Understanding Precession Effects in Inspiral Waveforms

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Plan

- I mostly work with post-Newtonian, inspiral-only precessing waveforms
- Have some neat analytic results, and a good understanding of what's going on
- Maybe these insights will be useful in the more difficult IMR and NR problems
- Plus I wanted to visit the awesome people here

What am I working on?

- Working towards an aLIGO NSBH search
- Derived template bank for aligned-spin
- Now testing aligned-spin search
  - Worries about background unfounded
  - Computing issues are the big problem
- Simultaneously working toward a single-spin precession search
  - Analytic amplitude and mismatch predictions
  - Efficient Fourier-domain precessing waveform

Collaborators

- Tito Dal Canton, Alex Nitz – two awesome grad students
- Ian Harry
- Drew Keppel
- Alex Nielsen
- Evan Ochsner
- Frank Ohme
- Richard O'Shaughnessy
- Duncan Brown, Ben Owen, Badri Krishnan

TaylorF2

Simplest inspiral waveform

$$\widetilde{h}(f) = \mathcal{A}\, f^{-7/6} e^{i\Psi(f)}$$

Phase:

$$\Psi(f) = \lambda_0 v^{-5} + \lambda_2 v^{-3} + \lambda_3 v^{-2} + \ldots + \phi_c + 2\pi f t_c$$

plus terms like $v^k \log^\ell v$ at higher order.

Plug in $v = (\pi M f)^{1/3}$ and get a completely frequency-domain waveform.

Time-domain waveforms

- TaylorT4 is the other common approximant
  - Time domain rather than frequency domain
  - ODE for dv/dt gives v(t) and Φ(t)
  - Not analytic, slow
- Easy to add precession in time domain
  - Just evolve **L**, spins, GW emission aligned with instantaneous orbit
  - LIGO calls this SpinTaylorT4

Unfaithfulness and Ineffectualness

- Compare TaylorF2 aligned-spin template
- Precessing SpinTaylorT4 signal
- They don't agree
  - The effect of the precession?
  - Or the difference between the approximants?
- Ajith et.al. 1210.6666 say it's the approximant
- We didn't agree – made SpinTaylorT2

SpinTaylorT2

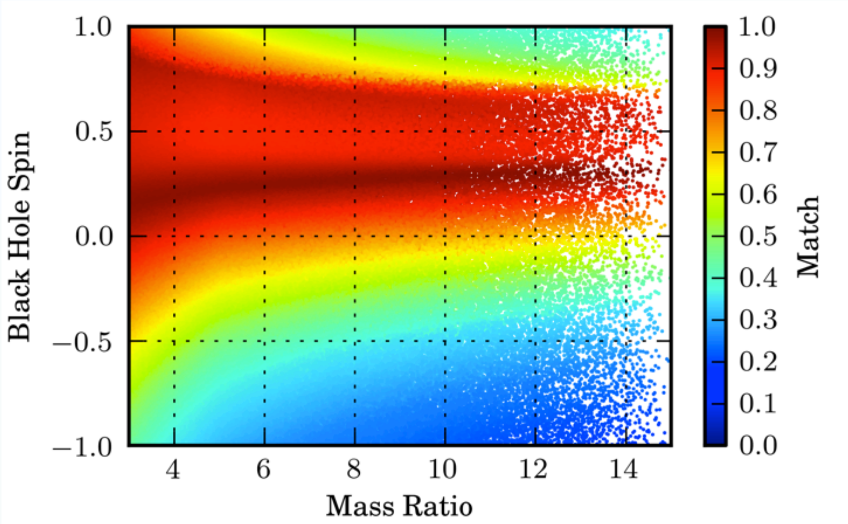
- T4 defined by power series for dv/dt
- F2 defined by power series for dt/dv
  - Direct integration gives t(v)
- So we used

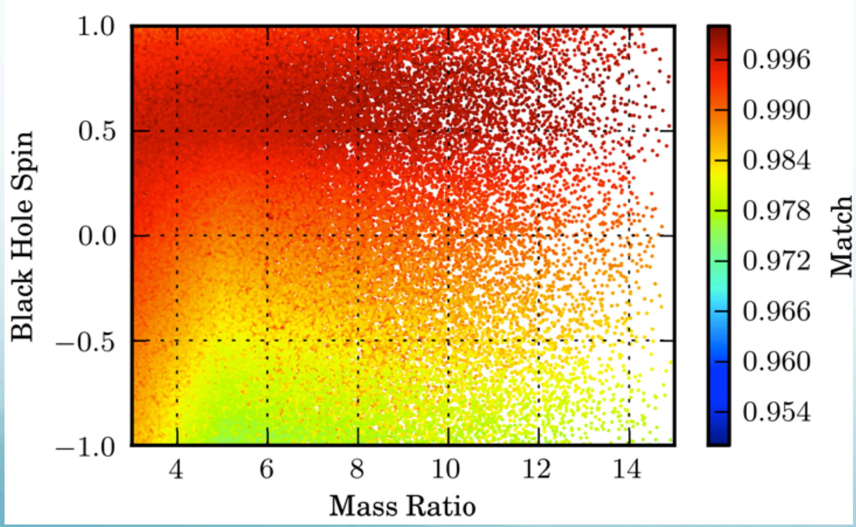
$$\frac{dv}{dt} = 1/\frac{dt}{dv}$$

- For aligned spins, T2 is faithful to F2, but can also do precession

Difference between Approximants

- Match of T4 and F2
  - Good for equal mass and non-spinning
  - Bad elsewhere





- Match of T2 and F2
  - Main mismatch is effect of termination

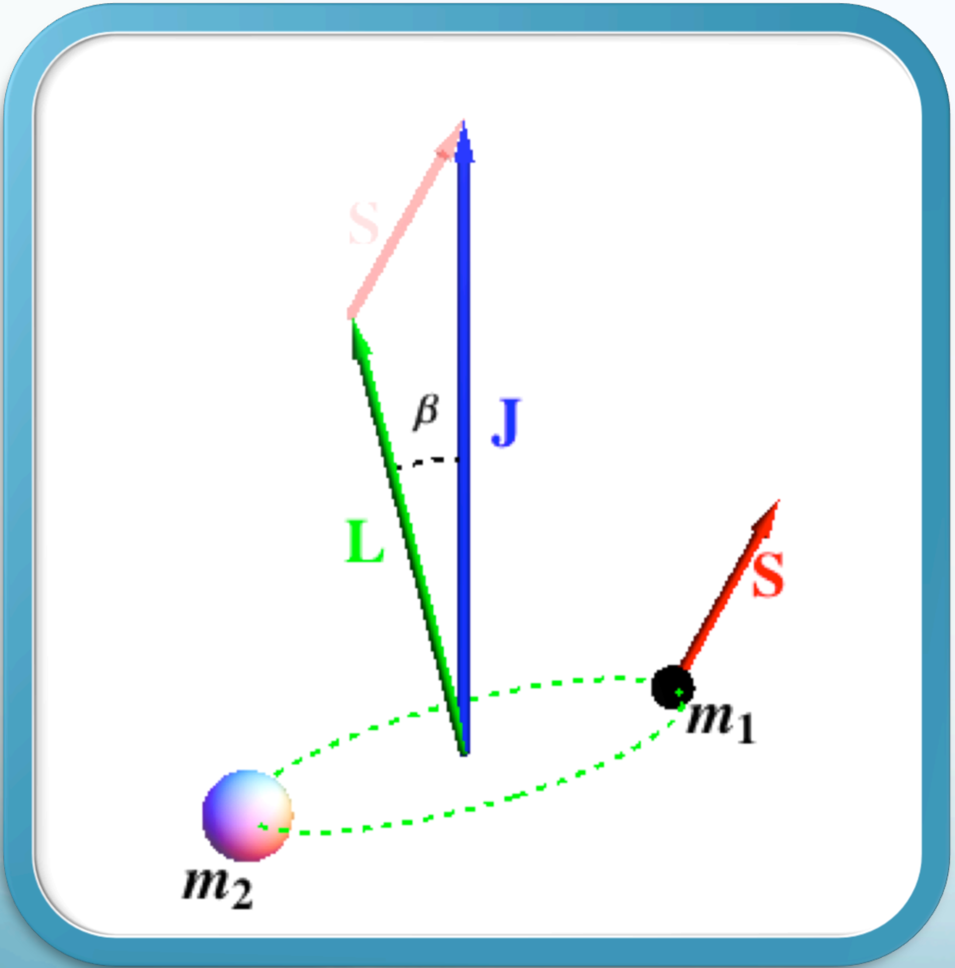
Nitz et.al.

PRD 88, 124039 (2013)
1307.1757

Onward to Precession

- Difference between approximants is important
  - Especially with high spin and low mass ratio
- Precession is a separate issue
- We've got aligned spin under control
  - Template banks for F2 and T4
  - Search background is not scary
- Now let's tackle precession

Dynamics of Spin



Simple precession

- Single spin
- **L** precesses around **J**
- β simple function of frequency
- Assume:
  - Fixed **J** direction

ACST: Apostolatos et.al.

PRD 49, 6274 (1994).

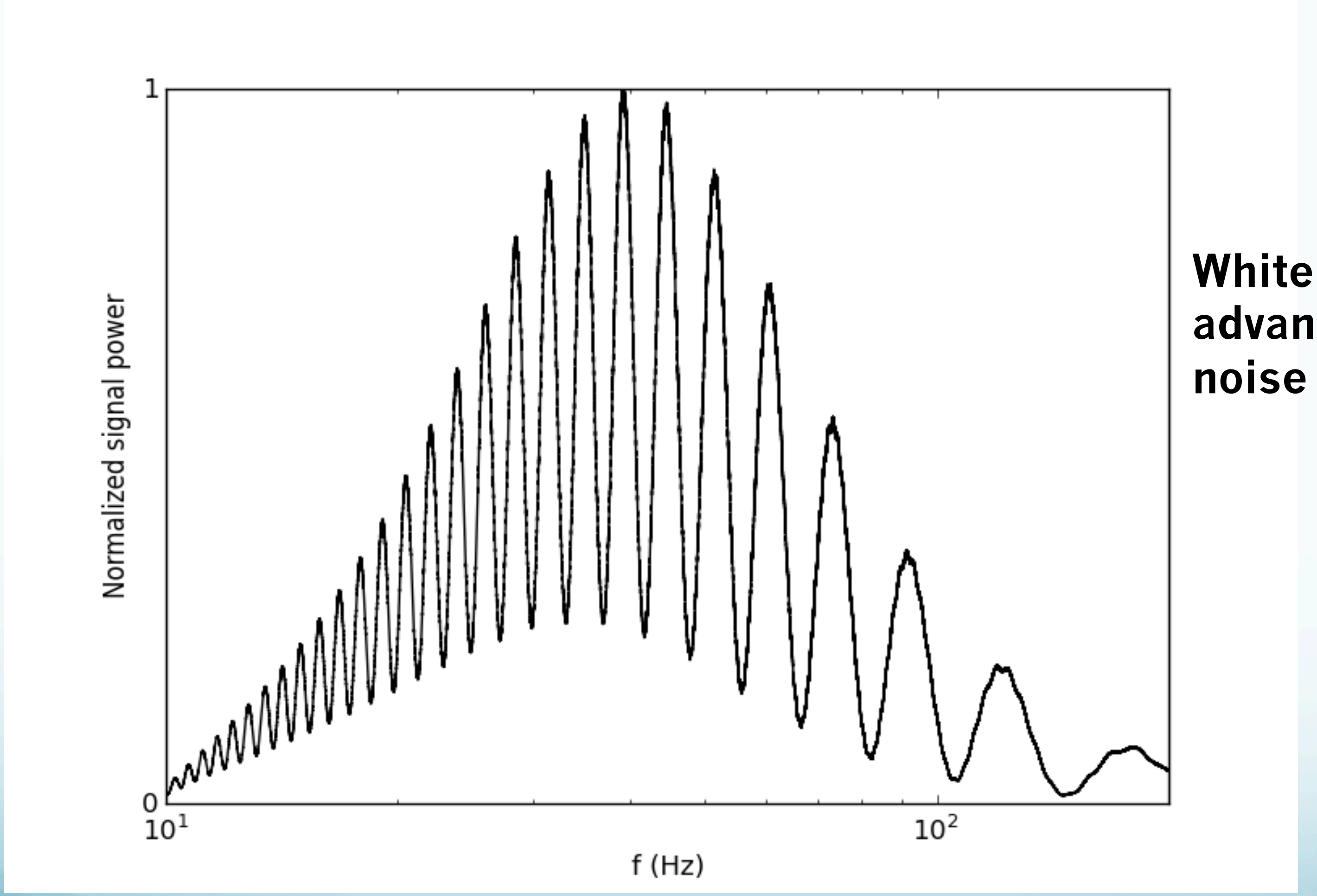
Cone Opening Angle β

$$\gamma = \frac{|\mathbf{S}|}{|\mathbf{L}|} = \frac{\chi_1 m_1^2}{m_1 m_2 (\pi M f)^{-1/3}}$$

$$\kappa = \hat{\mathbf{L}} \cdot \hat{\chi}_1$$

$$\cos\beta = \frac{1 + \kappa\gamma}{\sqrt{1 + 2\kappa\gamma + \gamma^2}}$$

Shape of the Waveform



Whitened with advanced detector noise spectrum

See Buonanno et.al. PRD 67, 104025 (2003).

Separation of Timescales

- Emission is aligned with instantaneous **L**
- **L** precesses in a cone around **J = L + S**
- Orbit is much faster than precession

Brown, Lundgren, O'Shaughnessy PRD 86, 064020 (2012) 1203.6060

Rotation Operator

$$h_{2\,m'}(t) = \sum_{m=-2}^{2} \mathcal{D}_{m',m}(\alpha,\beta,\zeta) h_{2\,m}(t)$$

h(t) on left is precessing, right is aligned

Schmidt et.al. PRD 84, 024046 (2011)

Boyle et.al. PRD 84, 124011 (2011)

Precessing Inspiral

- Write

$$\widetilde{h}(f) = \mathcal{A}(f) \sum_{m=-2}^{2} E_m \, e^{i(m\alpha(f)+2\psi(f)+2\zeta(f))}$$

- Rotation operator decomposes into sum
- Alpha is monotonically increasing
- Do stationary phase approximation for each term in sum

SpinTaylorF2

$$\widetilde{h}(f) = \mathcal{A}(f) \sum_{m=-2}^{2} E_m \, e^{i(m\alpha(f)+2\psi(f)+2\zeta(f))}$$

- E_m depends only on orientation of **J** and on β
- α , β , ζ are closed-form in velocity (or frequency)
  - Annoying: α and β are not pN expansions
- Like a sum of 5 non-precessing waveforms

Lundgren and O'Shaughnessy,

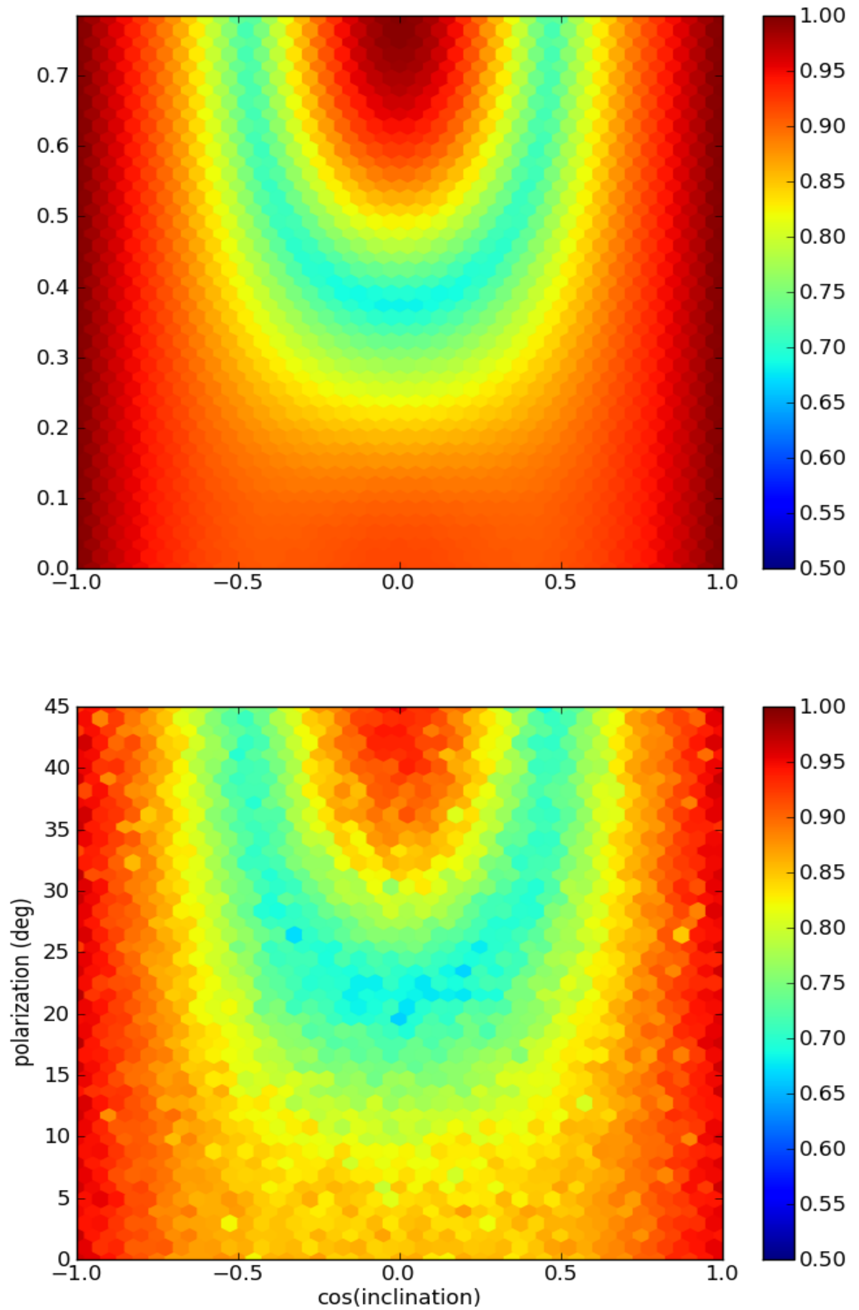
PRD, 1304.3332

What is SpinTaylorF2 for?

- Very fast to compute, so use as templates or for PE
- Everything closed-form and frequency domain, so derive Fisher matrix
  - Template banks, coincidence metric
  - Jump proposals and best variables for PE
- Predict mismatch of aligned-spin search
- Ideas for precessing search

Predicting Mismatch

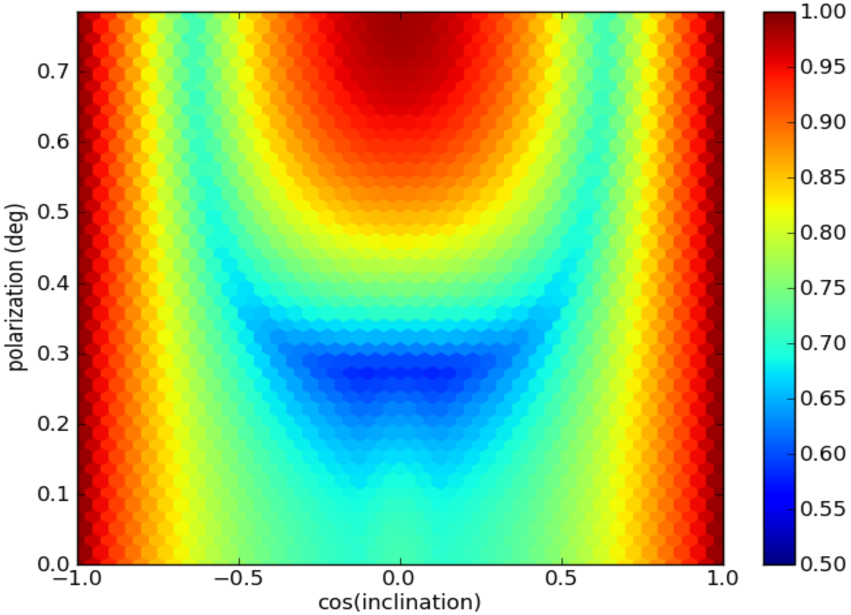
- Give me direction of **J**, and information to calculate β (masses, spin, **L•S**)
- I'll calculate the E_m – the amplitudes of the five 'sidebands'
- Aligned-spin search will lock onto the loudest
- Can also predict amplitude as a function of orientation

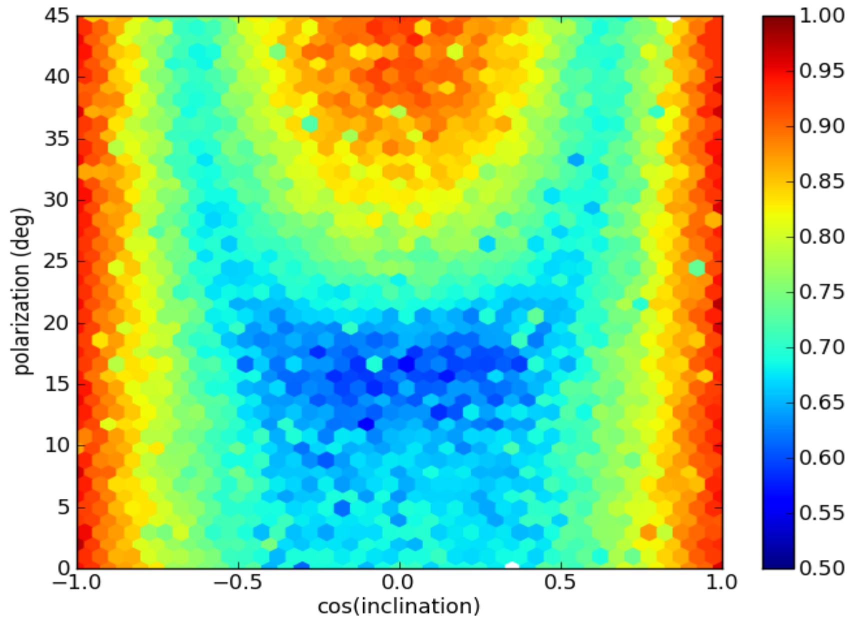


Prediction

Match at $\beta = 30°$

Simulation



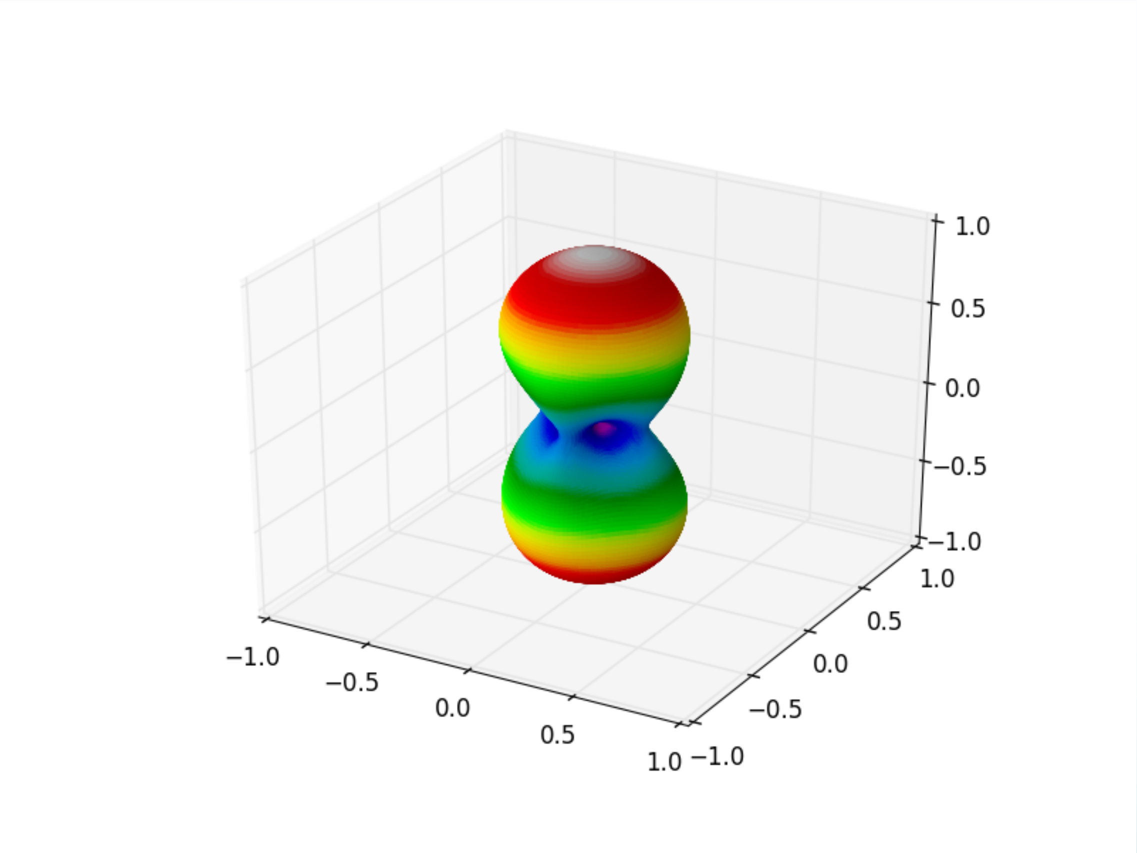


Prediction

Match at
β =45°

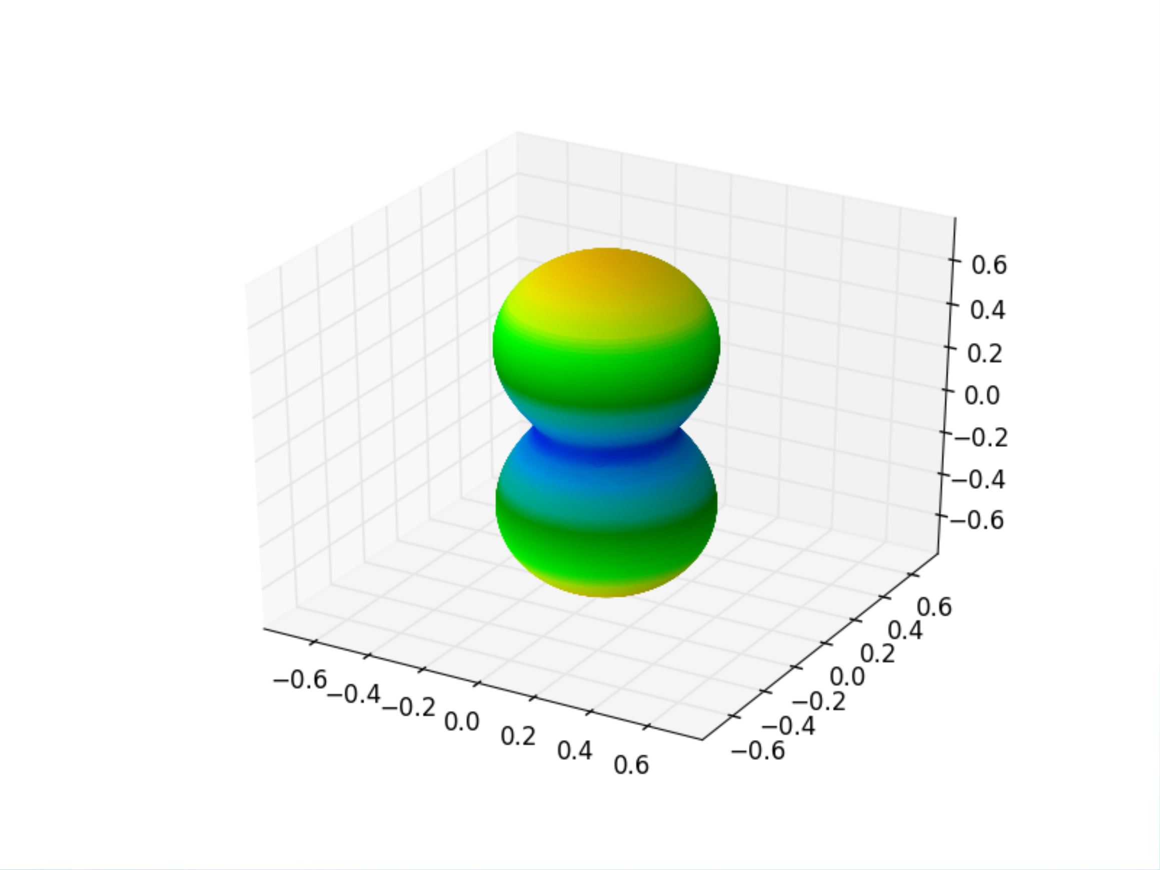
Simulation

Non-Precessing Amplitude



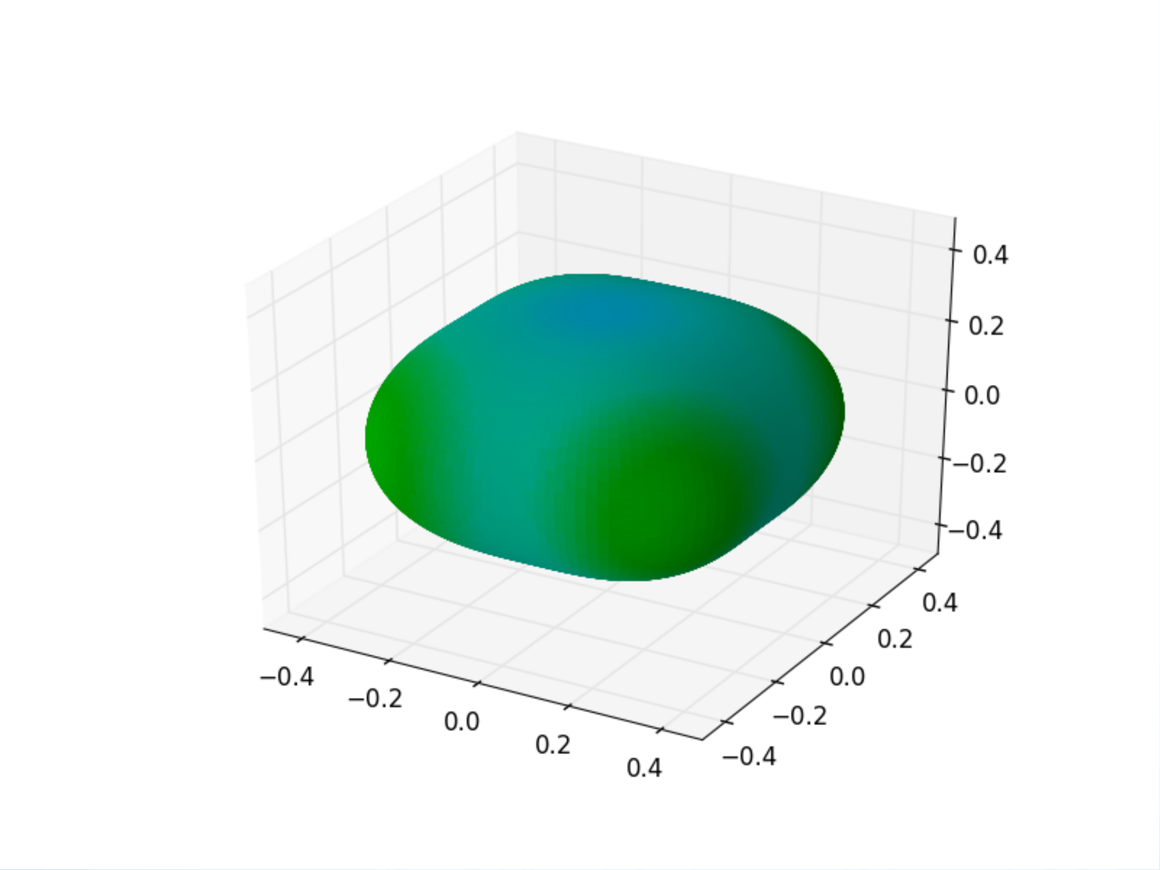
- Z axis toward observer
- Amplitude shown by color scale, distance to origin

Amplitude for 30° cone



- Z axis toward observer
- Amplitude shown by color scale, distance to origin

Amplitude for 60° cone



- Z axis toward observer
- Amplitude shown by color scale, distance to origin

Previous Precessing Searches

- What stops precessing searches from working?
  1. Phenomenological waveforms – didn't deal with non-precessing part well enough
  2. No template bank or coincidence criterion
  3. Wrong priors over orientation?
  4. Computing cost
- We've solved 1 and 2
- Amplitude predictions help with 3
- Moore's Law and parallelization help with 4

Precessing Search Ideas

- Non-precessing template bank, like aligned spin but more phase freedom
  - Pick up each sideband separately
  - Reassemble sidebands – usually only two dominate
- Use analytic Fisher matrix to make a stochastic bank
- Borrow ideas from the pulsar searches

Challenges

- Amplitude prior – configurations (**J** and β) with highest amplitude must be weighted more strongly
- Data quality
  - Real data is always more difficult
  - Go beyond chisq
  - Or get aggressive on the instrument side
- Double spin
- Stretching validity of post-Newtonian
- Also want to add merger and ringdown