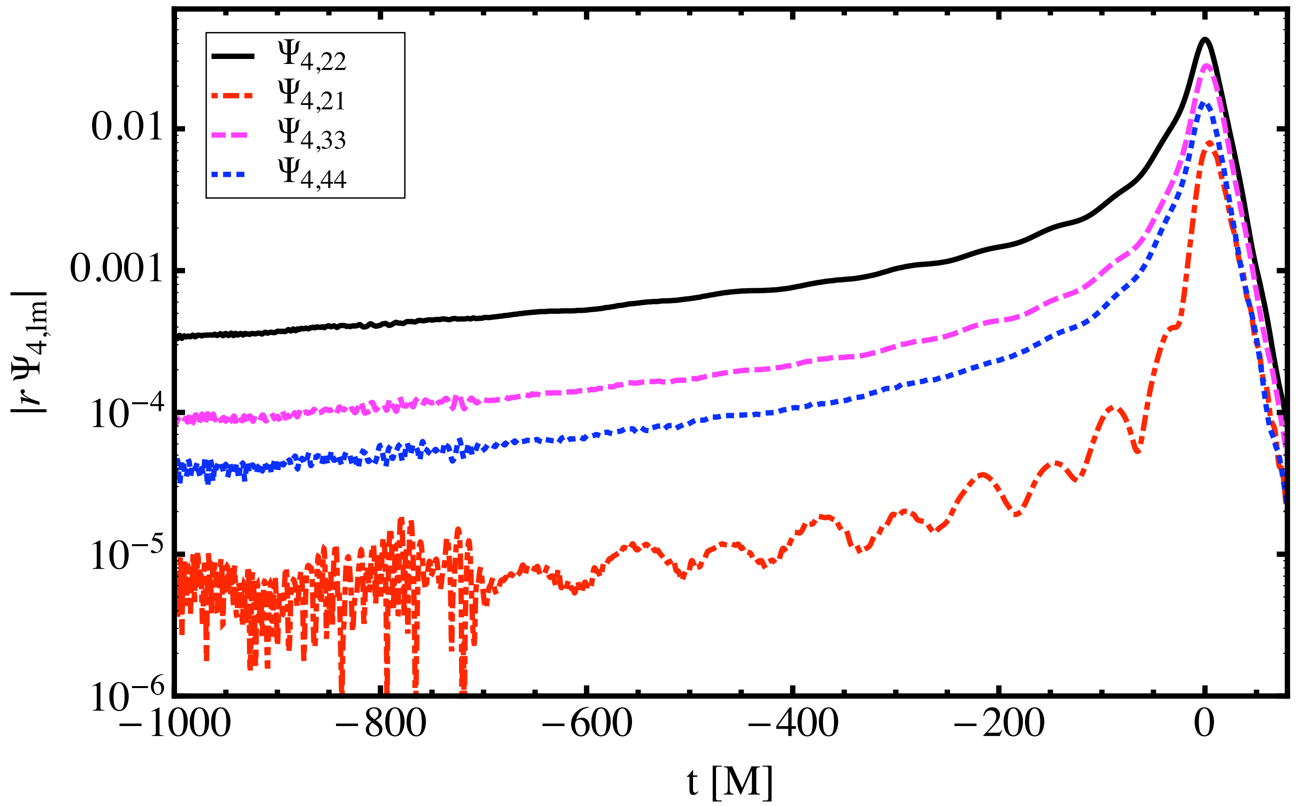


Frame aligned with initial direction of ***L***

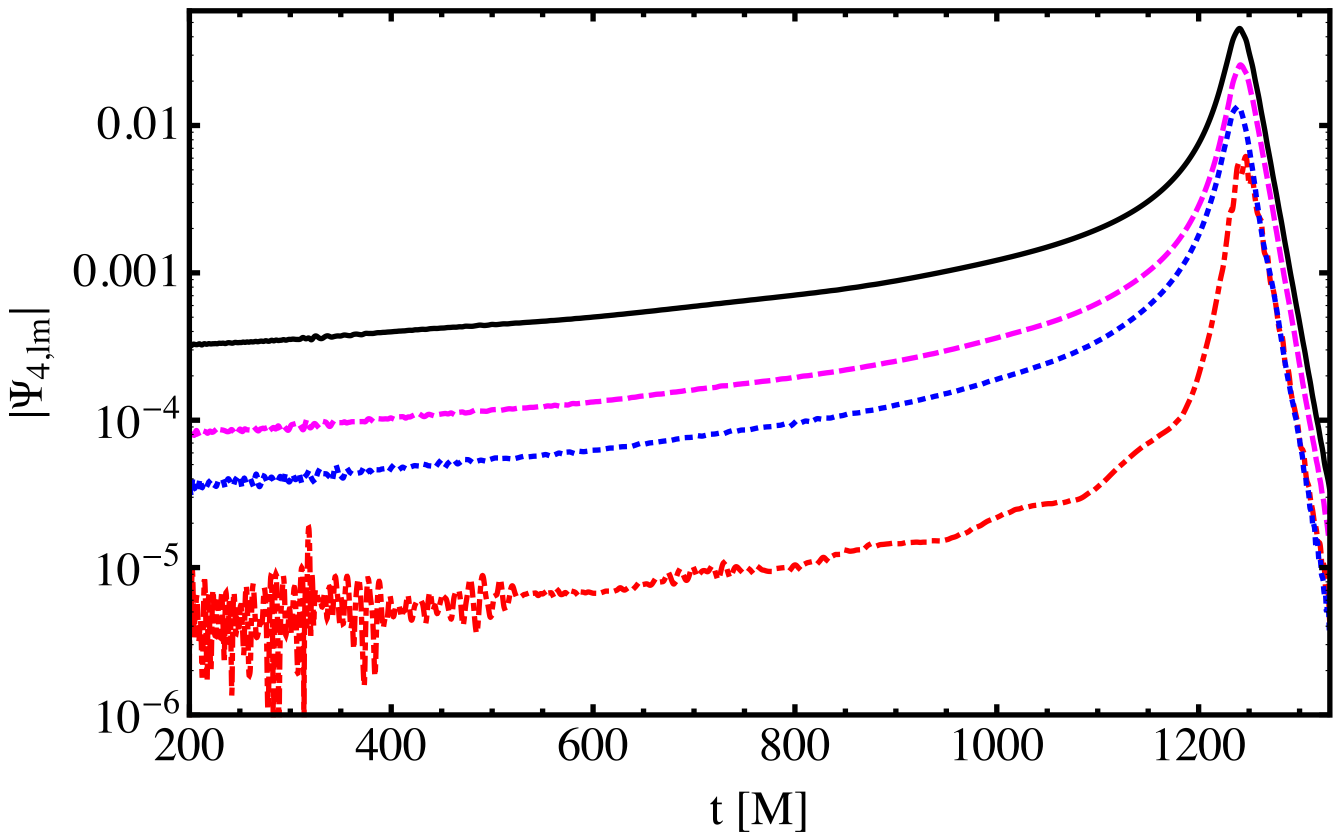


Co-precessing frame

Just a second...



q=3 precessing-binary waveform in co-precessing frame



q=3 non-precessing binary waveform

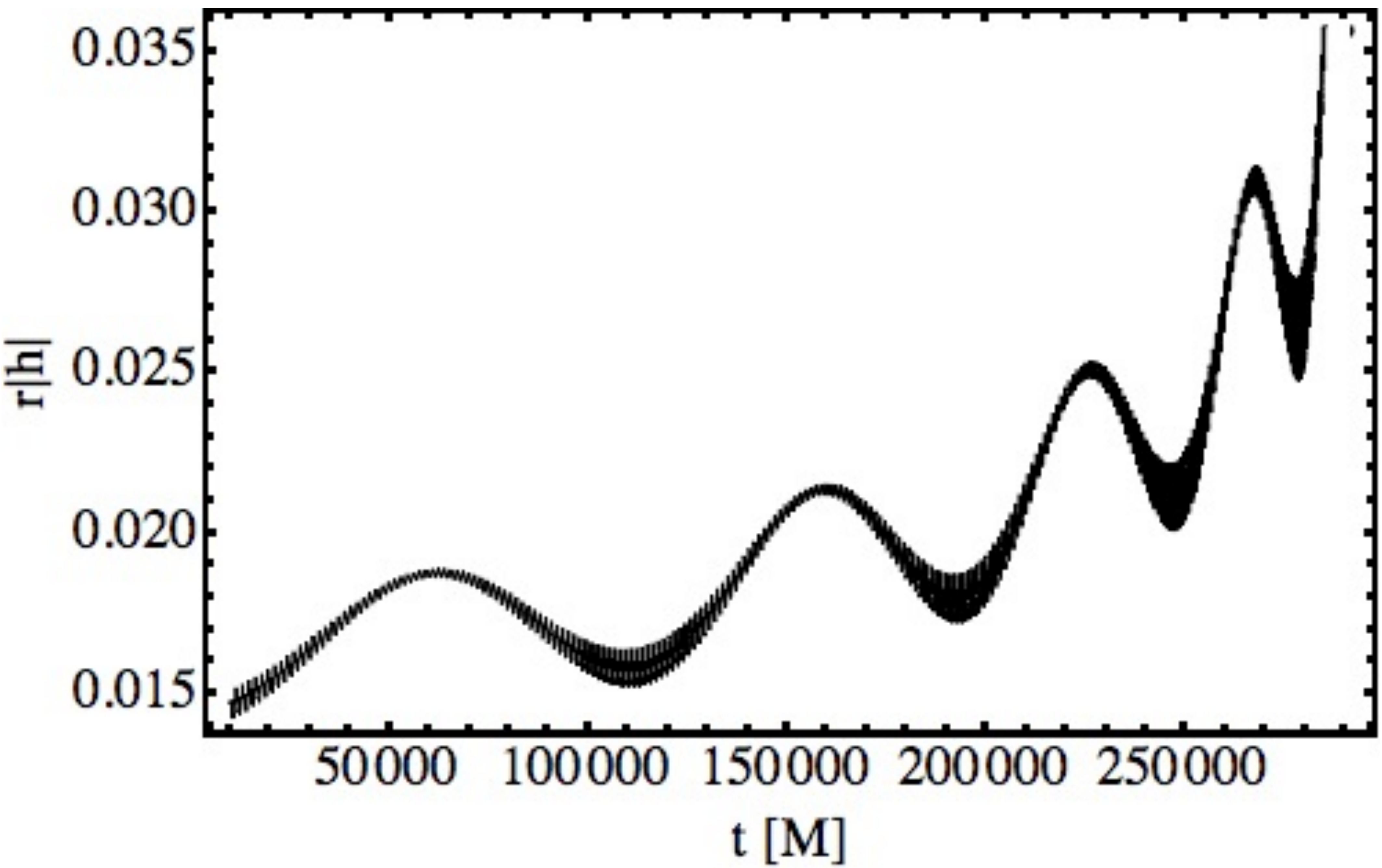
Precessing-binary waveforms
(in the co-precessing frame)
are
almost identical to
equivalent non-precessing-binary
waveforms

[Schmidt, et. al. (2012)]

Corollary:
We can approximate
precessing-binary waveforms
by
“twisting up”
equivalent non-precessing-binary
waveforms

[Schmidt, et. al. (2012)]

(Inclination 2.8 rad)



Precessing-binary waveform

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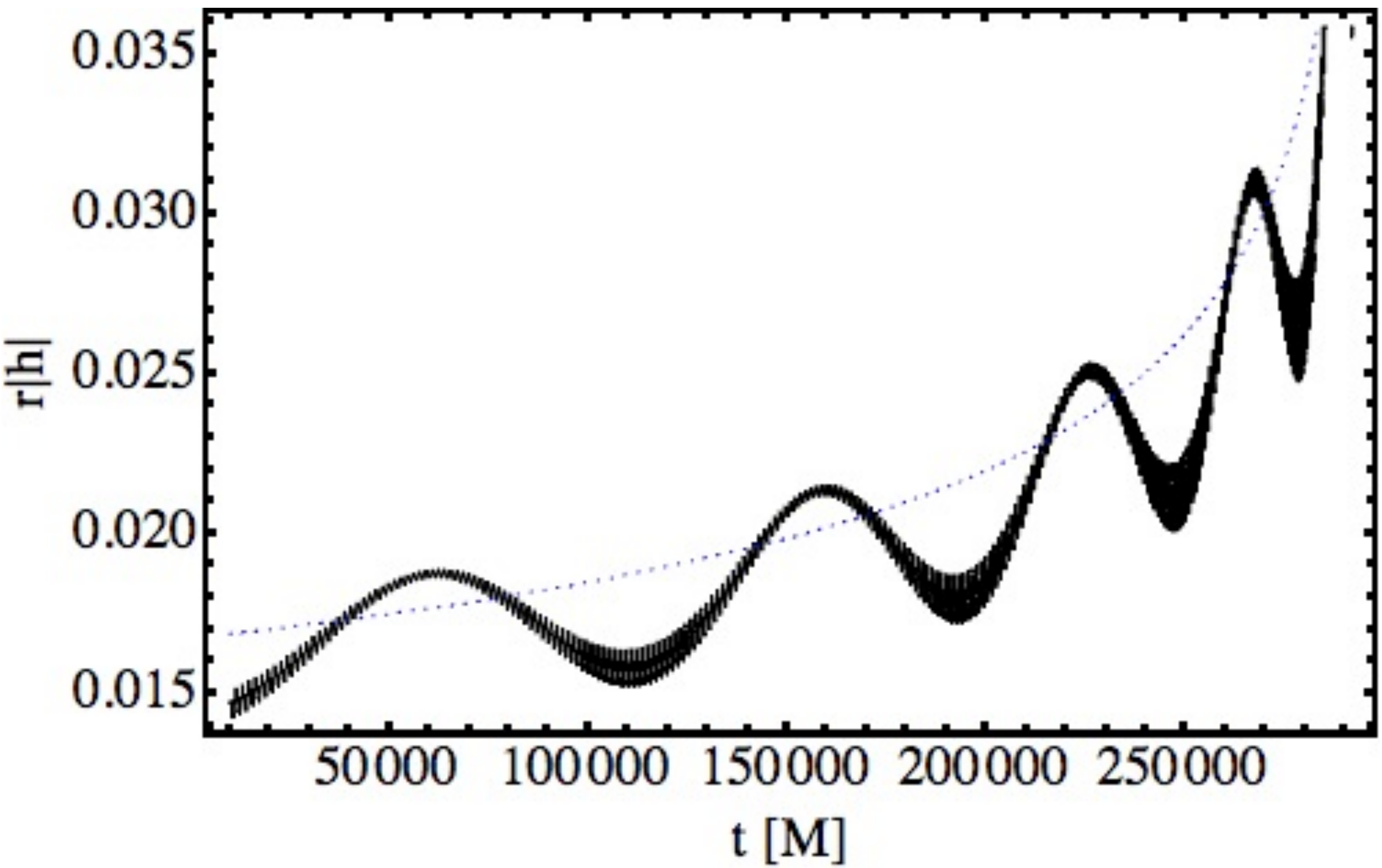

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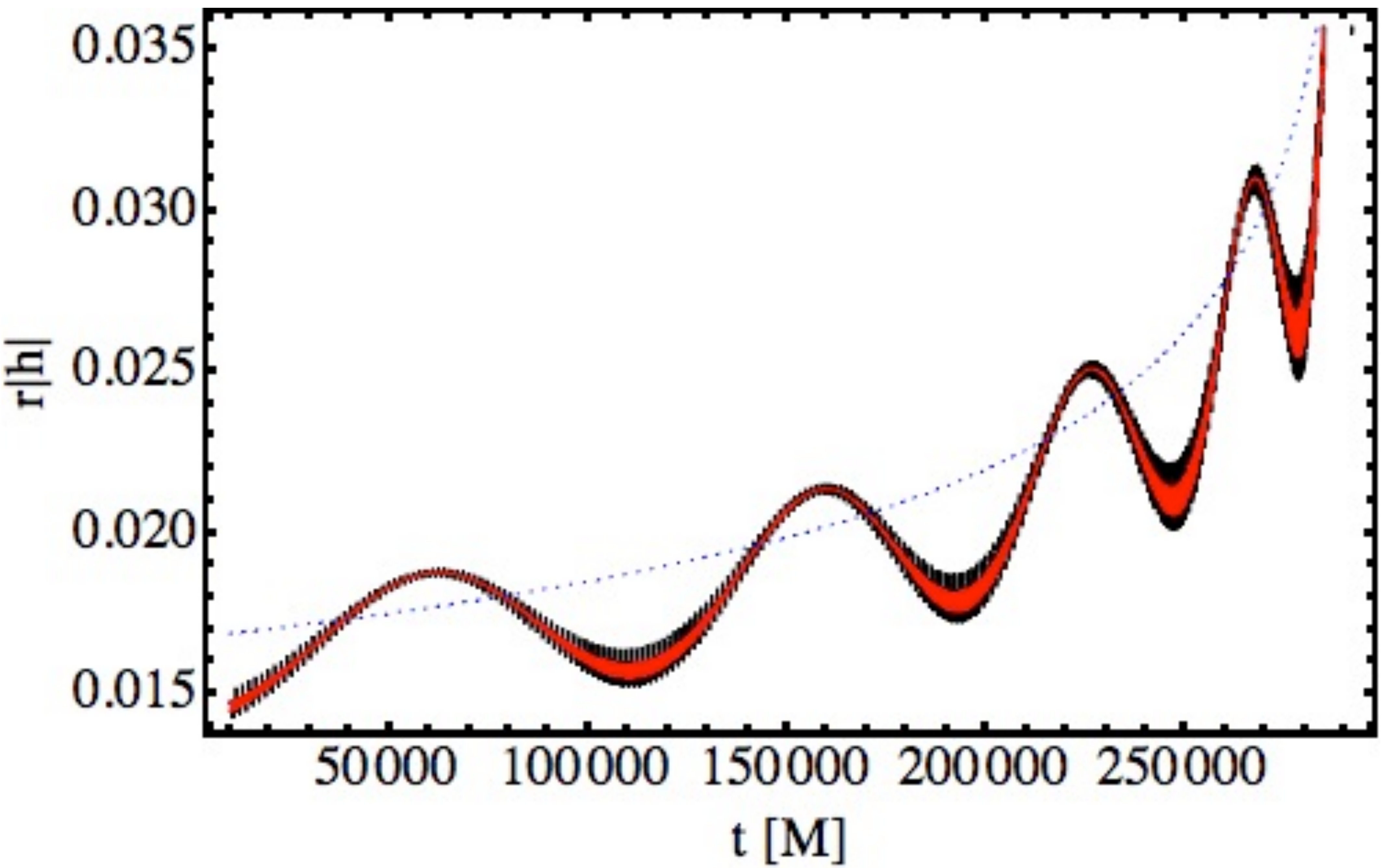


We still have to
model the precession angles!

We still have to deal
with a seven-dimensional
parameter space!

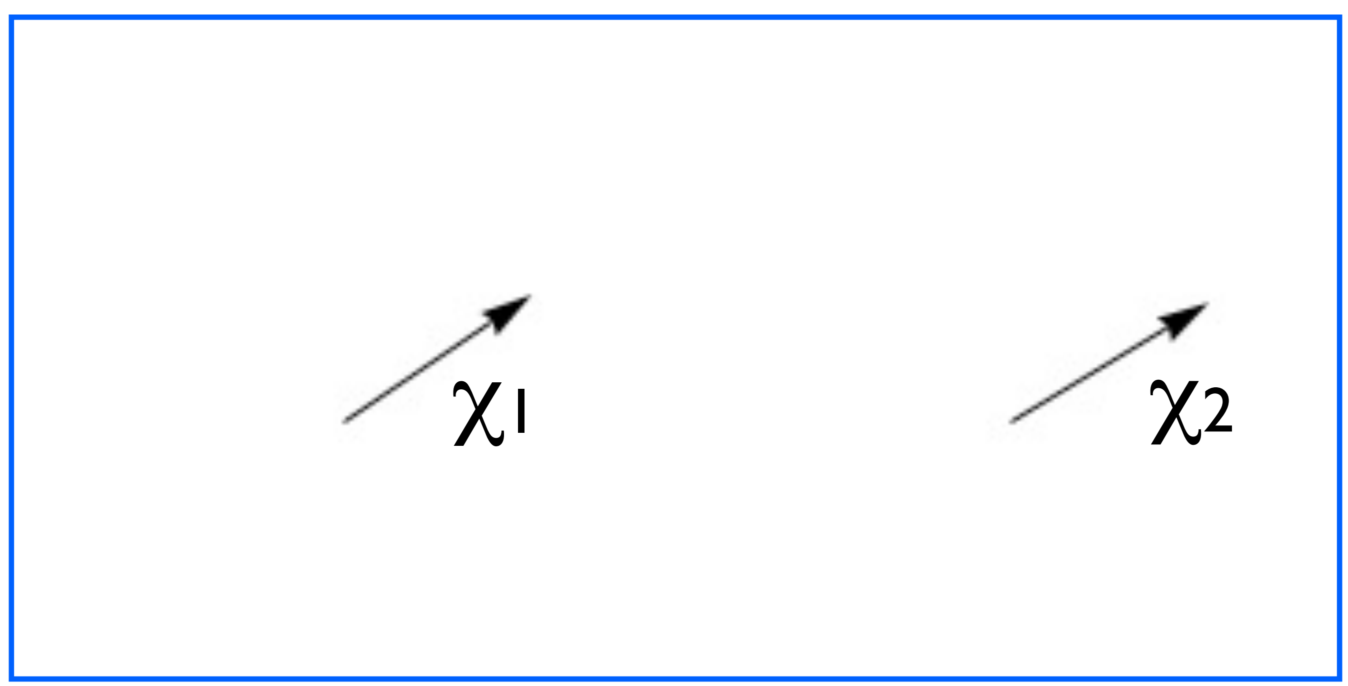
For non-precessing binaries we used only *one* spin parameter, χ_{eff}.

Can we use the same trick for precession?
i.e., replace
the four in-plane spin components
with *one* "precession spin"?

A precession parameter

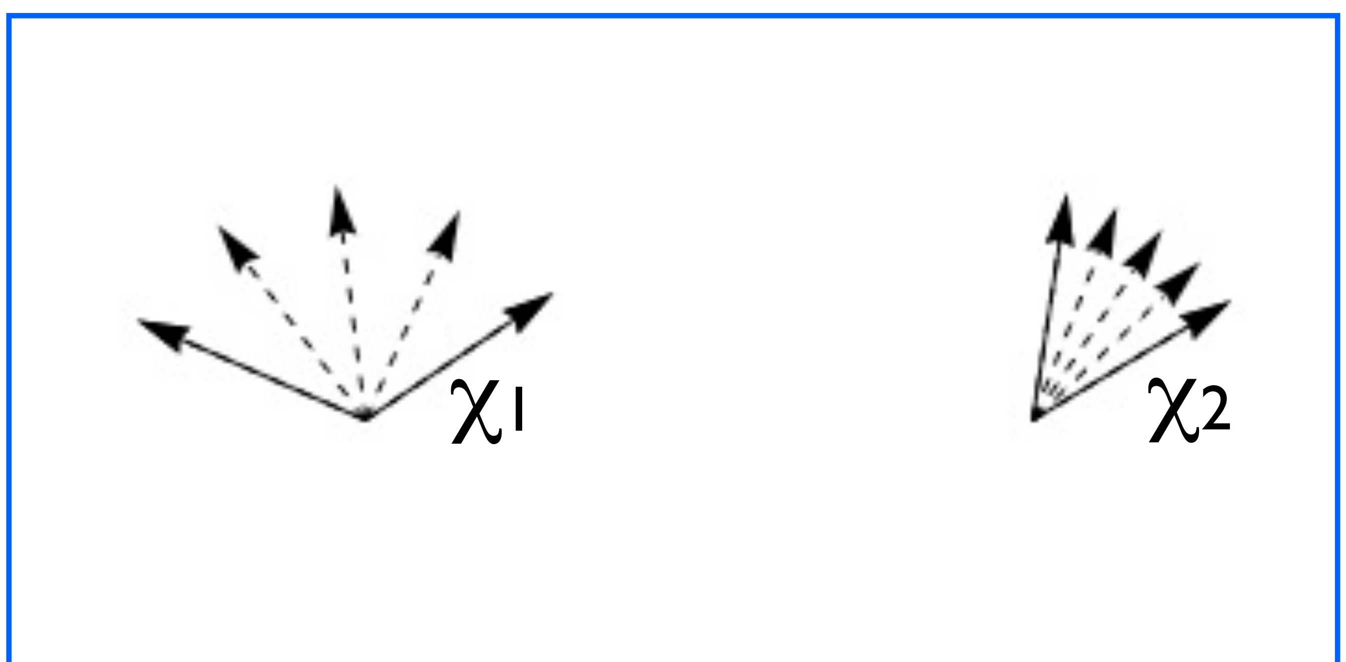
- First consider one spinning BH
- spin rotates in the plane during evolution
- Only in-plane spin *magnitude* matters!

Double spins?



Double spins?

- Spins rotate at different rates
- Consider only the average spin in the plane, "χ_p"!



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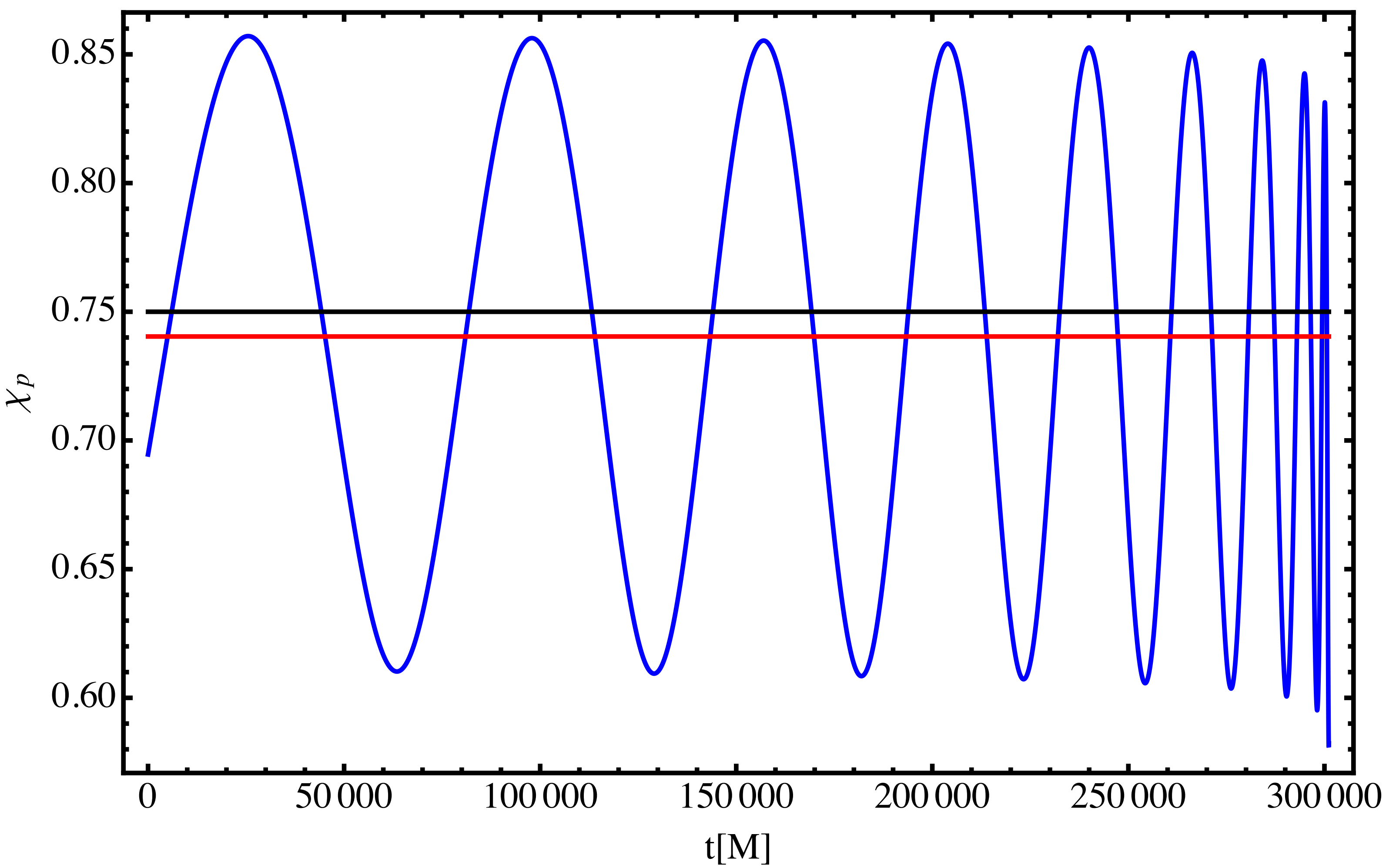
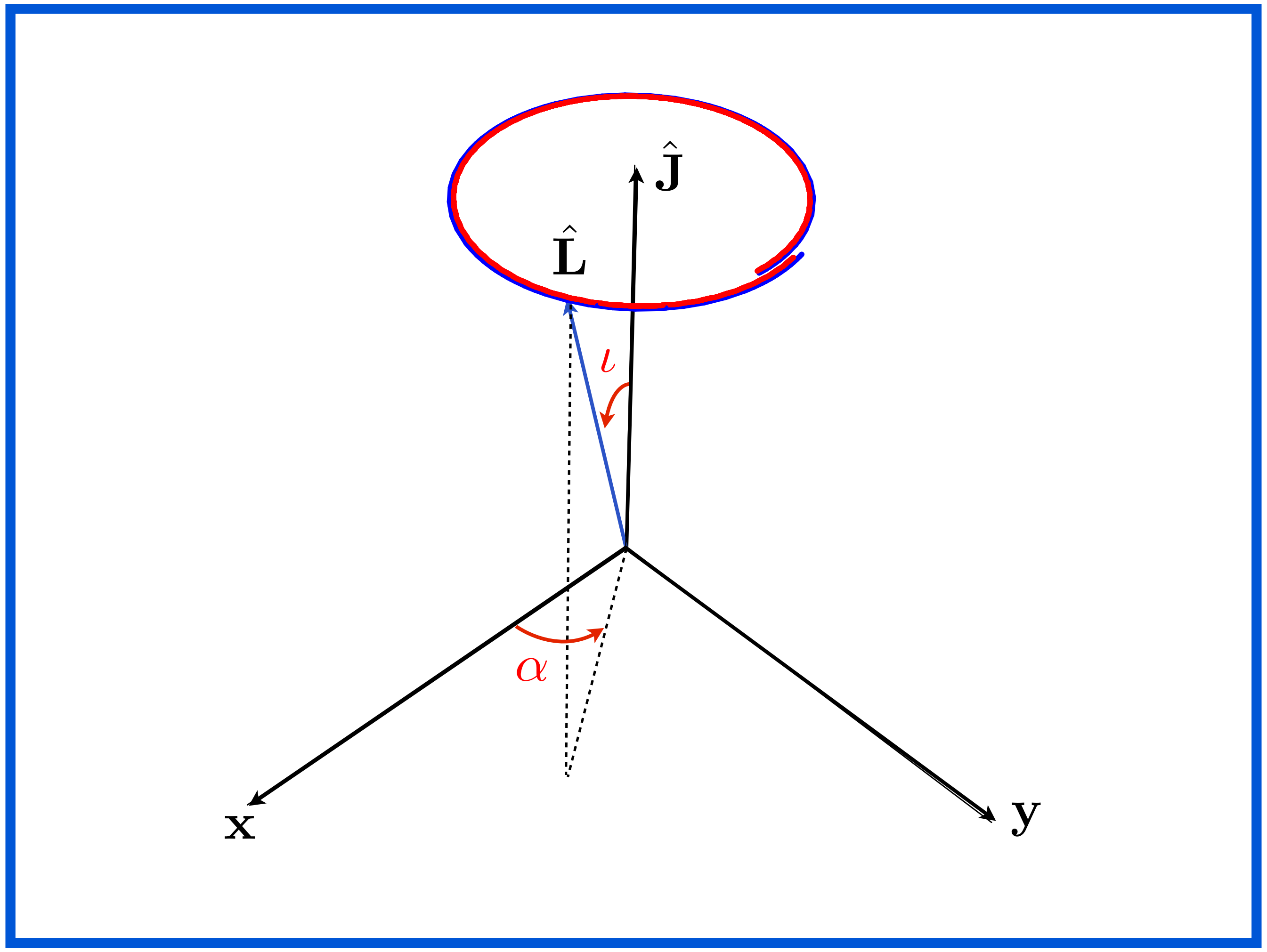


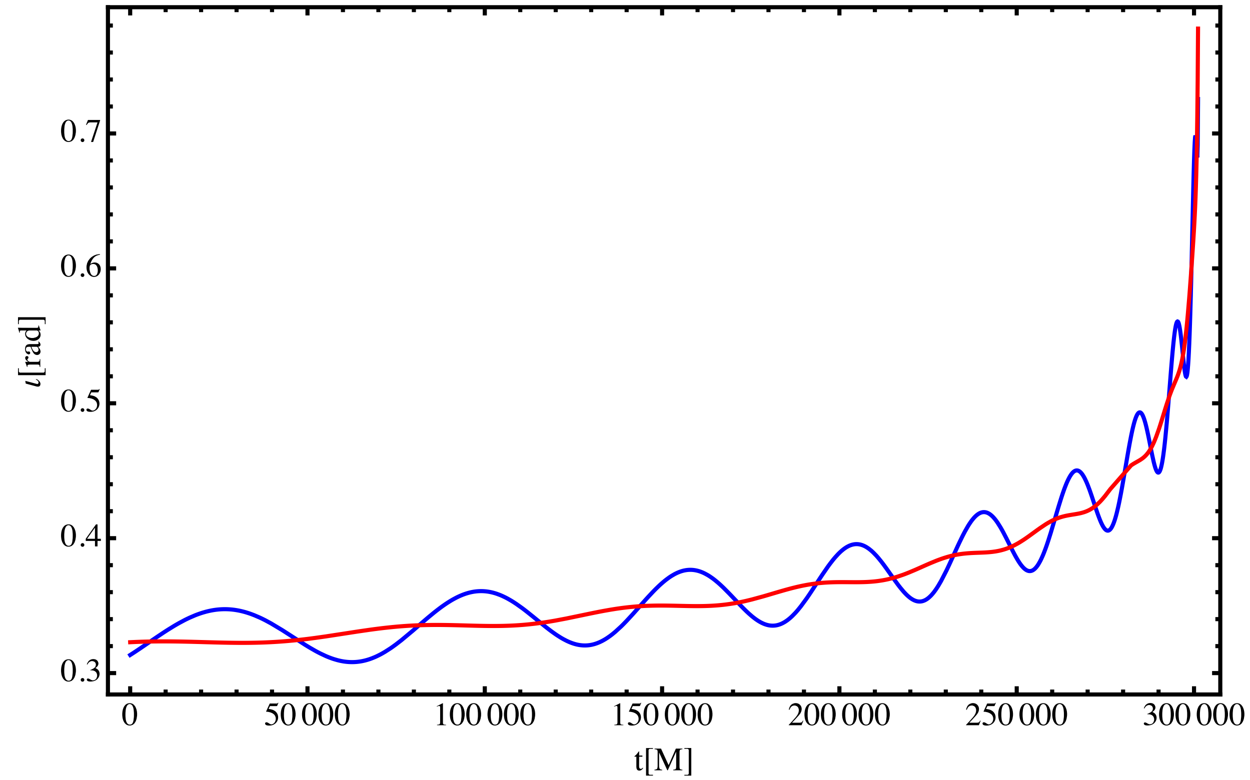
q=3,
χ_1=(-0.2,-0.4,0.3)
χ_2=(0.747, 0.045, 0.1)

χ_p = 0.75

Compare precession angles

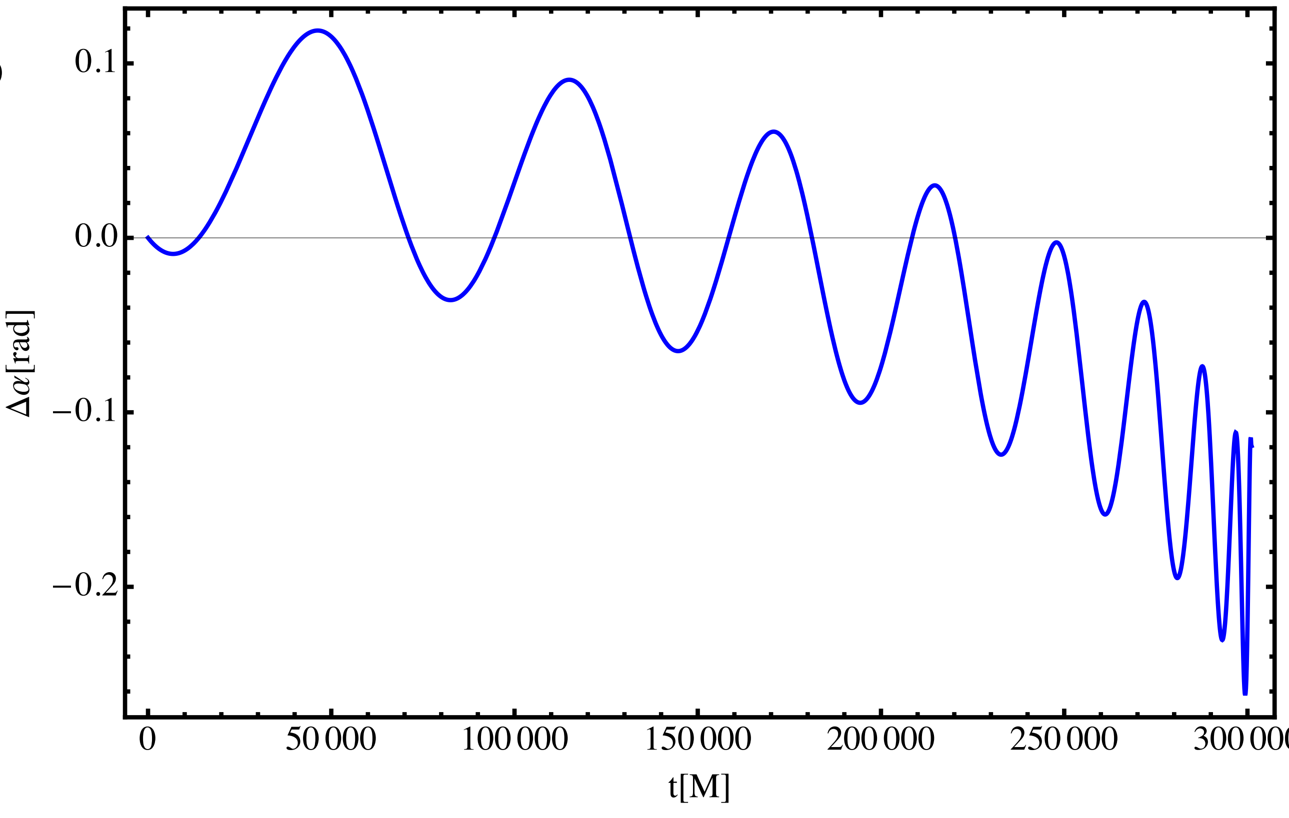


Compare precession angles



Inclination

Precession angle



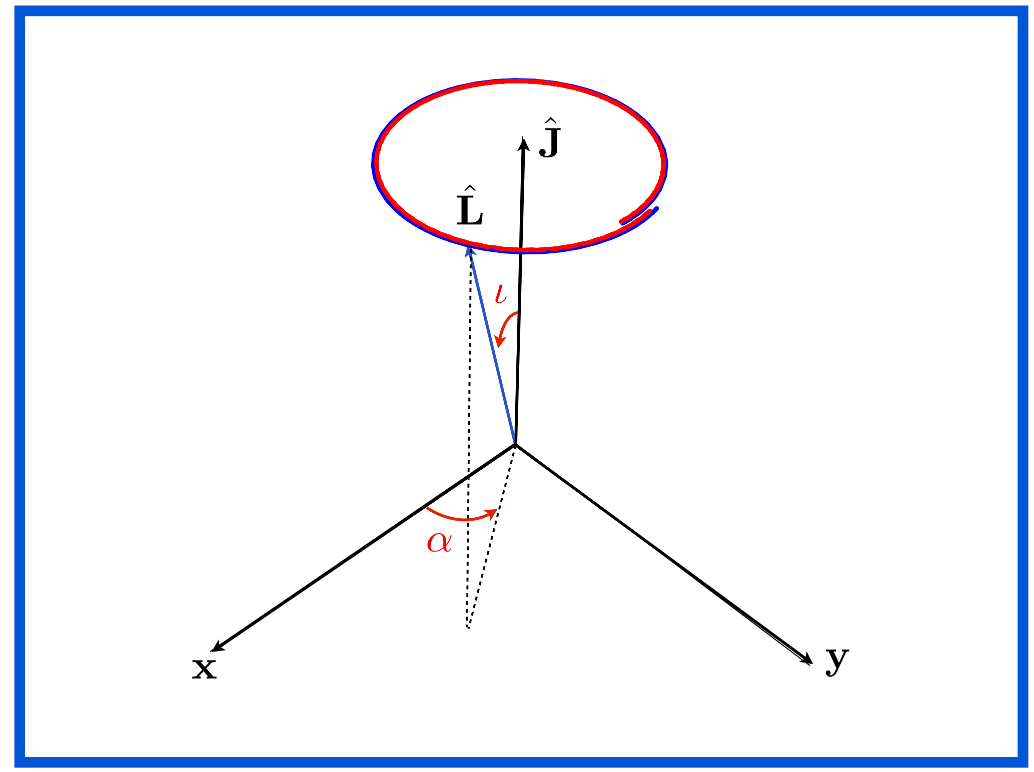
Now we have only three key parameters

Parameter space has been simplified!

What about a model?

Orbital plane tilt, ι(t)

$$\cos\iota = \hat{J}\cdot\hat{L} = \frac{L+S_{||}}{\sqrt{(L+S_{||})^2+S_{\perp}^2}}$$



L(t) (or L(f)) can be calculated from PN theory

ι(t) mostly affects mode amplitudes, not phases...

Precession angle, α(t)

- Strongly affects waveform phase
- For a single-spin model, to leading order:

$$\Omega_p = \left(2 + \frac{3m_1}{2m_2}\right)\frac{J}{r^3}$$

- Use next-to-next-to-leading-order in spin-orbit terms

Stationary phase approximation

$$h^P_{2m}(t) = e^{-im\alpha} \sum_{|m'|=2} e^{im'\epsilon} d^2_{m',m}(-\iota) h_{2,m'}(t)$$

- Assume waveform amplitude varies slowly:

$$\Psi(v) = 2\pi f t(v) - \phi(v)$$

- Precession angles also vary slowly
- See also [Lundgren and O'Shaughnessy (2013)]

Merger and ringdown

- ***J*** is approximately fixed
- Use final spin estimates [Barausse, *et. al.* (2009)]

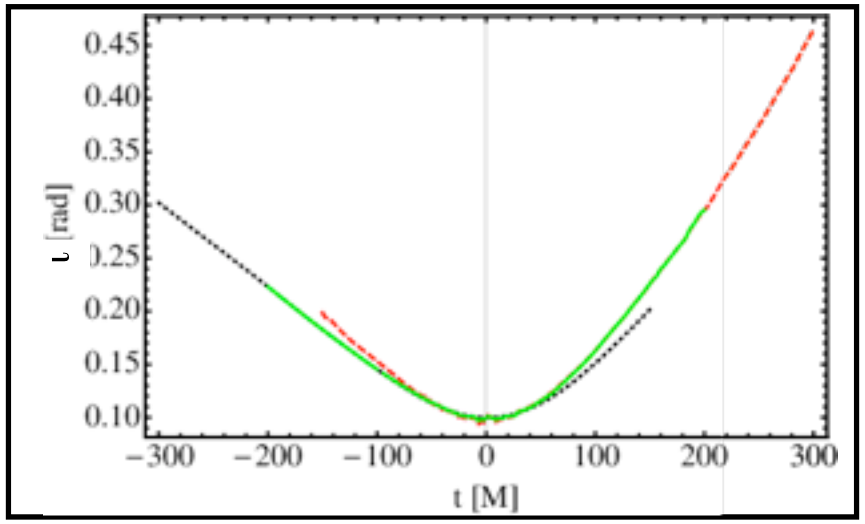
Crude approximations:

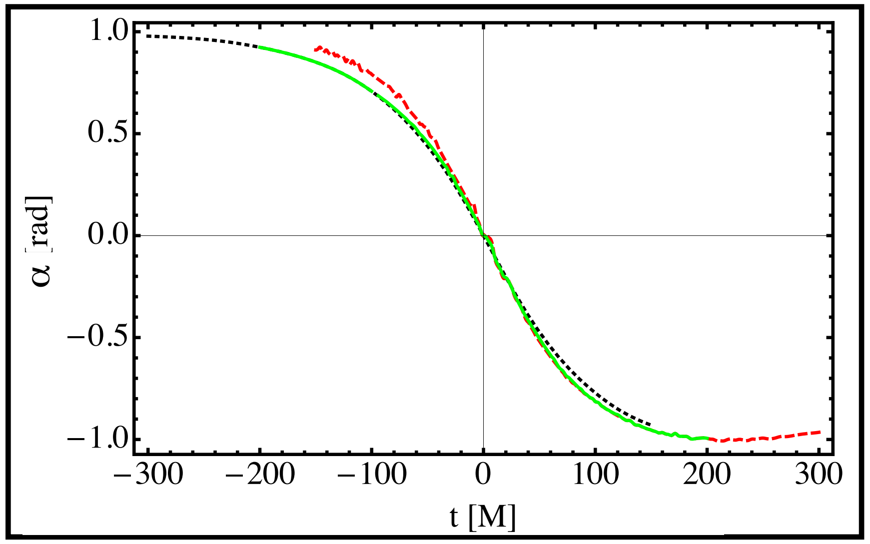
- Use PN angles through merger/ringdown
- Use SPA through merger/ringdown.

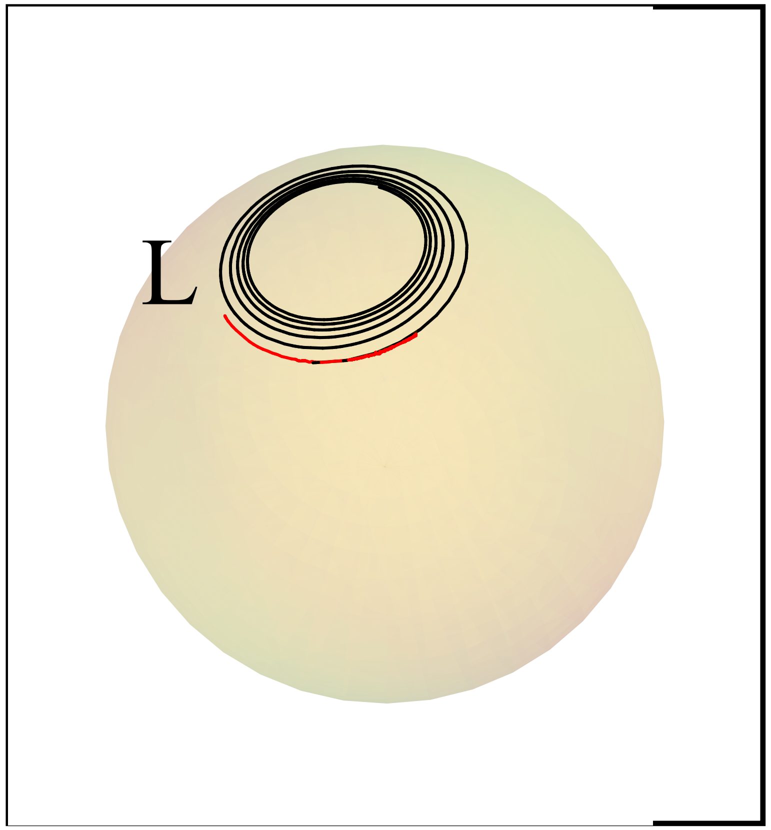
Testing the model: PN-NR hybrids

Hybridize waveforms in co-precessing frame [Schmidt, *et al* 2012]

(q = 1, 2, 3; single & double-spin cases)





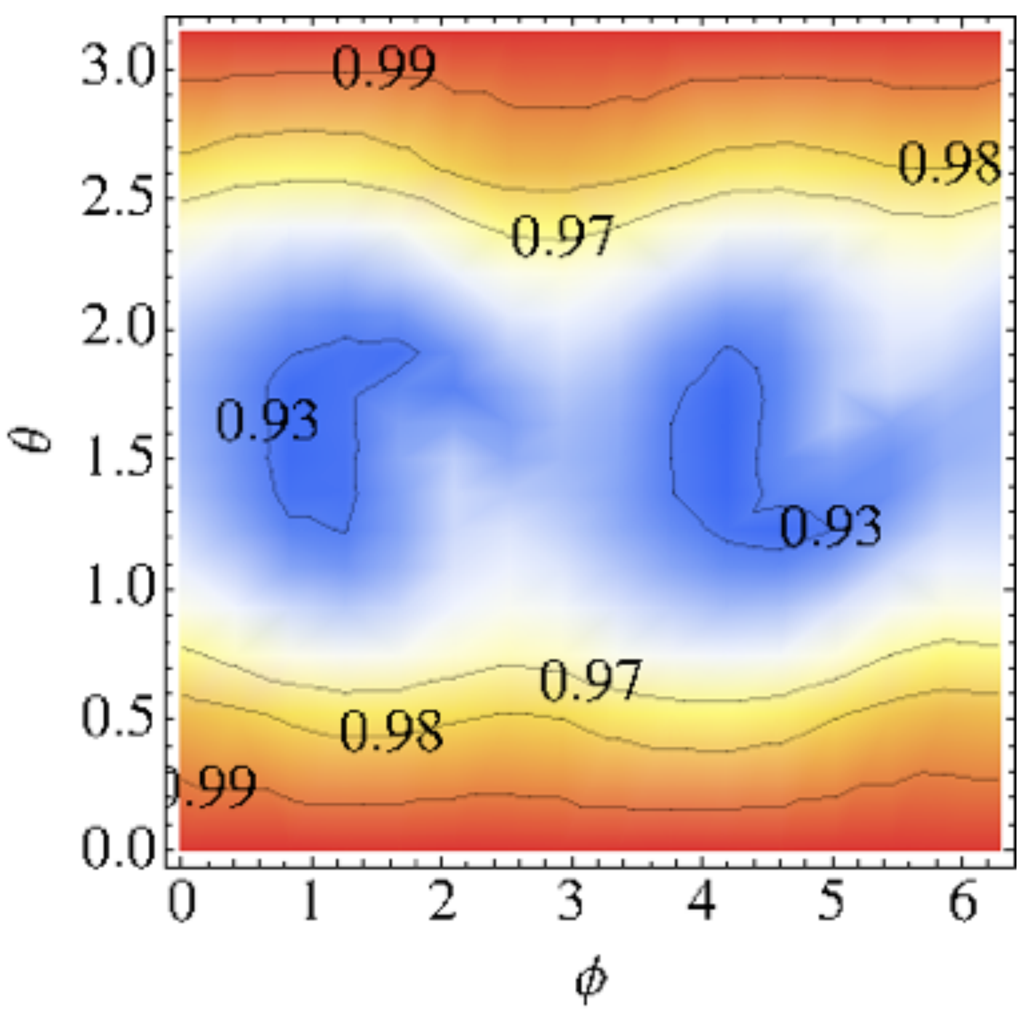


Also use NR initial parameters and evolve PN backwards in time

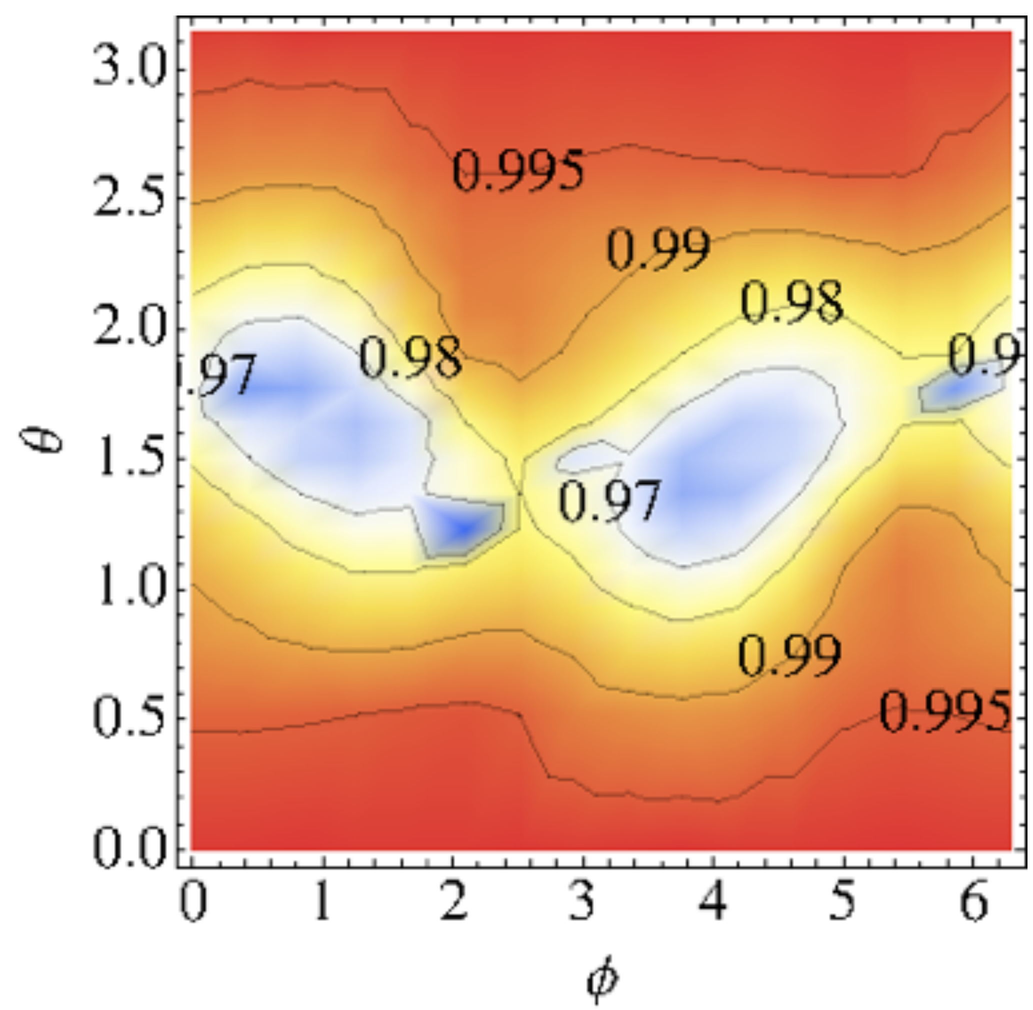
Comparisons

Most extreme comparison:
q=3, χ_p = 0.75, 50 $M_\odot$

[Hannam, et. al. (2013)]



Against PhenomC



Against PhenomP

Now implemented
in LALSimulation
for all your
GW Astronomy
needs!

To do list

- Perform simulations across (q, $\chi_{\rm eff}$, $\chi_{\rm p}$)
- Calibrate model to simulations
- Verify / improve assumptions
- Improve merger/ringdown model
- Parameter estimation capabilities/limitations
- Revolutionise our understanding of the universe