

From Neutron Star Structure to Compact Binary Mergers and Back

Francesco Pannarale



Cardiff University

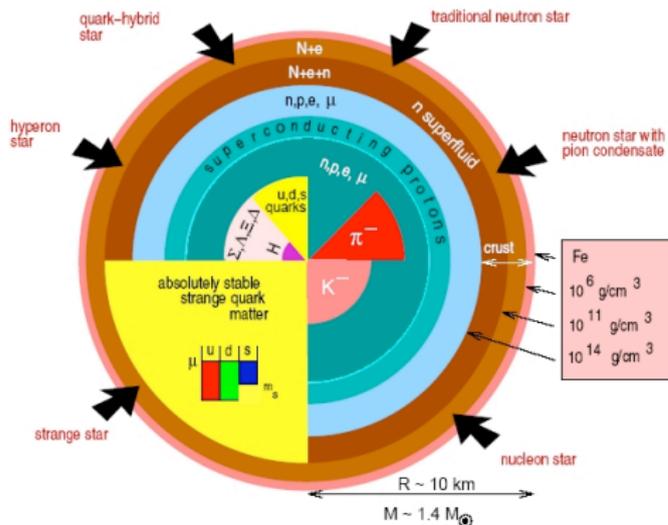
Cardiff University - December 13, 2013



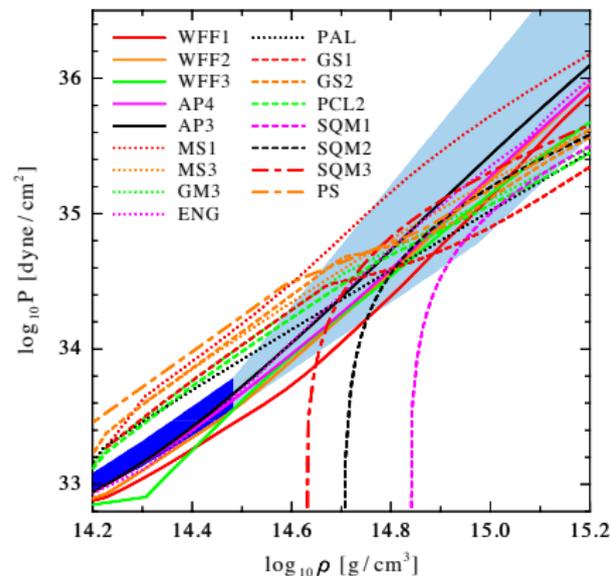
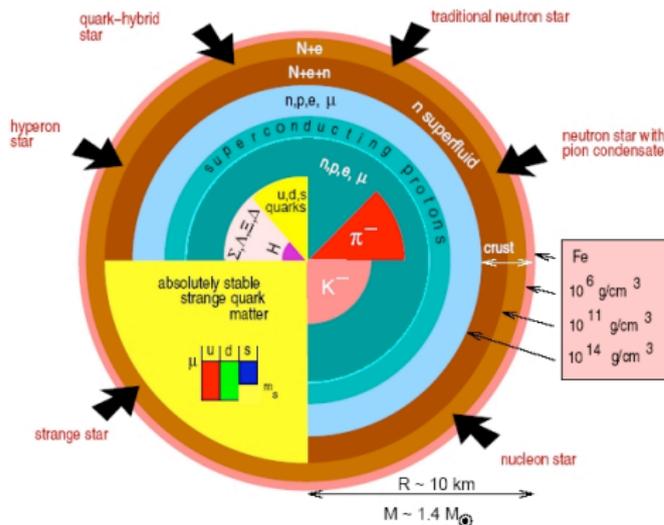
Neutron Star Structure

- Stellar structure equations require a bridge between macroscopic properties and microscopic dynamics
- This is the equation of state: closed expression for the thermodynamic variables of a physical system
- The NS EOS is highly uncertain
 1. We lack stringent NS radius measurements
 2. NSs reach supranuclear densities: we lack labs...
- Terrestrial densities \rightarrow Extrapolations \rightarrow “Realistic” EOS
 1. Reproduce properties of ordinary nuclear matter
 2. No causality violations

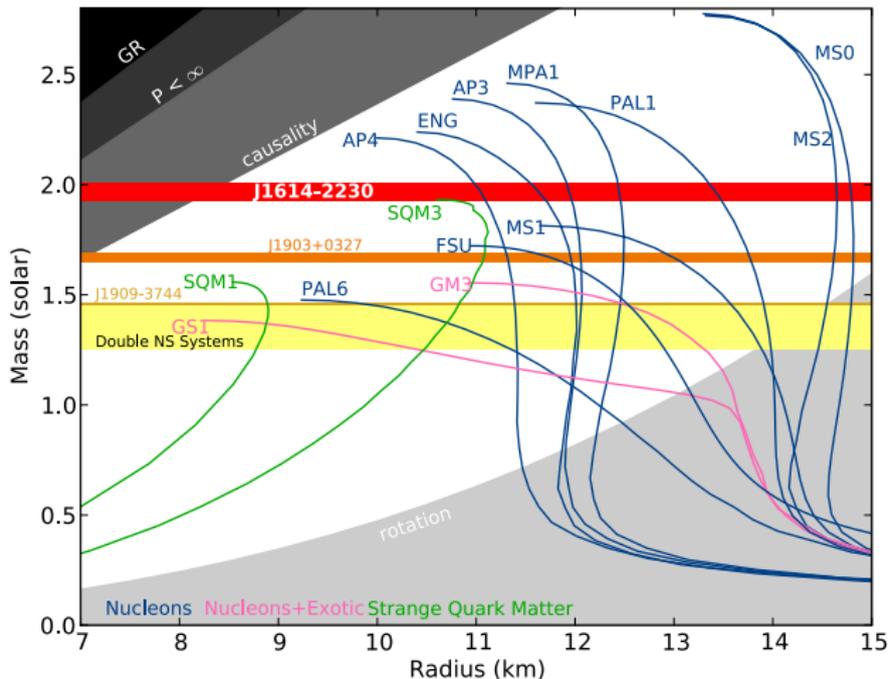
Neutron Star Structure



Neutron Star Structure

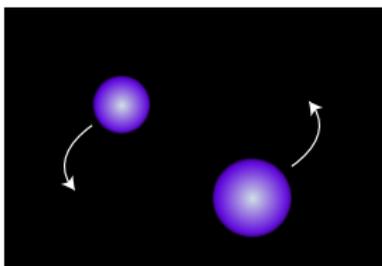
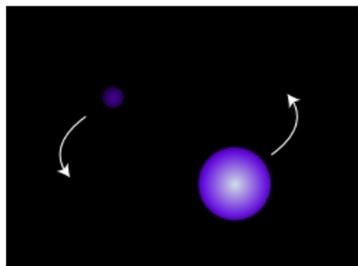


Neutron Star Structure



- Microphysics influences macroscopic properties and behaviour
- 98% of the mass is in the core, where our uncertainty is higher

Black Hole-Neutron Star and Double Neutron Star Binaries



- Progenitor binary
→ compact binary
[stellar evolution]

- Secular evolution
[GWs]

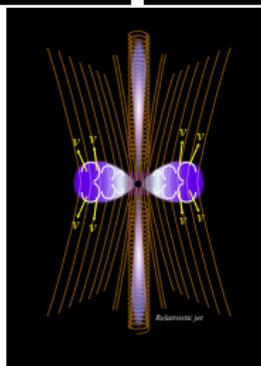
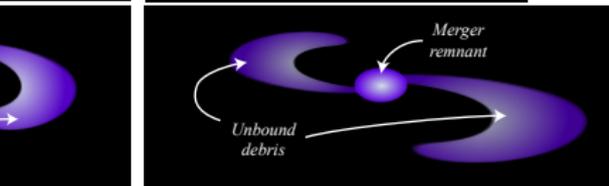
- Merger

- Ejecta
[*r*-processes, kilonovæ]

- Hypermassive NS
[EM- & ν -emission]

- Collapse

- Disk accretion
[SGRBs]



Black Hole-Neutron Star and Double Neutron Star Binaries

The EOS is everywhere!

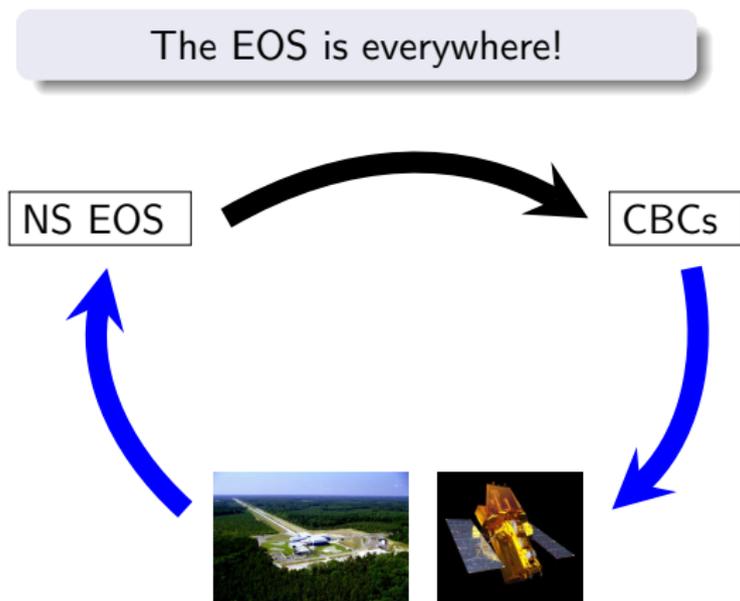
NS EOS



CBCs

- Progenitor binary
→ compact binary
[stellar evolution]
- Secular evolution
[GWs]
- Merger
- Ejecta
[*r*-processes, kilonovæ]
- Hypermassive NS
[EM- & ν -emission]
- Collapse
- Disk accretion
[SGRBs]

Black Hole-Neutron Star and Double Neutron Star Binaries



Can we constrain the EOS via observations of merger events?

- Progenitor binary
→ compact binary
[stellar evolution]
- Secular evolution
[GWs]
- Merger
- Ejecta
[*r*-processes, kilonovæ]
- Hypermassive NS
[EM- & ν -emission]
- Collapse
- Disk accretion
[SGRBs]

Black Hole-Neutron Star and Double Neutron Star Binaries

- 1 Focus only on selected quantities and a specific problem: not complete, but more general
- 2 Compute “everything”: complete, but with little generality



Black Hole-Neutron Star and Double Neutron Star Binaries

- 1 Focus only on selected quantities and a specific problem: not complete, but more general
- 2 Compute “everything”: complete, but with little generality



Black Hole-Neutron Star and Double Neutron Star Binaries

- 1 Focus only on selected quantities and a specific problem: not complete, but more general
- 2 Compute “everything”: complete, but with little generality



Black Hole-Neutron Star and Double Neutron Star Binaries

- 1 Focus only on selected quantities and a specific problem: not complete, but more **general**



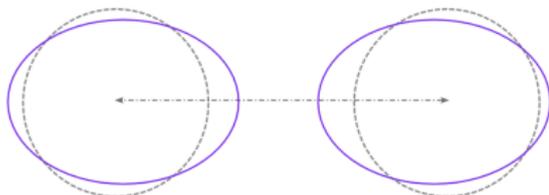
- 2 Compute “everything”: **complete**, but with little generality



- 3 ...try to make the best use of both worlds!

Tidal Deformability

The residual gravitational effect on two extended bodies in orbit or free-fall is a tidal deformation that



- depends on the NS size and EOS
- induces changes in the gravitational potential

Questions to answer

May NS tidal deformations affect the detection of BH-NS and NS-NS inspirals?

May we probe superdense matter indirectly, by looking at NS tidal deformation signatures in the GWforms?



FP, Rezzolla, Ohme, Read PRD **84**, 104017 (2011)

Tidal Effects in the GWforms

$\ell = 2, m = 0$ linear perturbation dominates tidal distortion effects

$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of induced quadrupole deformation}}{\text{tidal field strength}} \propto k_2 R_{\text{NS}}^5$$

Tidal Effects in the GWforms

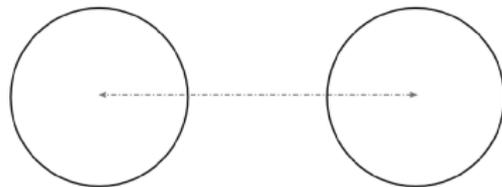
$\ell = 2, m = 0$ linear perturbation dominates tidal distortion effects

$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of induced quadrupole deformation}}{\text{tidal field strength}} \propto k_2 R_{\text{NS}}^5$$

$$E = -\frac{1}{2} M \eta x \left(1 + \sum_{\text{PP}}^{3\text{PN}} \right)$$

$$\dot{E} = -\frac{32}{5} \eta^2 x^5 \left(1 + \sum_{\text{PP}}^{3.5\text{PN}} \right)$$

Two point-particles or spheres



Tidal Effects in the GWforms

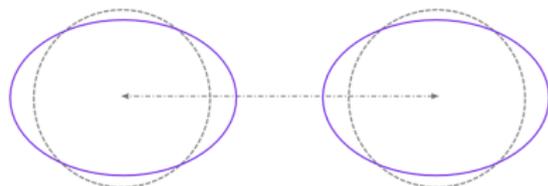
$\ell = 2, m = 0$ linear perturbation dominates tidal distortion effects

$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of induced quadrupole deformation}}{\text{tidal field strength}} \propto k_2 R_{\text{NS}}^5$$

$$E = -\frac{1}{2} M \eta x \left(1 + \sum_{\text{PP}}^{3\text{PN}} \right) + \frac{1}{2} Q_{ij}^2 \mathcal{E}_{ij}^1 + 1 \leftrightarrow 2$$

$$\dot{E} = -\frac{32}{5} \eta^2 x^5 \left(1 + \sum_{\text{PP}}^{3.5\text{PN}} \right) - \frac{1}{5} \sum_{a=1}^2 \langle \ddot{Q}_{ij}^a \ddot{Q}_{ij}^a \rangle$$

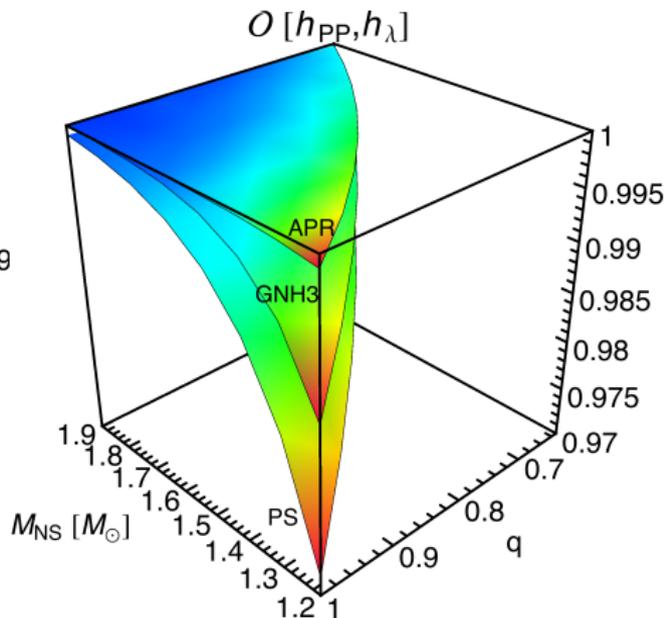
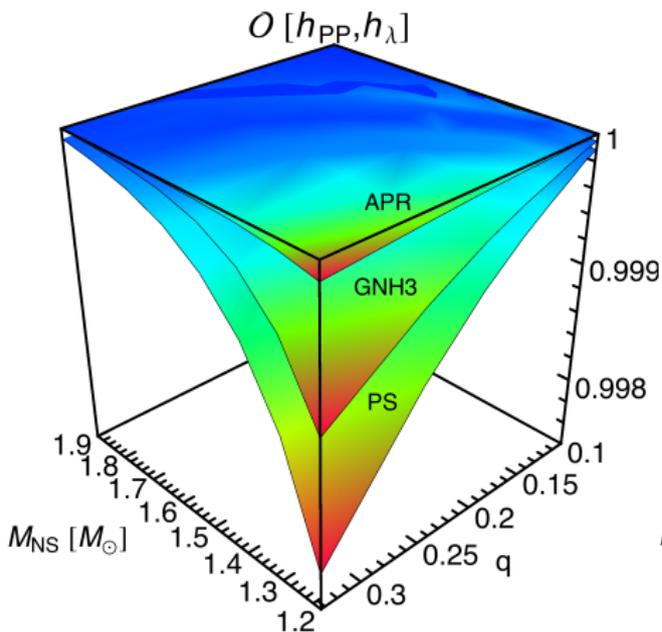
Linear in λ up to relative 1PN order



Vines, Flanagan, Hinderer, PRD **83**, 084051 (2011)

Overlap with Advanced LIGO/Virgo

Frequency domain; nonspinning BHs and NSs



Overlap with Advanced LIGO/Virgo

Will this affect the detection?

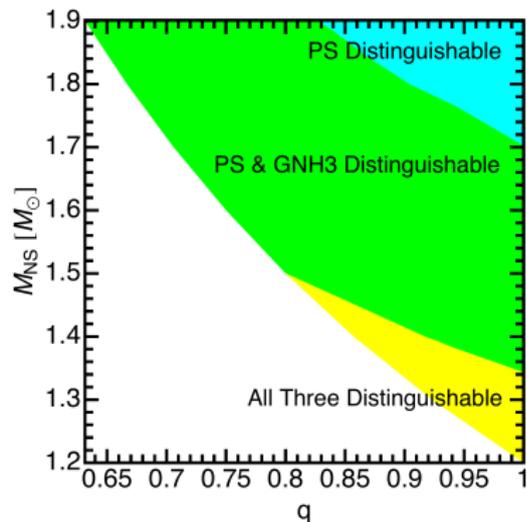
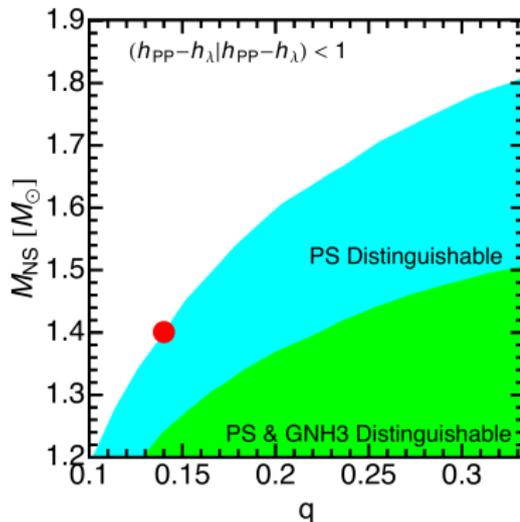
- $\mathcal{O}[h_{\text{PP}}, h_{\lambda}] > 0.997$ (the threshold for a 1% loss of signals)
- BH-BH template banks safe enough for mixed binary inspiral searches

Will this affect the detection?

- $\mathcal{O}[h_{\text{PP}}, h_{\lambda}] > 0.965$ (the threshold for a 10% loss of signals)
- BH-BH template banks safe enough for binary NS inspiral searches

Measurability with Advanced LIGO/Virgo

Sky location and binary orientation averaged; nonspinning BHs and NSs; sources at 100 Mpc



Measurability with Advanced LIGO/Virgo

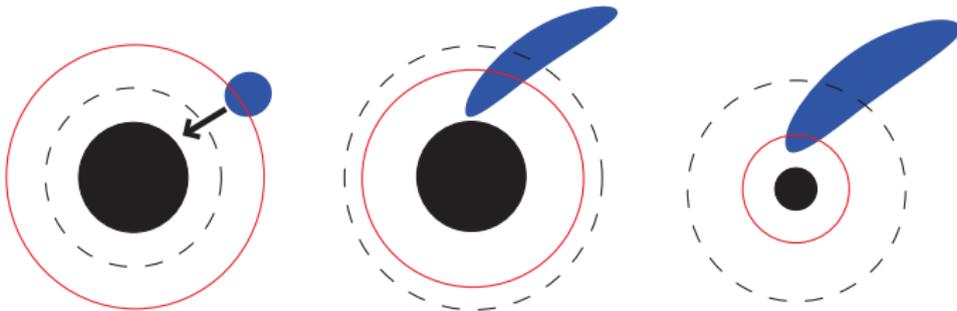
Will the EOS be measurable?

- Stiff EsOS are marginally distinguishable to distinguishable in a region of the parameter space
- The PS/GNH3 EOS for $1.2M_{\odot} + 4.2M_{\odot}$ BH-NS binaries is (marginally) distinguishable $\lesssim 200$ Mpc/300 Mpc

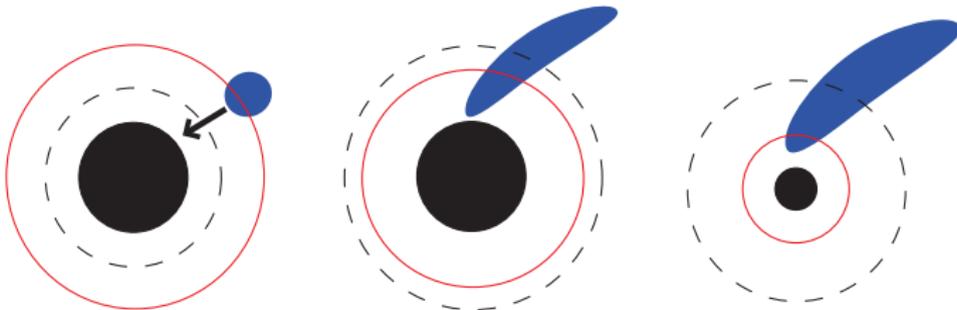
Will the EOS be measurable?

- Tidal effects for stiff and moderately stiff EsOS reach very high distinguishabilities
- There are chances of measuring tidal effects for NSs with small deformabilities, i.e. of distinguishing also a soft EOS like APR

BH-NS Phenomenology



BH-NS Phenomenology



Foucart, PRD **86**, 124007 (2012)

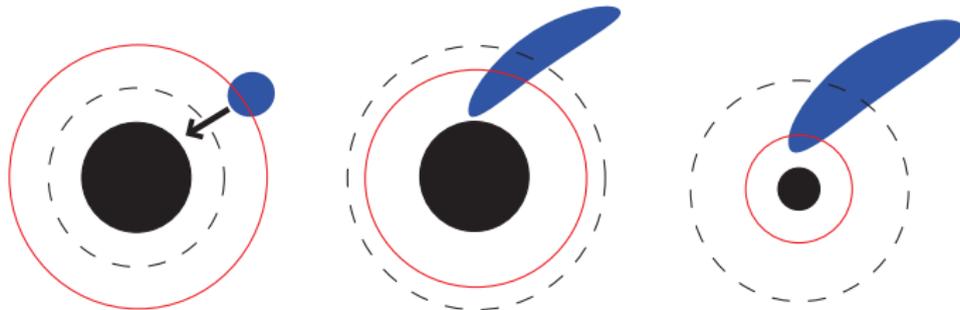
$$\frac{M_{b,\text{torus}}}{M_{b,\text{NS}}} = 0.296 \frac{r_{\text{tide}}}{R_{\text{NS}}} - 0.171 \frac{r_{\text{ISCO}}}{R_{\text{NS}}}$$

⇒ What are the spin and mass of the BH remnant?



FP, PRD **88**, 104025 (2013); FP, arXiv:1311.5931

BH-NS Phenomenology



Foucart, PRD **86**, 124007 (2012)

$$\frac{M_{b,\text{torus}}}{M_{b,\text{NS}}} = 0.296 \frac{r_{\text{tide}}}{R_{\text{NS}}} - 0.171 \frac{r_{\text{ISCO}}}{R_{\text{NS}}}$$

⇒ What are the spin and mass of the BH remnant?



FP, PRD **88**, 104025 (2013); FP, arXiv:1311.5931

- The tidal disruption frequency depends on the EOS

⇒ Can we build phenomenological GWforms?



FP, Berti, Kyutoku, Shibata, PRD **88**, 084011 (2013)

Extending the BKL Approach

Buonanno, Kidder, Lehner, PRD **77**, 026004 (2008)

$$S_f = S_1 + S_2 + L_{\text{orb}} - J_{\text{diss}}$$

Extending the BKL Approach

Buonanno, Kidder, Lehner, PRD **77**, 026004 (2008)

$$S_f \simeq S_1 + S_2 + L_{z, \text{ISCO}}$$

Extending the BKL Approach

Buonanno, Kidder, Lehner, PRD **77**, 026004 (2008)

$$S_f \simeq S_1 + S_2 + L_{z,\text{ISCO},f}$$

Extending the BKL Approach

Buonanno, Kidder, Lehner, PRD **77**, 026004 (2008)

$$a_f = \frac{a_1 M_1^2 + a_2 M_2^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_1 M_2}{(M_1 + M_2)^2}$$

Extending the BKL Approach

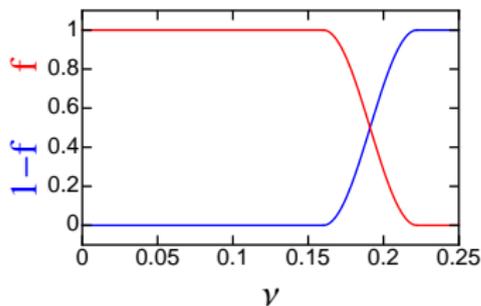
Buonanno, Kidder, Lehner, PRD **77**, 026004 (2008)

$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{BH}} M_{\text{NS}}}{(M_{\text{BH}} + M_{\text{NS}})^2}$$

Extending the BKL Approach

Modify for: possible full tidal disruption

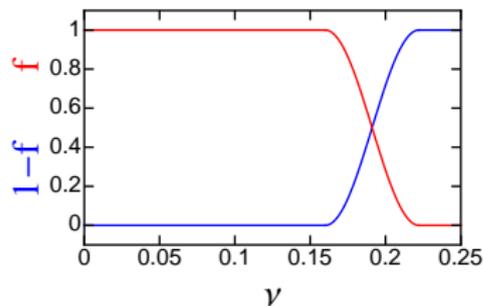
$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{\eta}_{\text{ISCO},f}, a_f) M_{\text{BH}} \{f(\nu) M_{\text{NS}} + [1 - f(\nu)] M_{\text{b,NS}}\}}{(M_{\text{BH}} + M_{\text{NS}})^2}$$



Extending the BKL Approach

Modify for: possible full tidal disruption, torus formation

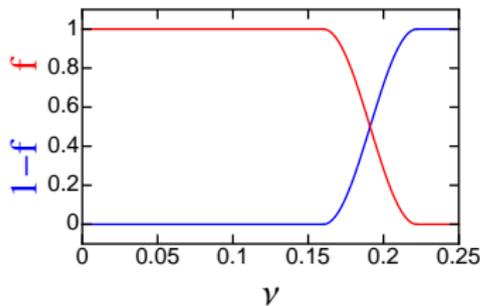
$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{BH}} \{f(\nu) M_{\text{NS}} + [1 - f(\nu)] M_{\text{b,NS}}\} - L_{z,\text{Torus}}}{(M - E_{\text{orb,Torus}})^2}$$



Extending the BKL Approach

Modify for: possible full tidal disruption, torus formation

$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{BH}} \{f(\nu) M_{\text{NS}} + [1 - f(\nu)] M_{\text{b,NS}} - M_{\text{b,torus}}\}}{[M - e(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{b,torus}}]^2}$$



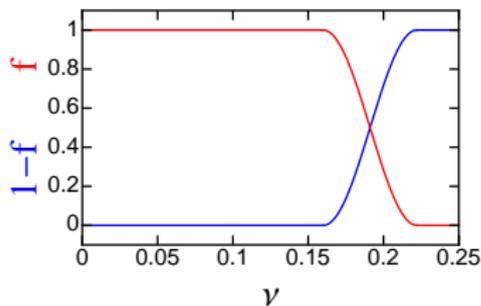
Linear fit for the torus mass

$$\frac{M_{\text{b,torus}}}{M_{\text{b,NS}}} = 0.296 \frac{r_{\text{tide}}}{R_{\text{NS}}} - 0.171 \frac{r_{\text{ISCO}}}{R_{\text{NS}}}$$

Extending the BKL Approach

Modify for: possible full tidal disruption, torus formation, GW emission

$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{BH}} \{f(\nu) M_{\text{NS}} + [1 - f(\nu)] M_{\text{b,NS}} - M_{\text{b,torus}}\}}{[M \{1 - [1 - e(\bar{r}_{\text{ISCO},i}, a_i)] \nu\} - e(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{b,torus}}]^2}$$



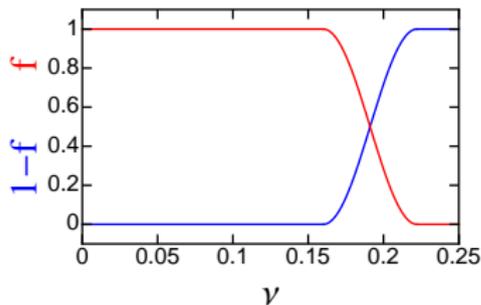
Linear fit for the torus mass

$$\frac{M_{\text{b,torus}}}{M_{\text{b,NS}}} = 0.296 \frac{r_{\text{tide}}}{R_{\text{NS}}} - 0.171 \frac{r_{\text{ISCO}}}{R_{\text{NS}}}$$

Extending the BKL Approach

Modify for: possible full tidal disruption, torus formation, GW emission

$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{BH}} \{f(\nu) M_{\text{NS}} + [1 - f(\nu)] M_{\text{b,NS}} - M_{\text{b,torus}}\}}{[M \{1 - [1 - e(\bar{r}_{\text{ISCO},i}, a_i)] \nu\} - e(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{b,torus}}]^2}$$



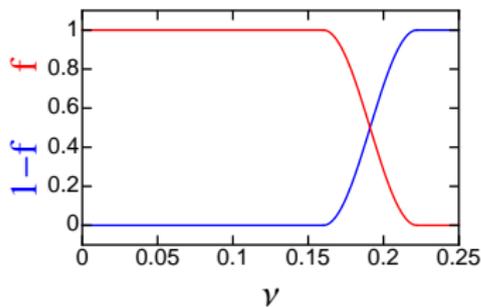
Linear fit for the torus mass

$$\frac{M_{\text{b,torus}}}{M_{\text{b,NS}}} = 0.296 \frac{r_{\text{tide}}}{R_{\text{NS}}} - 0.171 \frac{r_{\text{ISCO}}}{R_{\text{NS}}}$$

Extending the BKL Approach

Modify for: possible full tidal disruption, torus formation, GW emission

$$a_f = \frac{a_i M_{\text{BH}}^2 + \ell_z(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{BH}} \{f(\nu) M_{\text{NS}} + [1 - f(\nu)] M_{\text{b,NS}} - M_{\text{b,torus}}\}}{[M \{1 - [1 - e(\bar{r}_{\text{ISCO},i}, a_i)] \nu\} - e(\bar{r}_{\text{ISCO},f}, a_f) M_{\text{b,torus}}]^2}$$



Linear fit for the torus mass

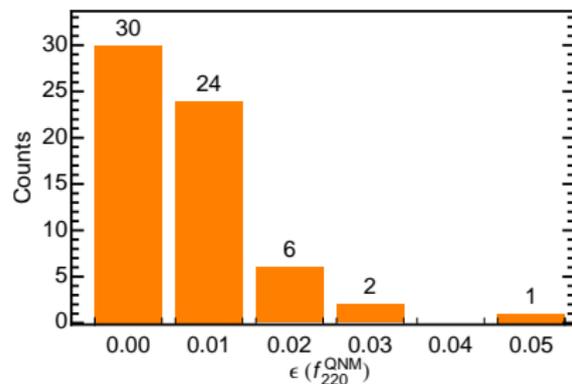
$$\frac{M_{\text{b,torus}}}{M_{\text{b,NS}}} = 0.296 \frac{r_{\text{tide}}}{R_{\text{NS}}} - 0.171 \frac{r_{\text{ISCO}}}{R_{\text{NS}}}$$

Can be extended to misaligned BH spin

Tests Against Numerical-Relativity Results

74 simulations with aligned BH spin

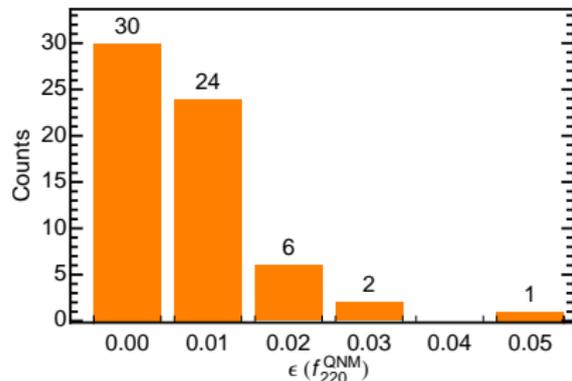
- $|a_f - a_f^{\text{NR}}| \leq 0.02$ in 72/74 cases
- $\epsilon(M_f) \leq 2\%$ [$\leq 1\%$ in 56/63 cases]
- $\epsilon(M_{\text{irr}}) \leq 2\%$ [$\leq 1\%$ in 62/63 cases]



Tests Against Numerical-Relativity Results

74 simulations with aligned BH spin

- $|a_f - a_f^{\text{NR}}| \leq 0.02$ in 72/74 cases
- $\epsilon(M_f) \leq 2\%$ [$\leq 1\%$ in 56/63 cases]
- $\epsilon(M_{\text{irr}}) \leq 2\%$ [$\leq 1\%$ in 62/63 cases]



7 simulations with misaligned BH spin

($M_{\text{NS}}/R_{\text{NS}} = 0.144$)

- $\epsilon(M_f) \leq 0.02$ [≤ 0.01 in 6/7 cases]

Q	a_i	θ_i	a_f^{NR}	a_f^{PN}	a_f
7	0.9	20°	0.91	0.91	0.93
7	0.9	40°	0.90	0.90	0.91
7	0.9	60°	0.86	0.87	0.87
3	0.0	0°	0.56	0.54	0.55
3	0.5	0°	0.77	0.70	0.76
3	0.9	0°	0.93	0.83	0.94
3	0.5	20°	0.76	0.70	0.75
3	0.5	40°	0.74	0.69	0.73
3	0.5	60°	0.71	0.67	0.69
3	0.5	80°	0.66	0.64	0.64

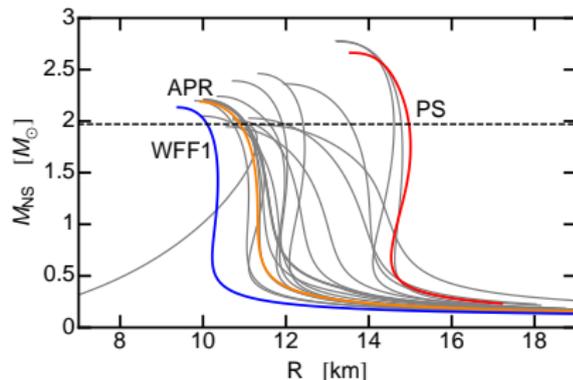


Stone, Loeb, Berger, PRD **87**, 084053 (2013)

Results

Systematically explore the BH-NS space of parameters:

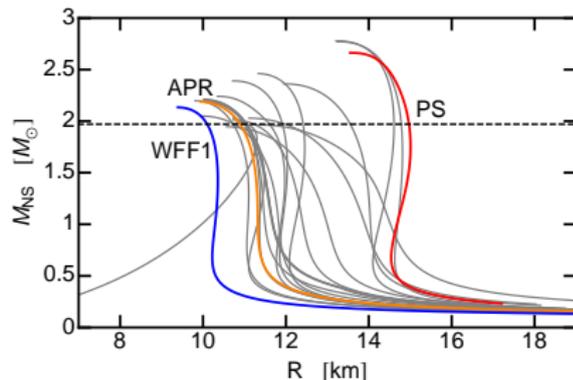
- $2 \leq Q \leq 10$
- $0 \leq a_i \leq 0.99$
- $0^\circ \leq \theta_i \leq 180^\circ$
- $1.2M_\odot \leq M_{\text{NS}} \leq 2.0M_\odot$



Results

Systematically explore the BH-NS space of parameters:

- $2 \leq Q \leq 10$
- $0 \leq a_i \leq 0.99$
- $0^\circ \leq \theta_i \leq 180^\circ$
- $1.2M_\odot \leq M_{\text{NS}} \leq 2.0M_\odot$

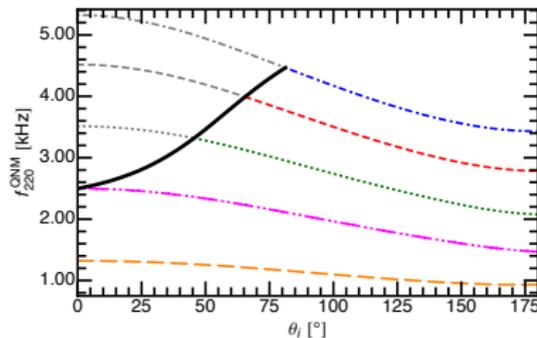
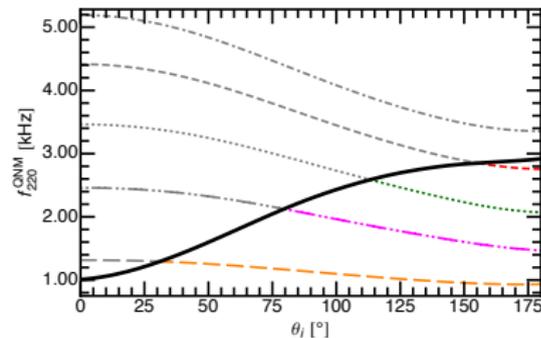
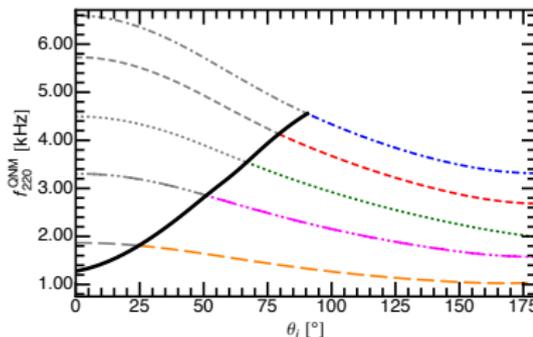
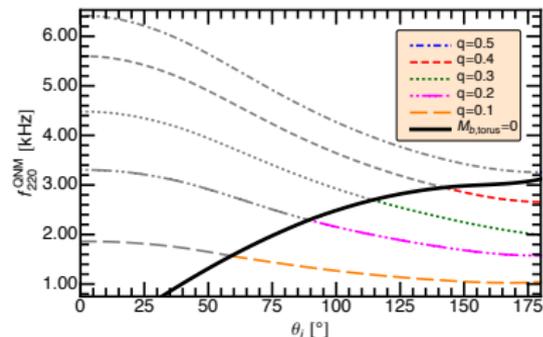


- The softer EOS, the higher a_f
- Indirect support to the *Cosmic censorship conjecture*: no overspinning BHs ($a_f > 1$) are formed
- $\max a_f = 0.997$: to be compared with the 0.998 limit of

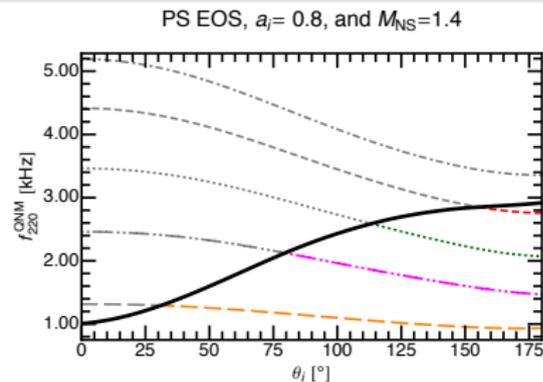
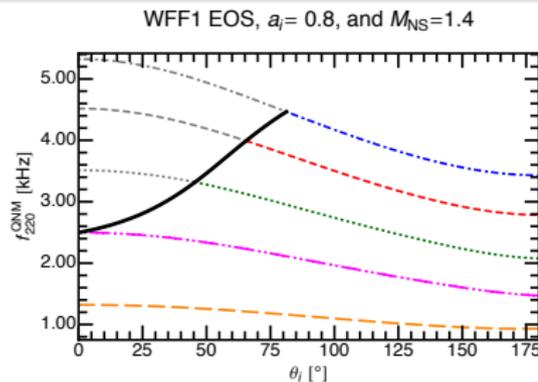


Thorne, ApJ **191**, 507 (1974)

Results: Black Hole Remnant Ringdown

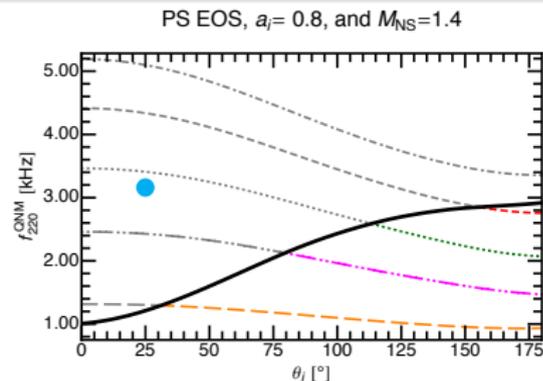
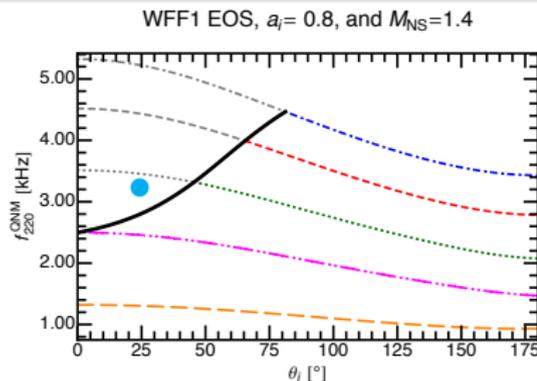
WFF1 EOS, $a_i = 0.8$, and $M_{\text{NS}} = 1.4$ PS EOS, $a_i = 0.8$, and $M_{\text{NS}} = 1.4$ WFF1 EOS, $a_i = 0.99$, and $M_{\text{NS}} = 1.4$ PS EOS, $a_i = 0.99$, and $M_{\text{NS}} = 1.4$ 

Results: Black Hole Remnant Ringdown



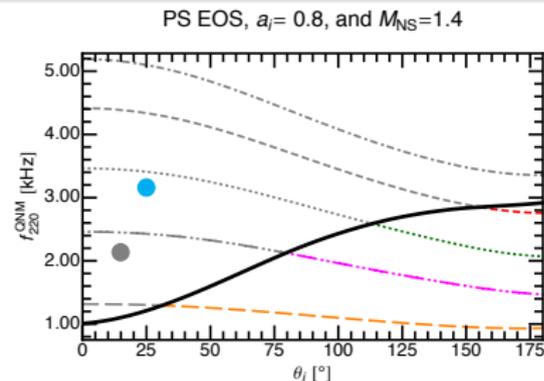
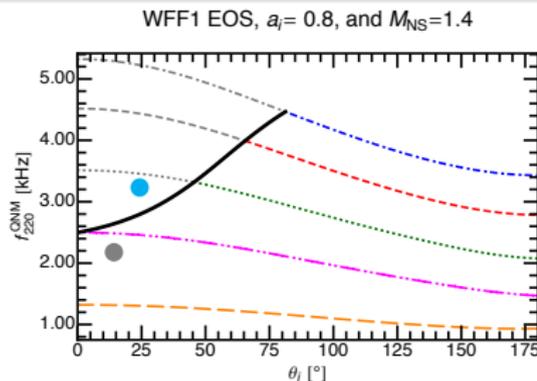
- BH-BH IMR vs. BH-NS templates

Results: Black Hole Remnant Ringdown



- BH-BH IMR vs. BH-NS templates
- BH-BH vs. BH-NS source

Results: Black Hole Remnant Ringdown



- BH-BH IMR vs. BH-NS templates
- BH-BH vs. BH-NS source
- Constraints on the NS EOS from repeated detections

BH-NS Gravitational Waveform Amplitude Model

- BH binaries have a history of Phenom GW modelling
 - The number of BH-NS simulations is increasing
- ⇒ Quick inspiral-merger-ringdown model: accurate phenomenology, SNRs, and cutoff frequencies



FP, Berti, Kyutoku, Shibata, PRD **88**, 084011 (2013)

BH-NS Gravitational Waveform Amplitude Model

- BH binaries have a history of Phenom GW modelling
 - The number of BH-NS simulations is increasing
- ⇒ Quick inspiral-merger-ringdown model: accurate phenomenology, SNRs, and cutoff frequencies



FP, Berti, Kyutoku, Shibata, PRD **88**, 084011 (2013)

38 numerical simulations

- SACRA code
- $Q \in [2, 5]$
- 2-piecewise polytrope EOS
- $M_{\text{NS}} \in \{1.2M_{\odot}, 1.35M_{\odot}\}$
- $\mathcal{C} \in [0.131, 0.194]$
- $\Gamma_{\text{core}} \in \{2.4, 2.7, 3.0, 3.3\}$

2 groups of runs

- 14 runs to build the model:
 - $\Gamma_{\text{core}} = 3.0$
 - $\text{EOS} \in \{2\text{H}, \text{H}, \text{HB}, \text{B}\}$
 - $M_{\text{NS}} = 1.35M_{\odot}$
- 24 runs as test cases

BH-NS Gravitational Waveform Amplitude Model

Starting tools

- Foucart's fit: tells us the torus mass ($M_{b,\text{torus}}$) and NS tidal disruption frequency (f_{tide})
- Model for the mass and spin of the BH remnant: we can compute the ringdown frequency (f_{RD})

Strategy

Exploit $M_{b,\text{torus}}$, f_{tide} , f_{RD} (and the initial physical parameters) to extend the PhenomC BH-BH gravitational waveform model

PhenomC Amplitude

Pure PN inspiral + a higher order term + ringdown Lorentzian

BH-NS Gravitational Waveform Amplitude Model

Tidal disruption

None: $f_{\text{tide}} > f_{\text{RD}}$

Mild: $f_{\text{RD}} \geq f_{\text{tide}}$ and $M_{\text{b,torus}} = 0$

Strong: $f_{\text{RD}} \geq f_{\text{tide}}$ and $M_{\text{b,torus}} > 0$

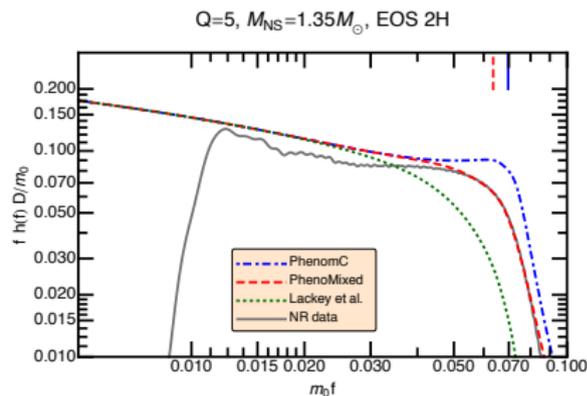
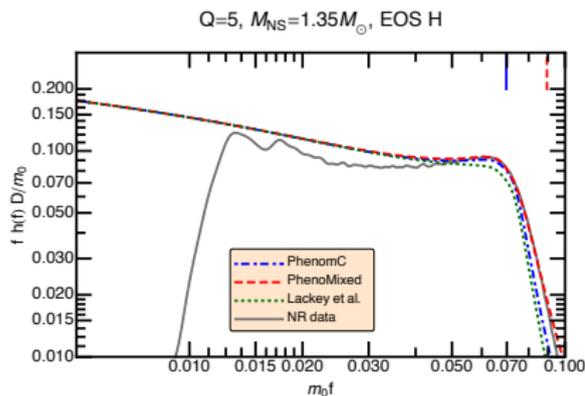
BH-NS Gravitational Waveform Amplitude Model

Tidal disruption

None: $f_{\text{tide}} > f_{\text{RD}}$

Mild: $f_{\text{RD}} \geq f_{\text{tide}}$ and $M_{\text{b,torus}} = 0$

Strong: $f_{\text{RD}} \geq f_{\text{tide}}$ and $M_{\text{b,torus}} > 0$

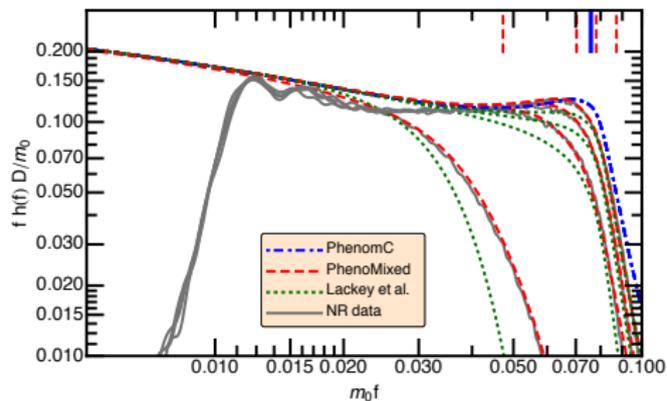


Lackey, Kyutoku, Shibata, Brady, Friedman, arXiv:1303.6298

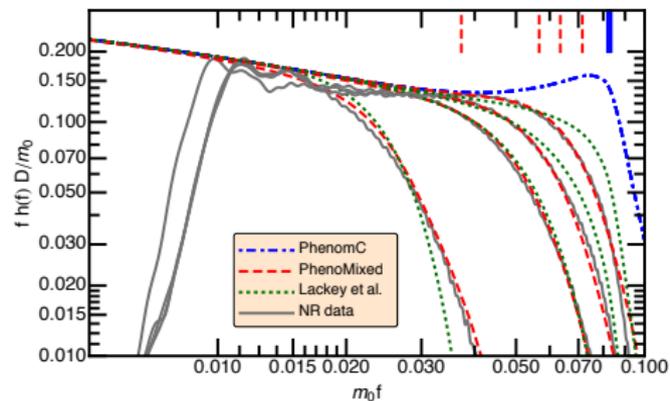


BH-NS Gravitational Waveform Amplitude Model

$Q=3, M_{NS}=1.35M_{\odot}$



$Q=2, M_{NS}=1.35M_{\odot}$



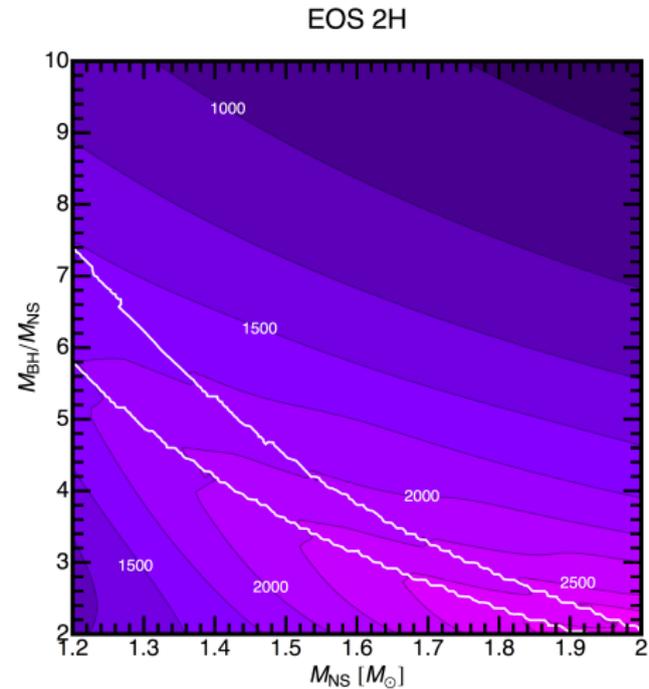
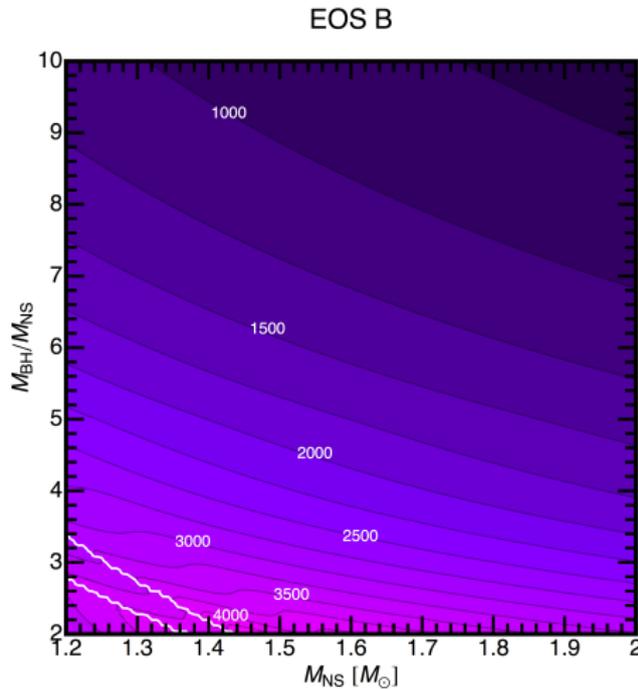
BH-NS Gravitational Waveform Amplitude Model

- Detectors: AdvLIGO, AdvZDHP, AdvVirgo, KAGRA, varBRSE, maxBRSE, and varDRSE, ET B and D

$$\left(1 - \frac{\rho_{\text{BHNS}}}{\rho_{\text{NR}}}\right) - \left(1 - \frac{\rho_{\text{BHBH}}}{\rho_{\text{NR}}}\right) = \frac{\rho_{\text{BHBH}} - \rho_{\text{BHNS}}}{\rho_{\text{NR}}}$$

- EOS2HQ2M12 and AdvZDHP LIGO yield $\sim 18\%$
- For Lackey+: 6%
- $|1 - \rho_{\text{RPN}}/\rho_{\text{BHNS}}|$ up to $\sim 10\%$
- $|1 - \rho_{\text{BHBH}}/\rho_{\text{BHNS}}|$ and $|1 - \rho_{\text{BHNS}}^{\text{Lackey+}}/\rho_{\text{BHNS}}| \lesssim 1\%$

Gravitational Wave Cutoff Frequency



Non-Universality of I-Love-Q Relations

- Slowly rotating unmagnetized NSs: quadrupole moment (Q), moment of inertia (I), and tidal Love number (λ) are in unique relations
- They are “universal”: EOS-independent
- Could be used to break degeneracies between parameters in GW signals



Yagi, Yunes, *Science* **341**, 365 (2013)

Non-Universality of I-Love-Q Relations

- **Slowly** rotating **unmagnetized** NSs: quadrupole moment (Q), moment of inertia (I), and tidal Love number (λ) are in unique relations
- They are “universal”: EOS-independent
- Could be used to break degeneracies between parameters in GW signals



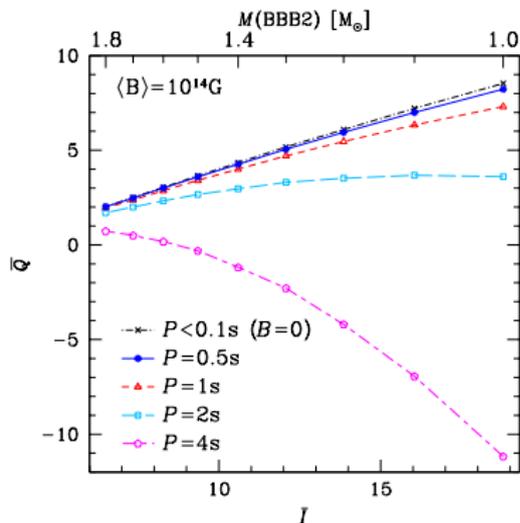
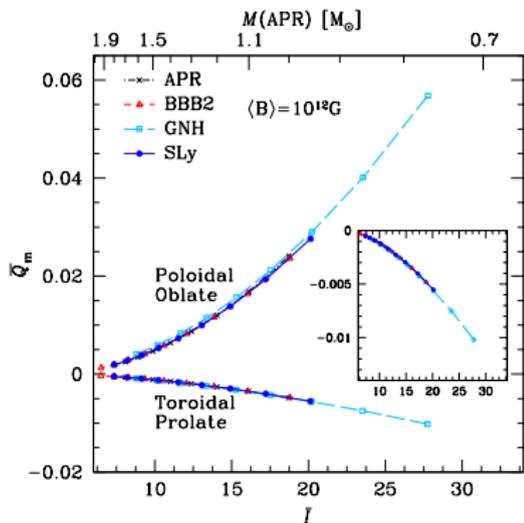
Yagi, Yunes, *Science* **341**, 365 (2013)

⇒ What happens in strongly magnetized NSs?



Haskell, Ciolfi, FP, Rezzolla, to appear in *MNRAS* (2013)

Non-Universality of I-Love-Q Relations

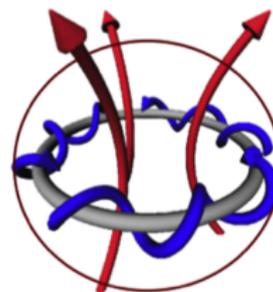
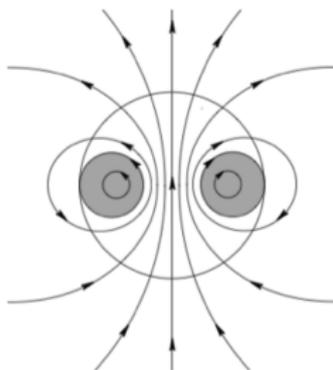


[LORENE]

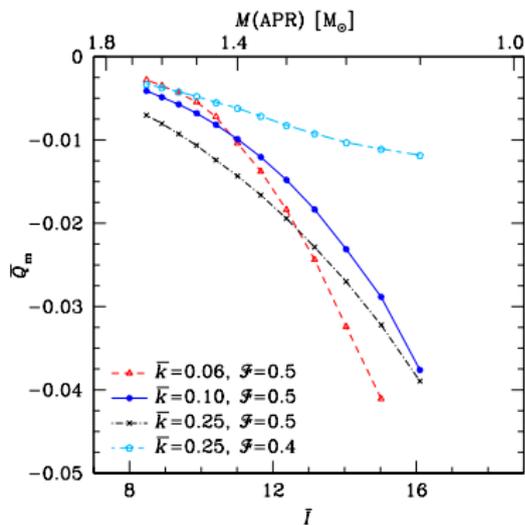
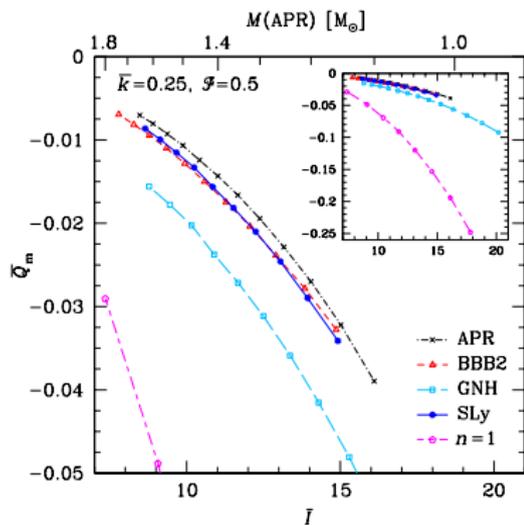
- Simple magnetic field configuration \Rightarrow weak EOS influence
- Different field geometries \Rightarrow different $I-Q$ relations
- Magnetic deformations dominate \Rightarrow universality broken

Non-Universality of I-Love-Q Relations

Twisted-torus configuration



Non-Universality of I-Love-Q Relations



[Perturbative code]

- Twisted-torus configuration \Rightarrow EOS-dependent I - Q relation

Non-Universality of I-Love-Q Relations

- The “universal” $I - \lambda - Q$ relations are not applicable to highly magnetized, slowly spinning NSs
- Any inferred parameter will not be reliable unless it is known that $B \lesssim 10^{12} \text{G}$ $P \lesssim 10 \text{s}$
- Pulsar B in PSR J0737-3039: $B_p \simeq 1.2 \times 10^{12} \text{G}$ and $P_{\text{merger}} \sim 4 \text{s}$. Possibly, $\langle B_{\text{internal}} \rangle \sim 10^{14} \text{G}$: the quadrupole of this star may deviate significantly from that of an unmagnetized rotating star, and the NS could even be prolate!
- Tests of GR: unobserved strong interior magnetic field component may cause deviations from the expected trend
- Can we constrain the internal magnetic field structure via deviations from the “universal” $I - \lambda - Q$?

Summary

- Microphysics has a dramatic impact on the properties and evolution of NSs
- (How) can we probe NS interiors via CBC GW observations? Something we cannot do otherwise!
- Inspiral-merger-ringdown: all three stages have a lot to tell us
- EOS signatures are stronger in NS-NS binaries, but cleaner in BH-NS mergers
- BH-BH tools may be adapted more easily to the BH-NS case
- Magnetic fields yield exciting counterparts, but complicate even the simplest scenario: we must be aware of this