Analytical and numerical modeling of precessing binary black holes

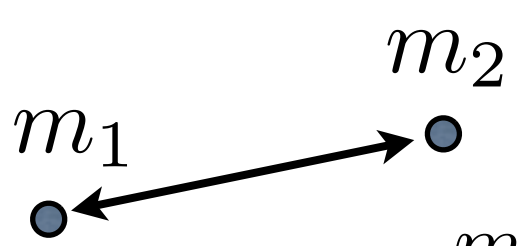
Harald Pfeiffer
Canadian Institute for Theoretical Astrophysics

Physics Seminar, Cardiff University, July 2, 2014

Simulations of Extreme Spacetimes (SXS) collaboration

Buonanno group

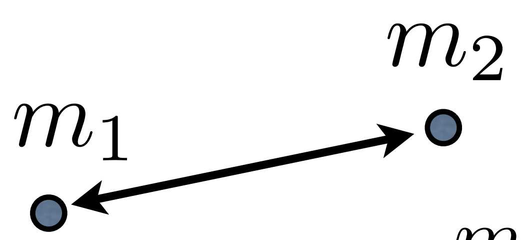
Two Newtonian two-body problem



$$F = \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x_2}|}$$

$$H = \frac{1}{2} m_1 \dot{\mathbf{x}}_1^2 + \frac{1}{2} m_2 \dot{\mathbf{x}}_2^2 - \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

Two Newtonian two-body problem

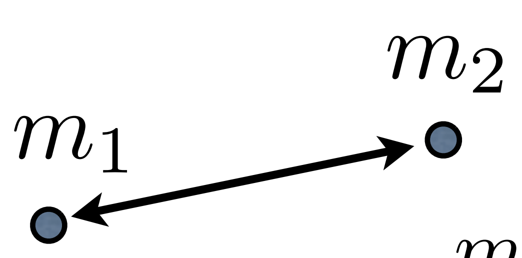


$$F = \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$H = \frac{1}{2} m_1 \dot{\mathbf{x}}_1^2 + \frac{1}{2} m_2 \dot{\mathbf{x}}_2^2 - \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$H = \frac{1}{2} \frac{m_1 m_2}{M} \left(\dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_2\right)^2 + \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|} + \frac{1}{2} M \left(\frac{m_1}{M}\dot{\mathbf{x}}_1 + \frac{m_2}{M}\dot{\mathbf{x}}_2\right)^2$$

Two Newtonian two-body problem



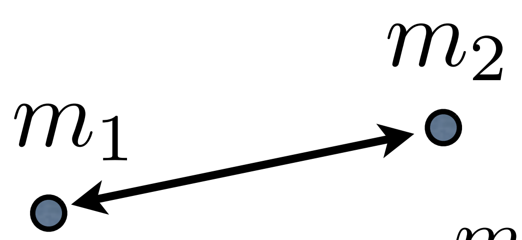
$$F = \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$H = \frac{1}{2} m_1 \dot{\mathbf{x}}_1^2 + \frac{1}{2} m_2 \dot{\mathbf{x}}_2^2 - \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$H = \frac{1}{2}\frac{m_1 m_2}{M}\left(\dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_2\right)^2 + \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|} + \frac{1}{2} M \left(\frac{m_1}{M}\dot{\mathbf{x}}_1 + \frac{m_2}{M}\dot{\mathbf{x}}_2\right)^2$$

Center of mass

Two Newtonian two-body problem



$$F = \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$H = \frac{1}{2} m_1 \dot{\mathbf{x}}_1^2 + \frac{1}{2} m_2 \dot{\mathbf{x}}_2^2 - \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

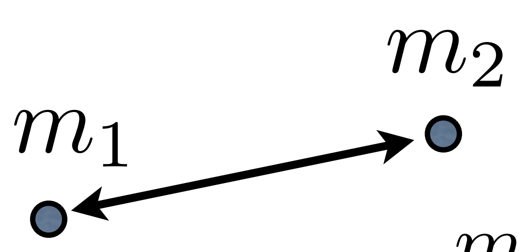
$\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ moves in central potential

$$H = \frac{1}{2} \frac{m_1 m_2}{M} \left(\dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_2\right)^2 + \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$+ \frac{1}{2} M \left(\frac{m_1}{M} \dot{\mathbf{x}}_1 + \frac{m_2}{M} \dot{\mathbf{x}}_2 \right)^2$$

Center of mass

Two Newtonian two-body problem



$$F = \frac{m_1 m_2}{|\mathbf{x_1} - \mathbf{x_2}|}$$

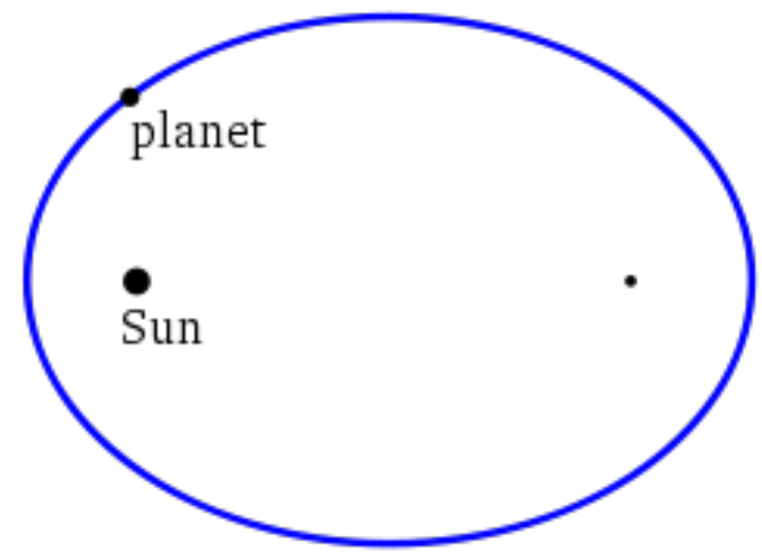
$$H = \frac{1}{2} m_1 \dot{\mathbf{x}}_1^2 + \frac{1}{2} m_2 \dot{\mathbf{x}}_2^2 - \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ moves in central potential

$$H = \frac{1}{2}\frac{m_1 m_2}{M}\left(\dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_2\right)^2 + \frac{m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|} + \frac{1}{2} M \left(\frac{m_1}{M}\dot{\mathbf{x}}_1 + \frac{m_2}{M}\dot{\mathbf{x}}_2\right)^2$$

Center of mass

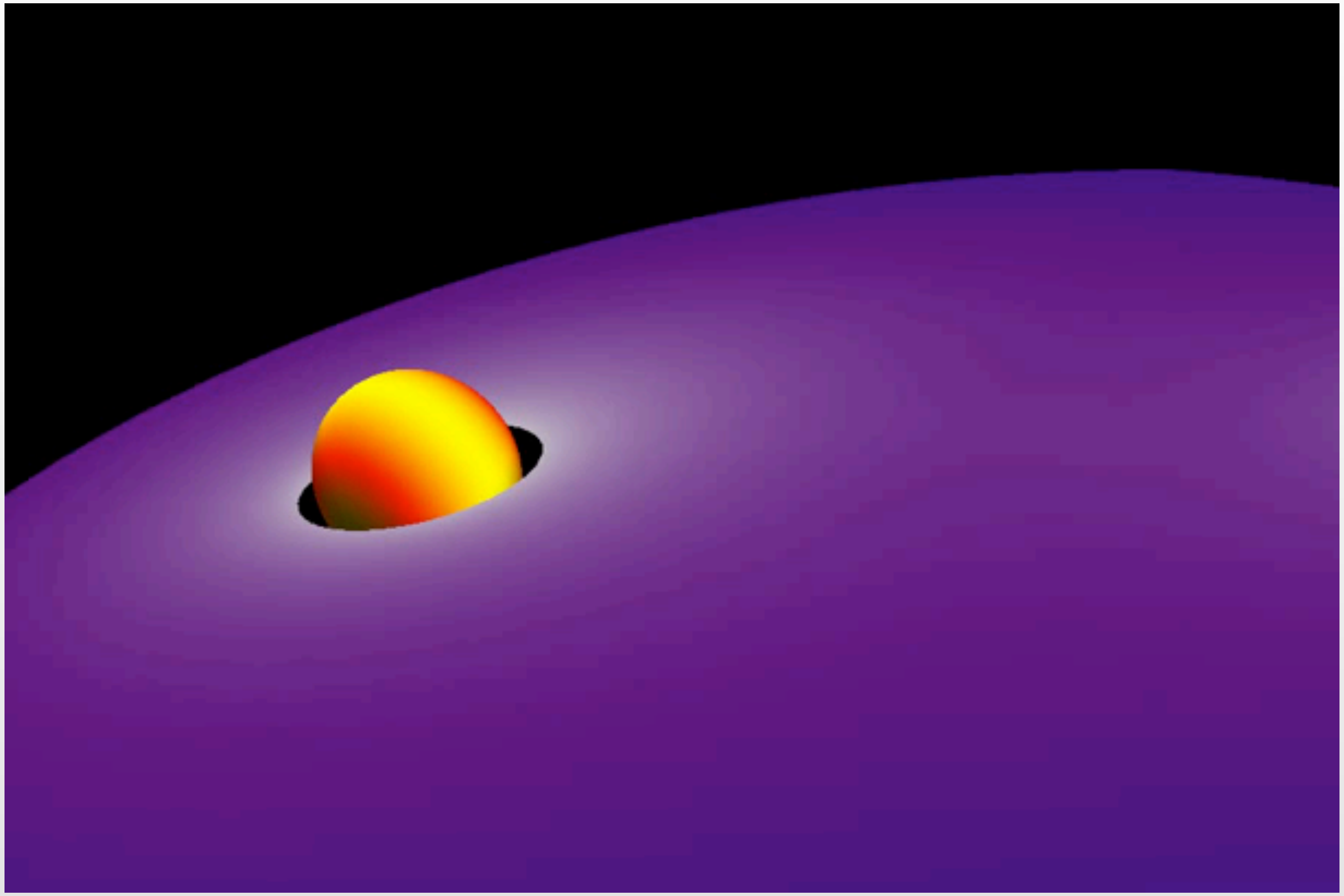
$$\mathbf{r}(\phi) = \frac{p}{1 + \varepsilon \cos\phi}$$



Wednesday, July 2, 14

General relativistic two-body problem

- ❖ Black holes
- ❖ Event horizons
- ❖ Black hole spin
- ❖ Orbit precession
- ❖ Spin precession
- ❖ Periastron advance
- ❖ Gravitational waves
- ❖ Merger
- ❖ Remnant black hole kick
- ❖ Ten 3-dim. partial differential equations

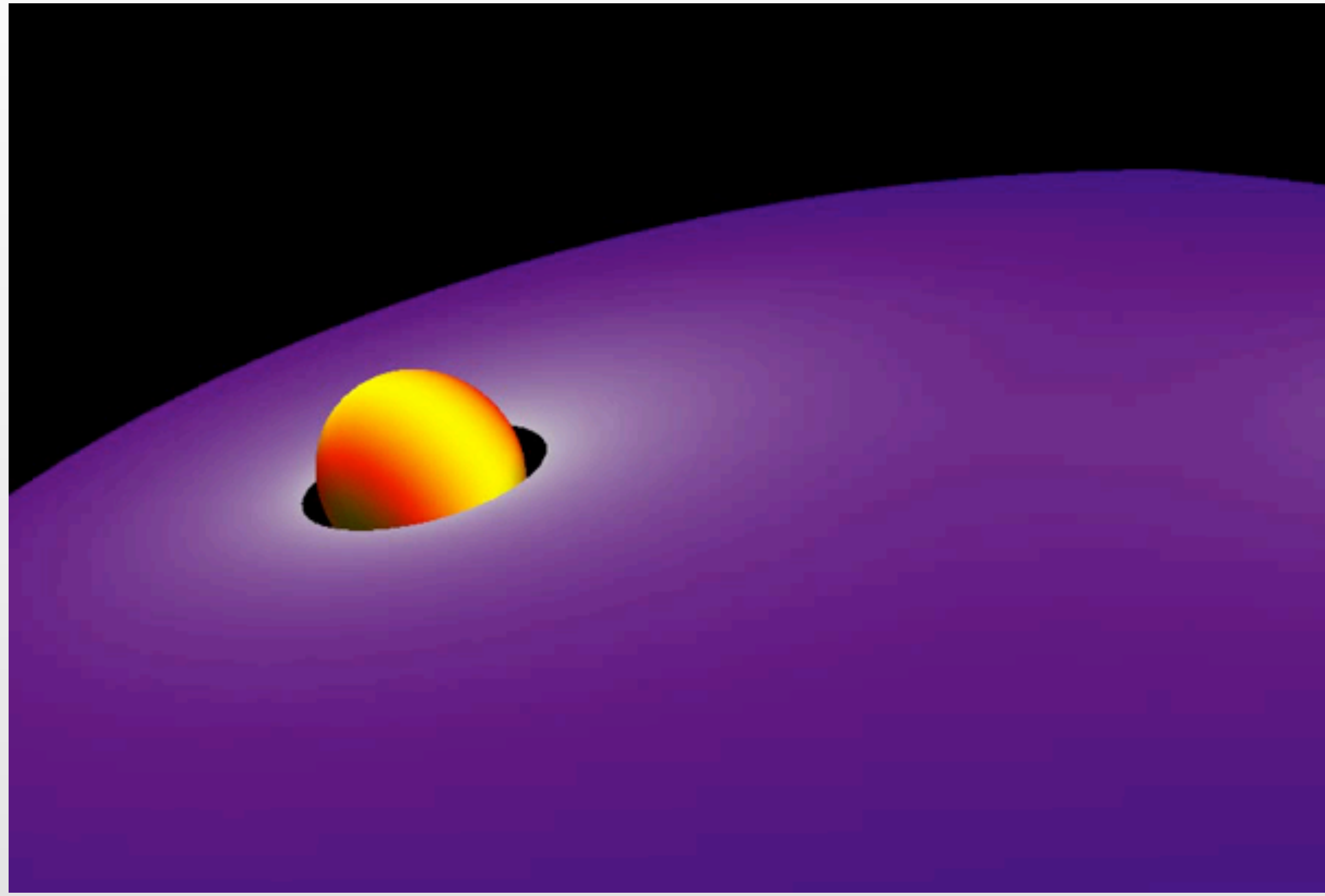


Patrick Fraser, University of Toronto

Combine analytical *and* numerical calculations to investigate BBH

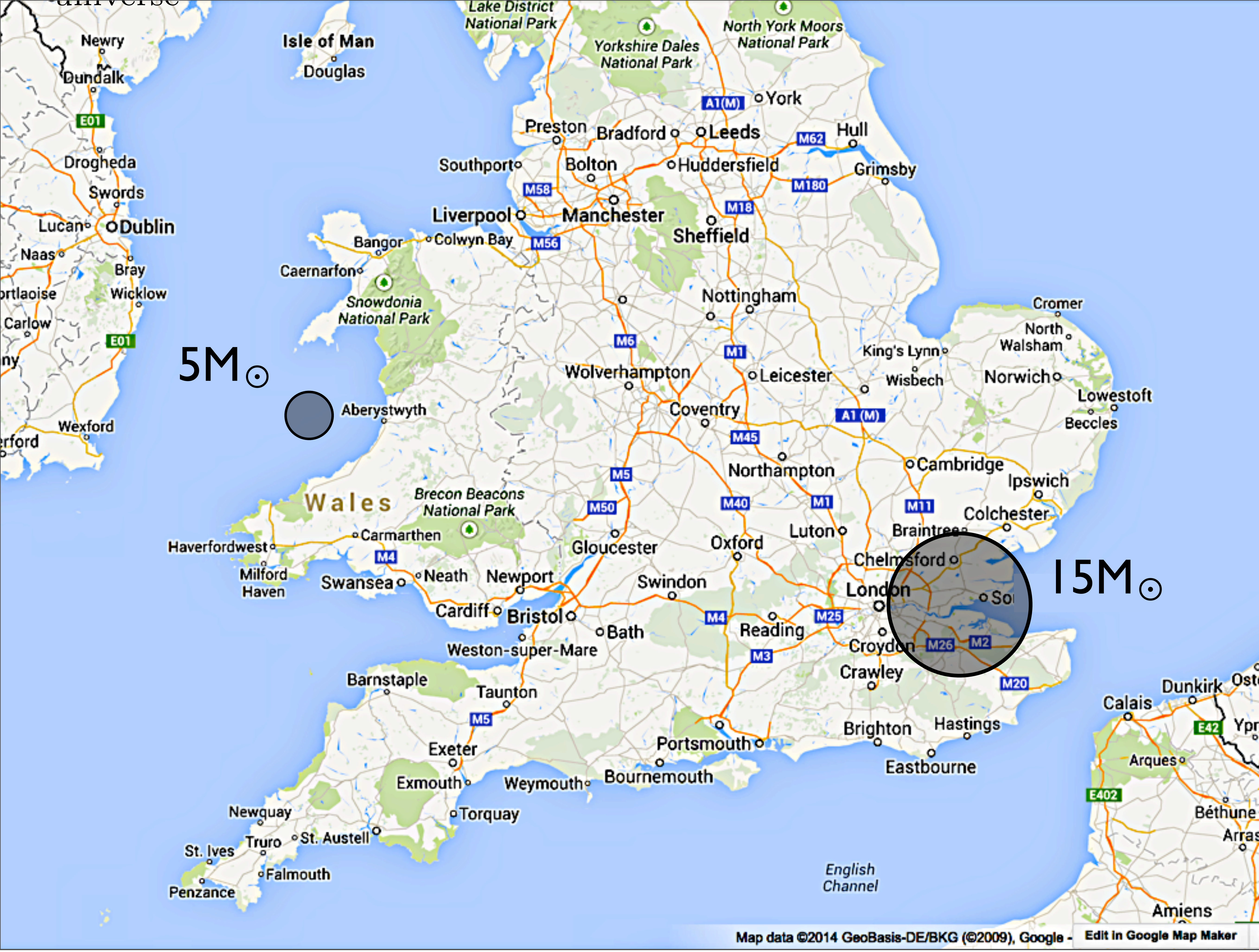
General relativistic two-body problem

- ❖ Black holes
- ❖ Event horizons
- ❖ Black hole spin
- ❖ Orbit precession
- ❖ Spin precession
- ❖ Periastron advance
- ❖ Gravitational waves
- ❖ Merger
- ❖ Remnant black hole kick
- ❖ Ten 3-dim. partial differential equations

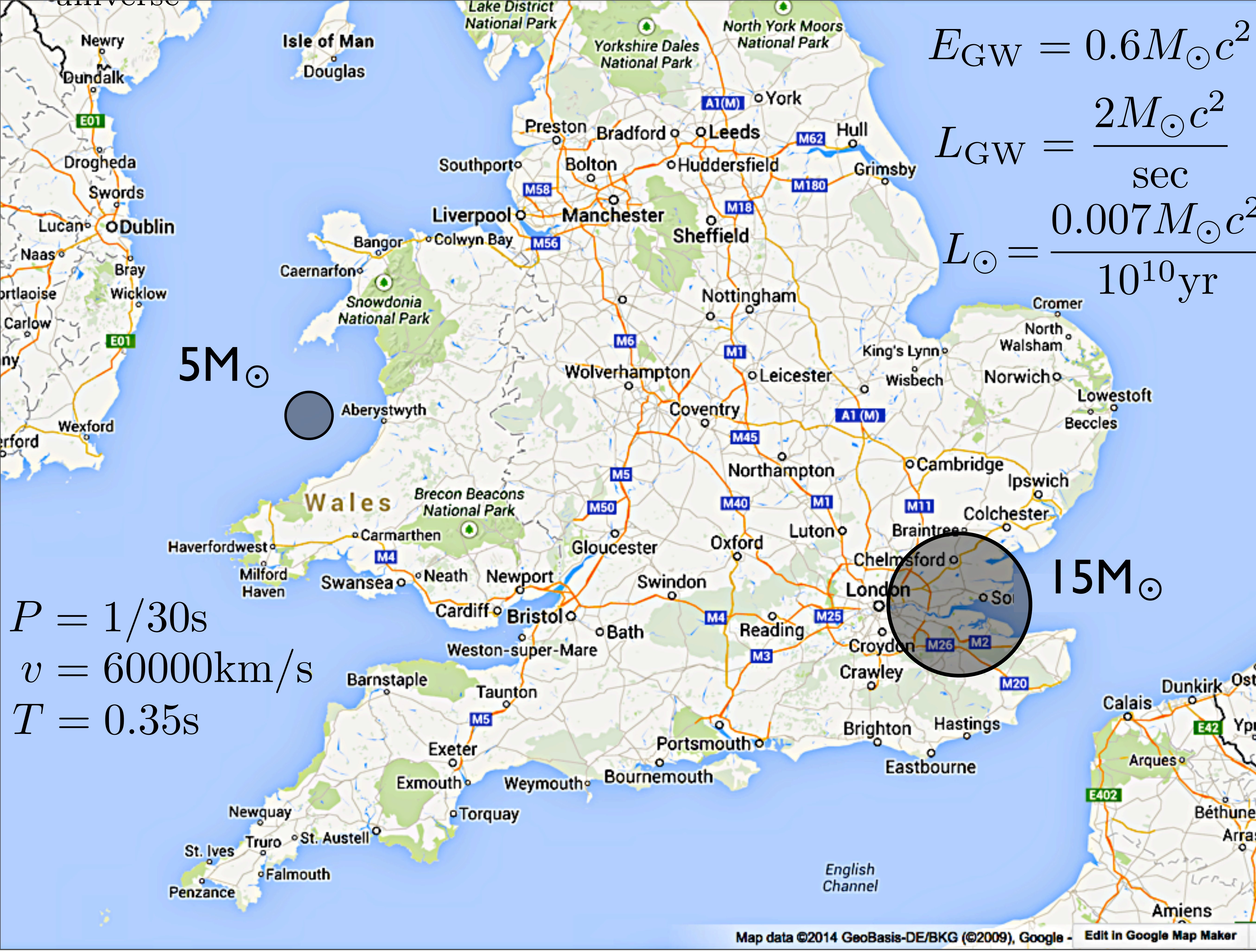


Patrick Fraser, University of Toronto

Combine analytical *and* numerical calculations to investigate BBH


5M☉
15M☉
Newry
Isle of Man
Dundalk
Douglas
Lake District National Park
Yorkshire Dales National Park
North York Moors National Park
York
Preston
Bradford
Leeds
Hull
Drogheda
Swords
Southport
Bolton
Huddersfield
Grimsby
Lucan
Dublin
Liverpool
Manchester
Sheffield
Naas
Bray
Bangor
Colwyn Bay
Caernarfon
Wicklow
Carlow
Snowdonia National Park
Nottingham
Cromer
North Walsham
King's Lynn
Wisbech
Wolverhampton
Leicester
Norwich
Lowestoft
Beccles
Aberystwyth
Coventry
Wexford
Cambridge
Northampton
Ipswich
Wales
Brecon Beacons National Park
Colchester
Braintree
Carmarthen
Luton
Haverfordwest
Oxford
Chelmsford
Gloucester
Milford Haven
Swansea
Neath
Newport
Swindon
London
Cardiff
Bristol
Bath
Reading
Croydon
Weston-super-Mare
Crawley
Barnstaple
Taunton
Dunkirk
Calais
Exeter
Portsmouth
Brighton
Hastings
Eastbourne
Exmouth
Weymouth
Bournemouth
Arques
Newquay
Torquay
Béthune
Truro
St. Austell
Arras
St. Ives
Falmouth
Penzance
English Channel
Amiens
Map data ©2014 GeoBasis-DE/BKG (©2009), Google
Edit in Google Map Maker

$$P = 1/30\mathrm{s}$$
$$v = 60000\mathrm{km/s}$$
$$T = 0.35\mathrm{s}$$



$$E_{\rm GW} = 0.6 M_\odot c^2$$

$$L_{\rm GW} = \frac{2 M_\odot c^2}{\rm sec}$$

$$L_\odot = \frac{0.007 M_\odot c^2}{10^{10}{\rm yr}}$$

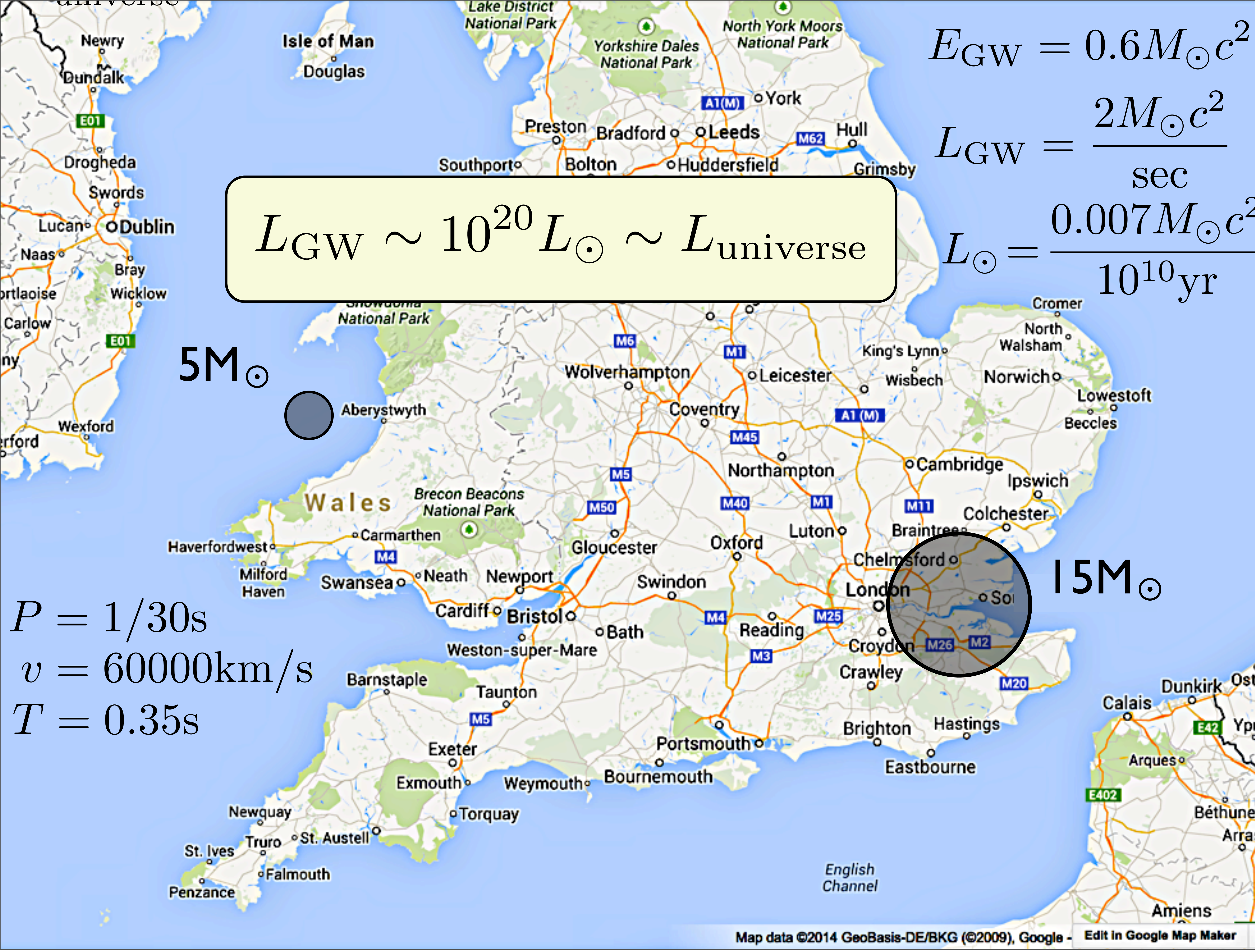
$5M_\odot$

$15M_\odot$

$$P = 1/30{\rm s}$$

$$v = 60000{\rm km/s}$$

$$T = 0.35{\rm s}$$



$$E_{\rm GW} = 0.6 M_\odot c^2$$

$$L_{\rm GW} = \frac{2 M_\odot c^2}{\rm sec}$$

$$L_\odot = \frac{0.007 M_\odot c^2}{10^{10}\rm yr}$$

$$L_{\rm GW} \sim 10^{20} L_\odot \sim L_{\rm universe}$$

$5 M_\odot$

$15 M_\odot$

$$P = 1/30\rm s$$

$$v = 60000 \rm km/s$$

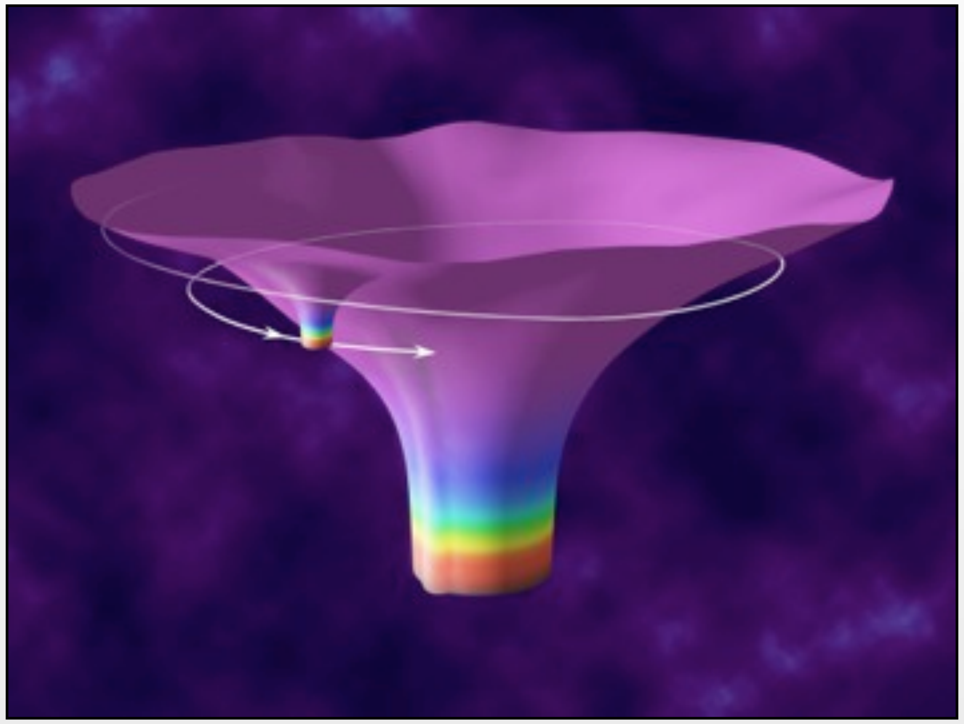
$$T = 0.35\rm s$$

Black Holes, Event Horizons

Black hole

❖ Made entirely of warped space-time

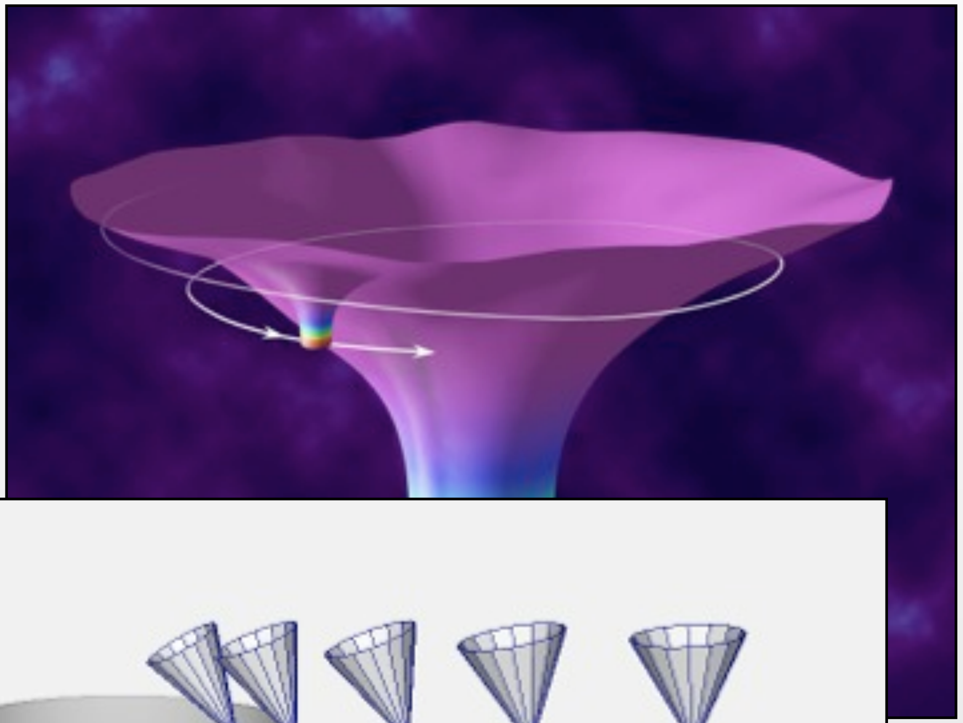
- Curvature of space
- Slowing of flow of time
- Dragging of space around BH

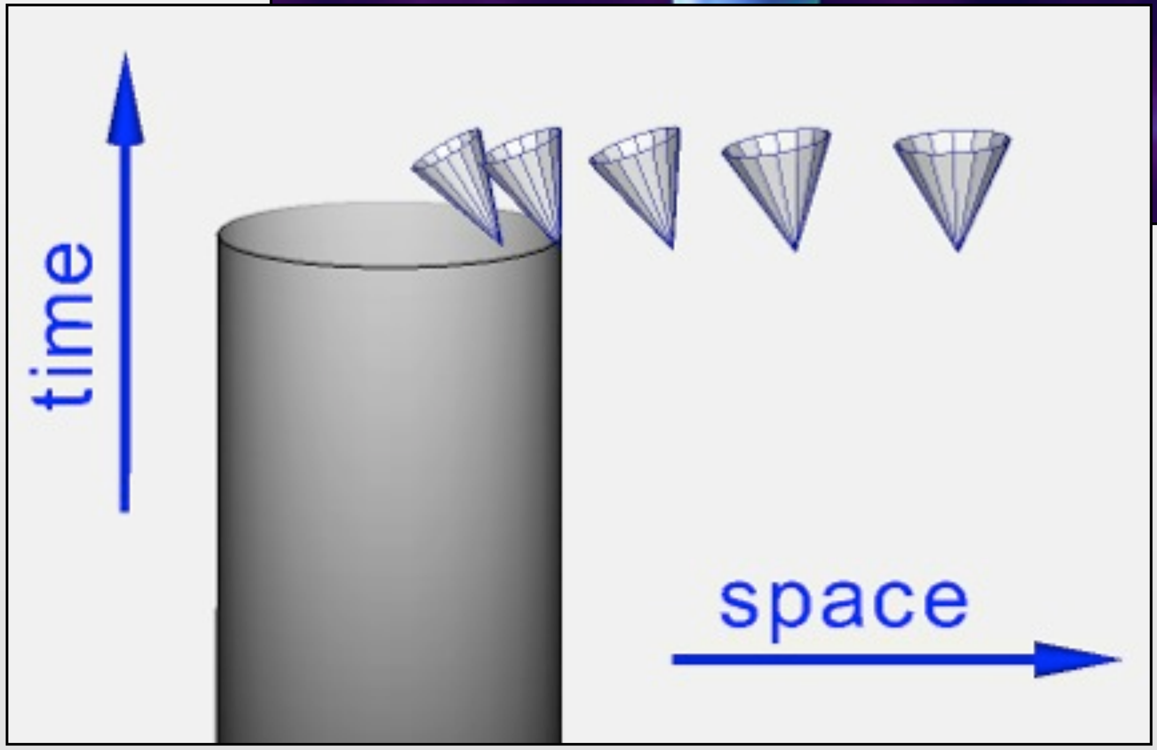


Courtesy Kip Thorne

Black hole

- Made entirely of warped space-time
  - Curvature of space
  - Slowing of flow of time
  - Dragging of space around BH

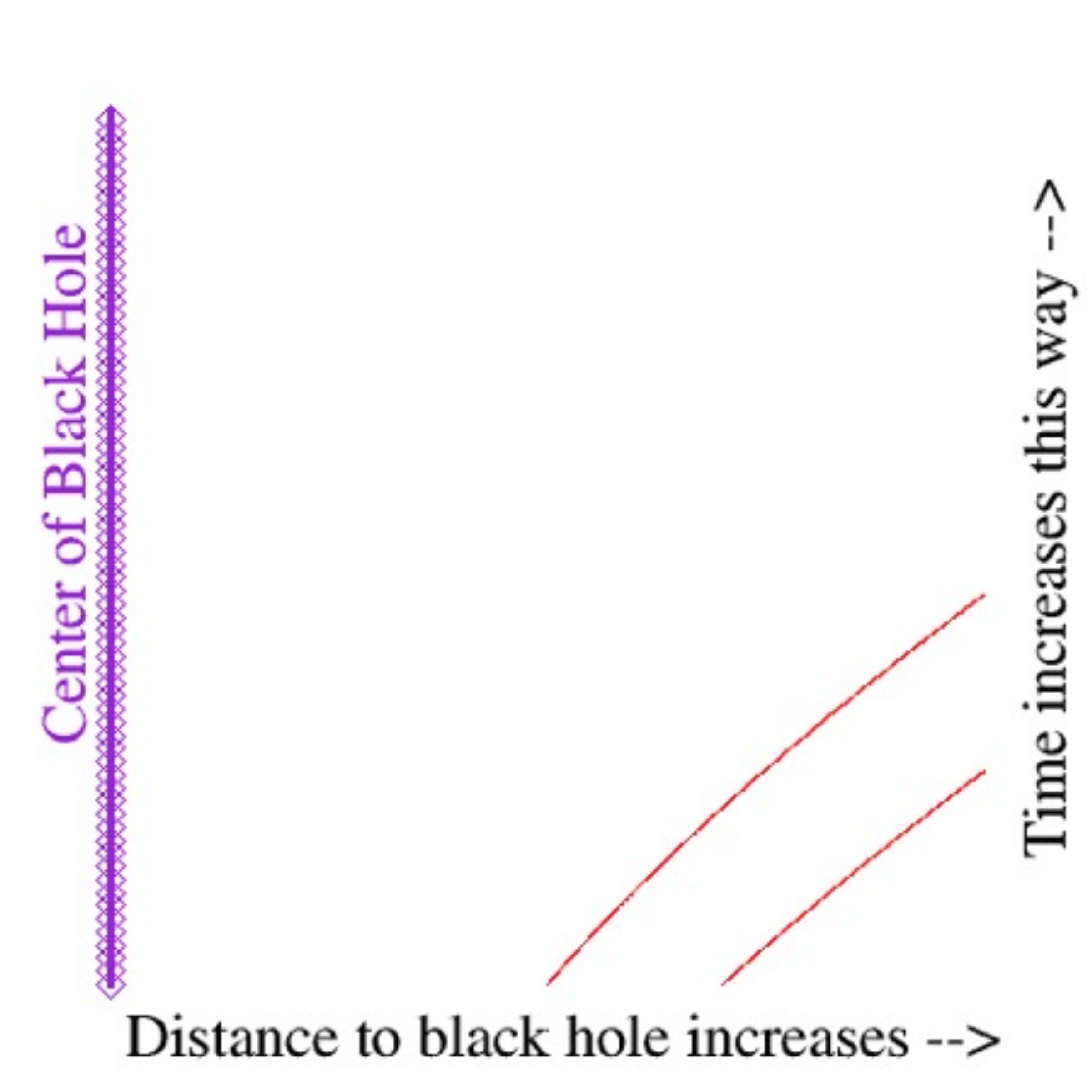




- Curved geometry changes causal structure
  - Tipping of light-cones
  - Event horizon

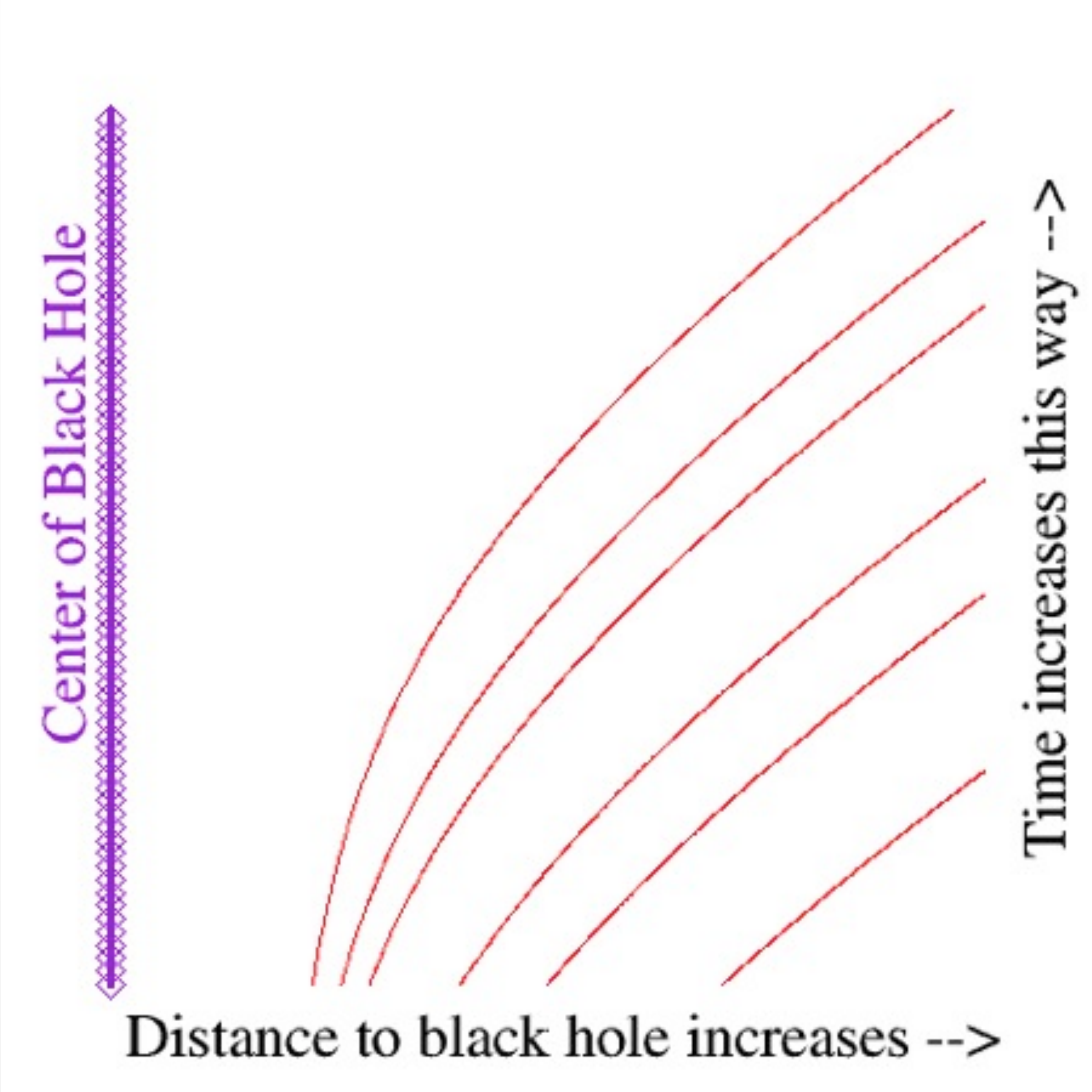
$$A_{\rm EH} = 4\pi r_S^2, \qquad r_S = \frac{2GM}{c^2} = 3\frac{M}{M_\odot}\,{\rm km}$$

Event Horizon



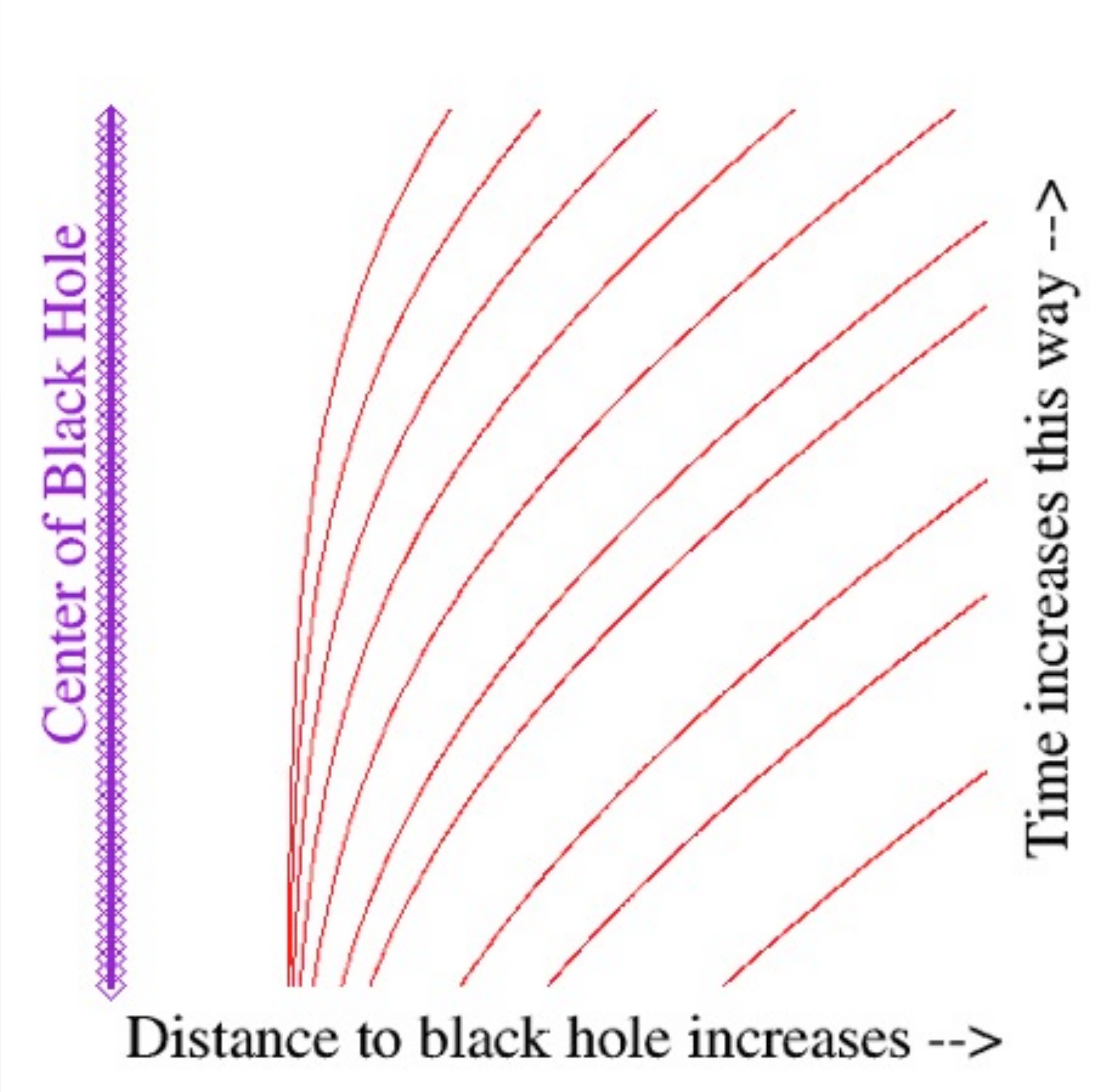
Light moving **away** from center of black hole

Event Horizon



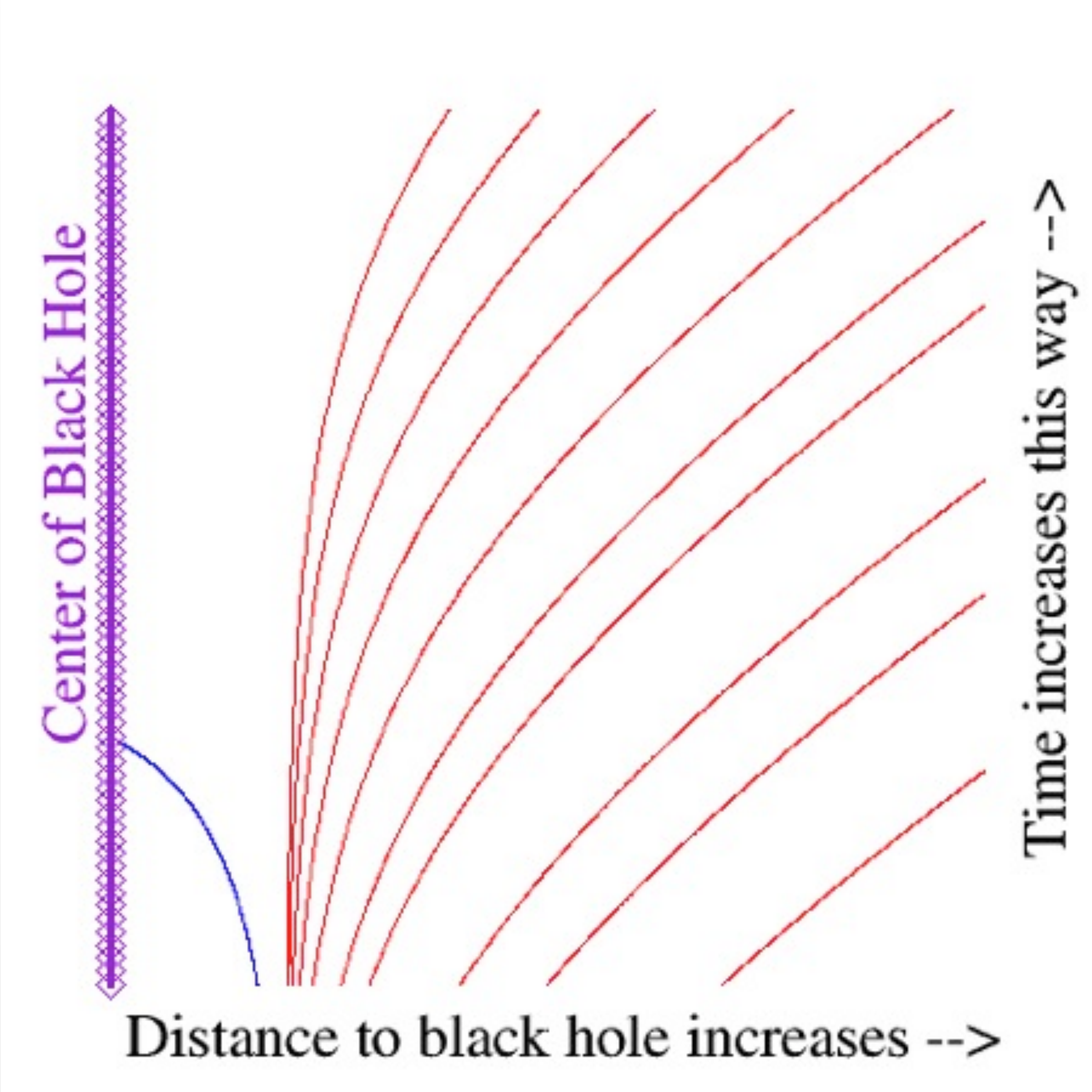
Light moving **away** from center of black hole

Event Horizon



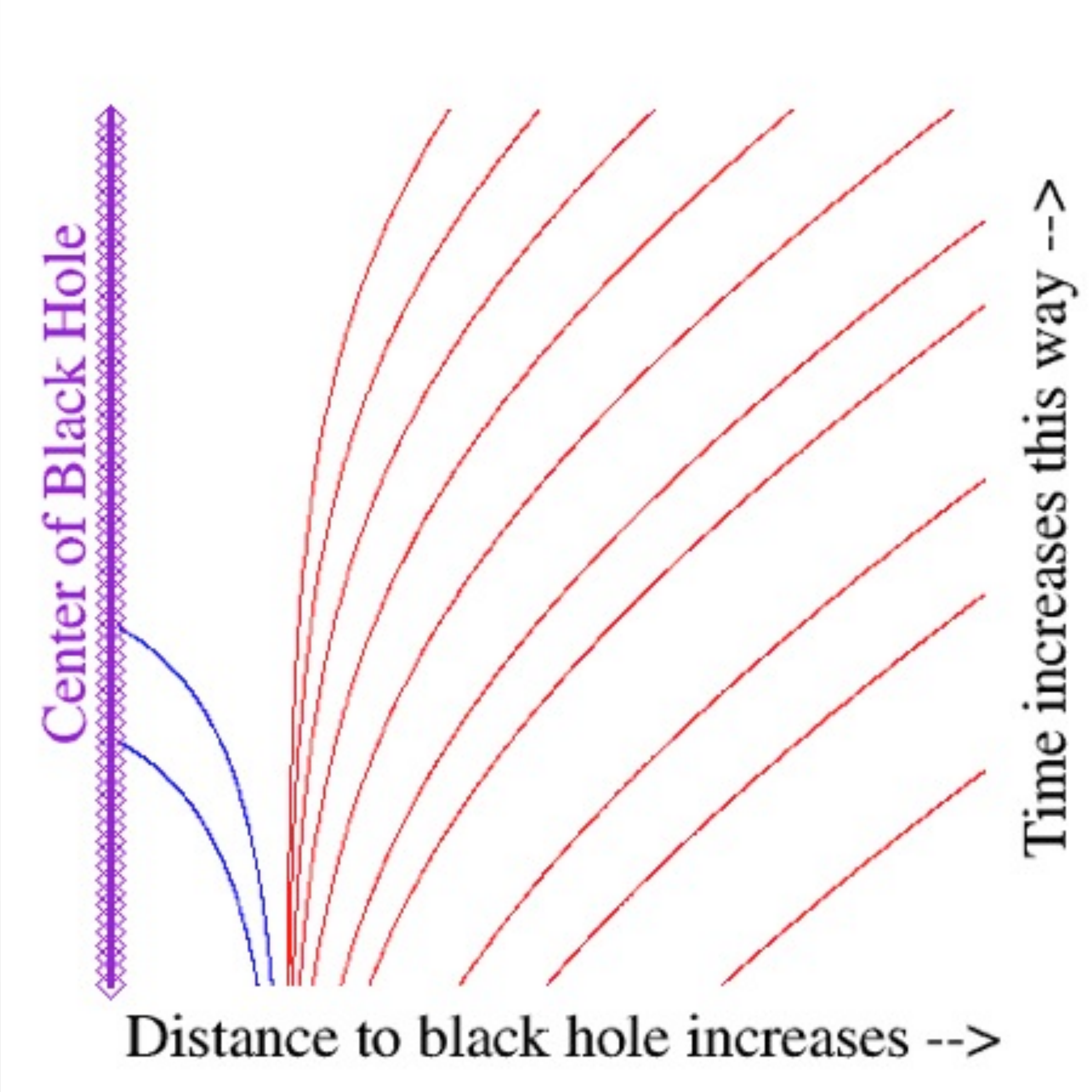
Light moving **away** from center of black hole

Event Horizon



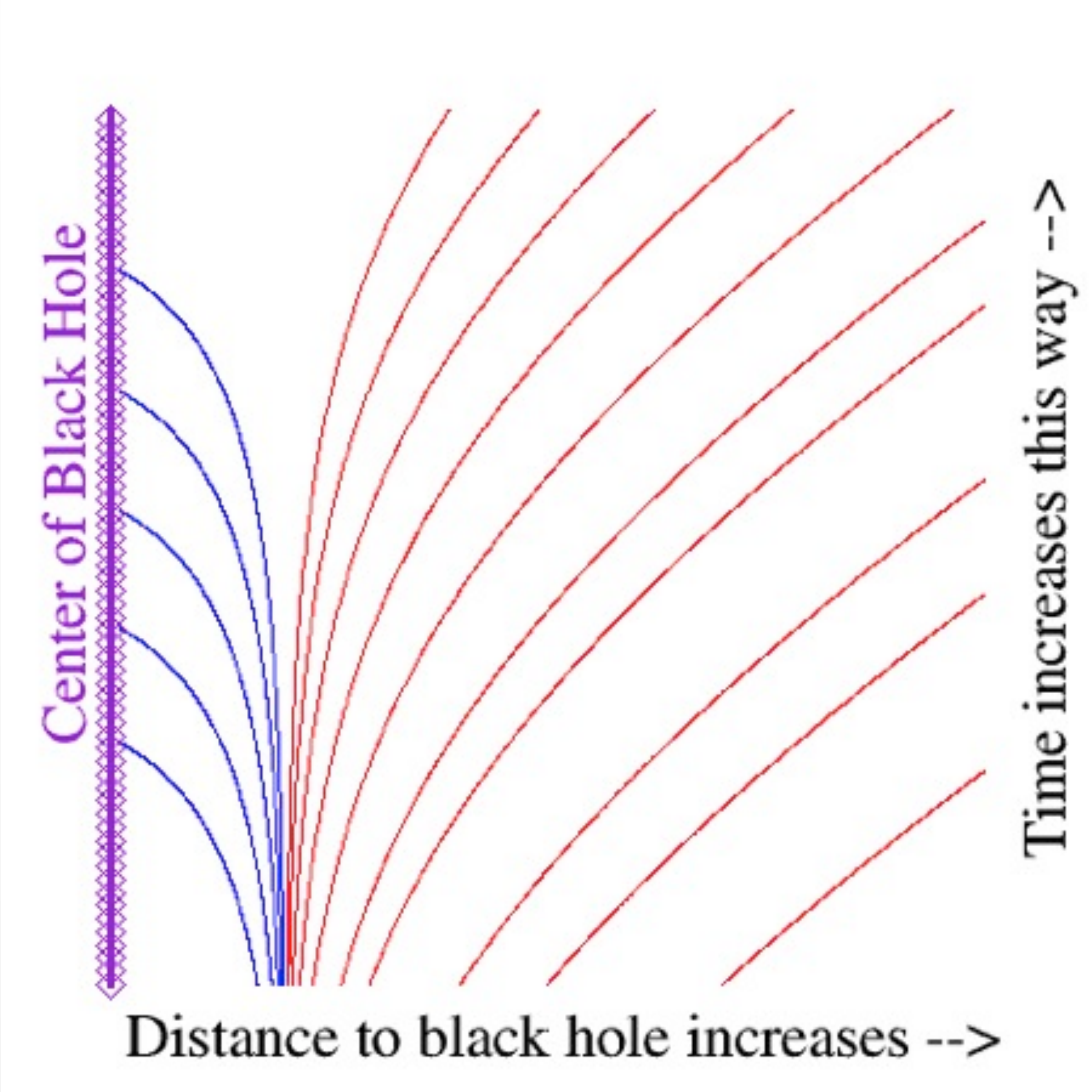
Light moving **away** from center of black hole

Event Horizon



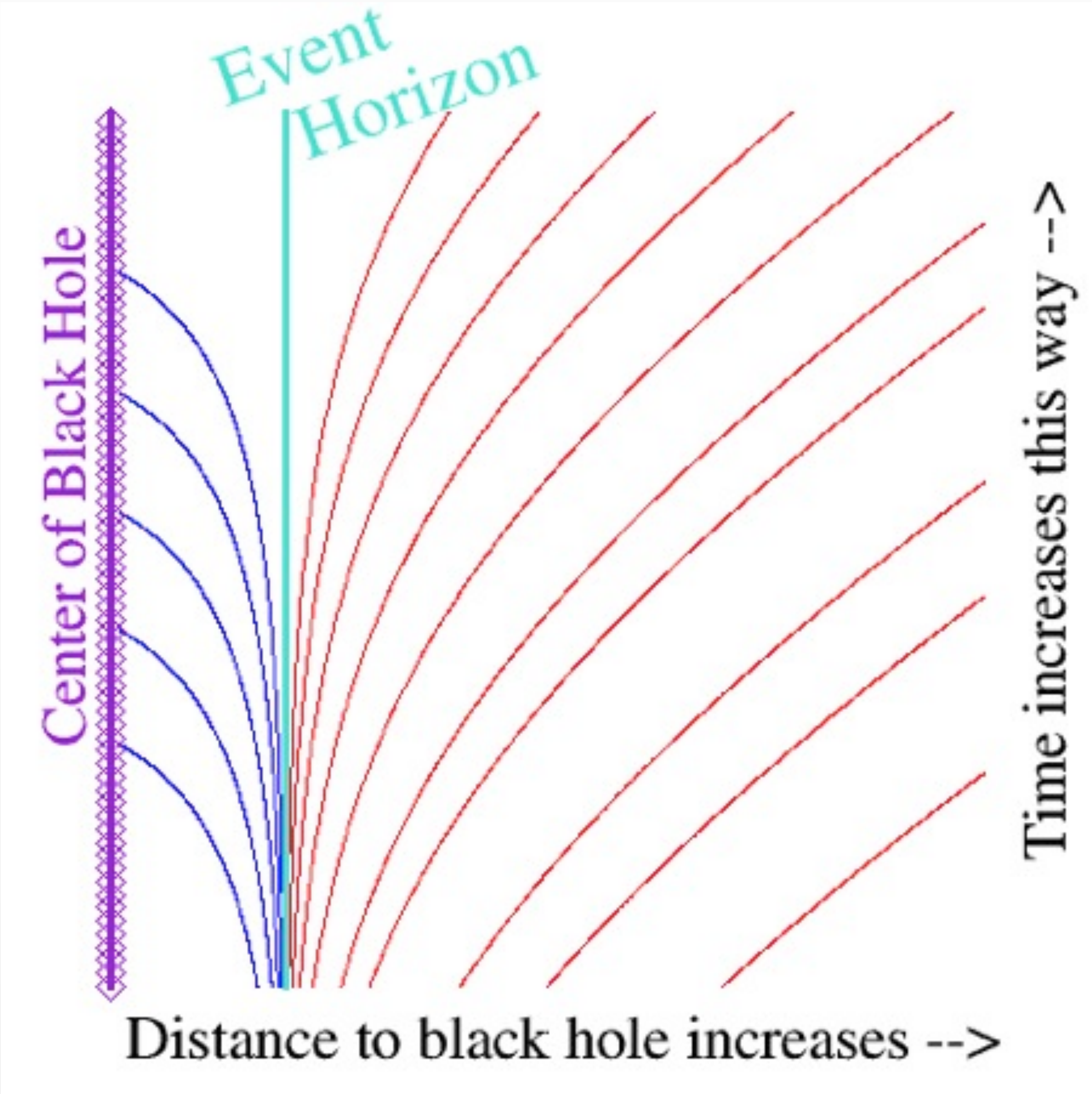
Light moving **away** from center of black hole

Event Horizon



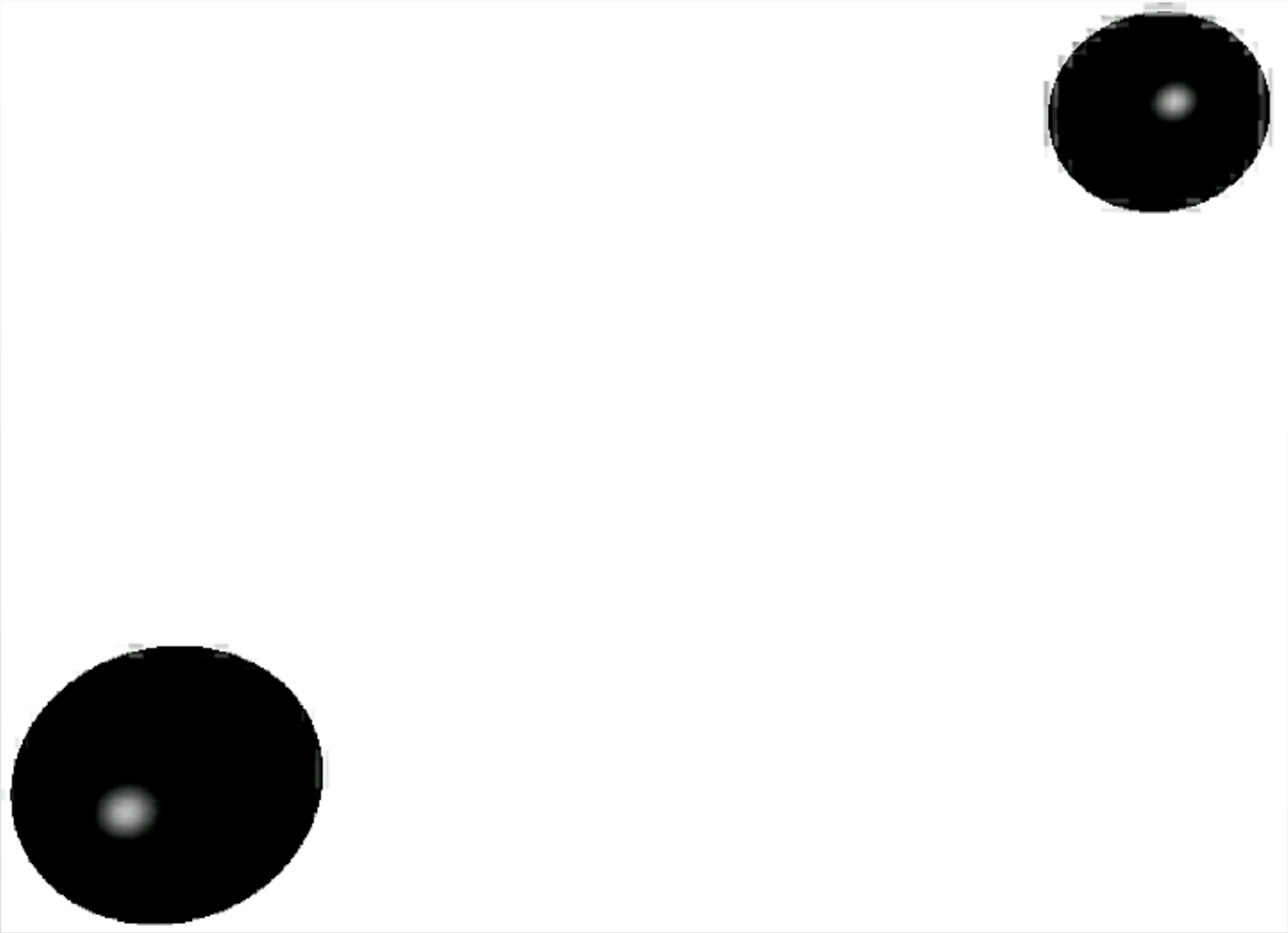
Light moving **away** from center of black hole

Event Horizon

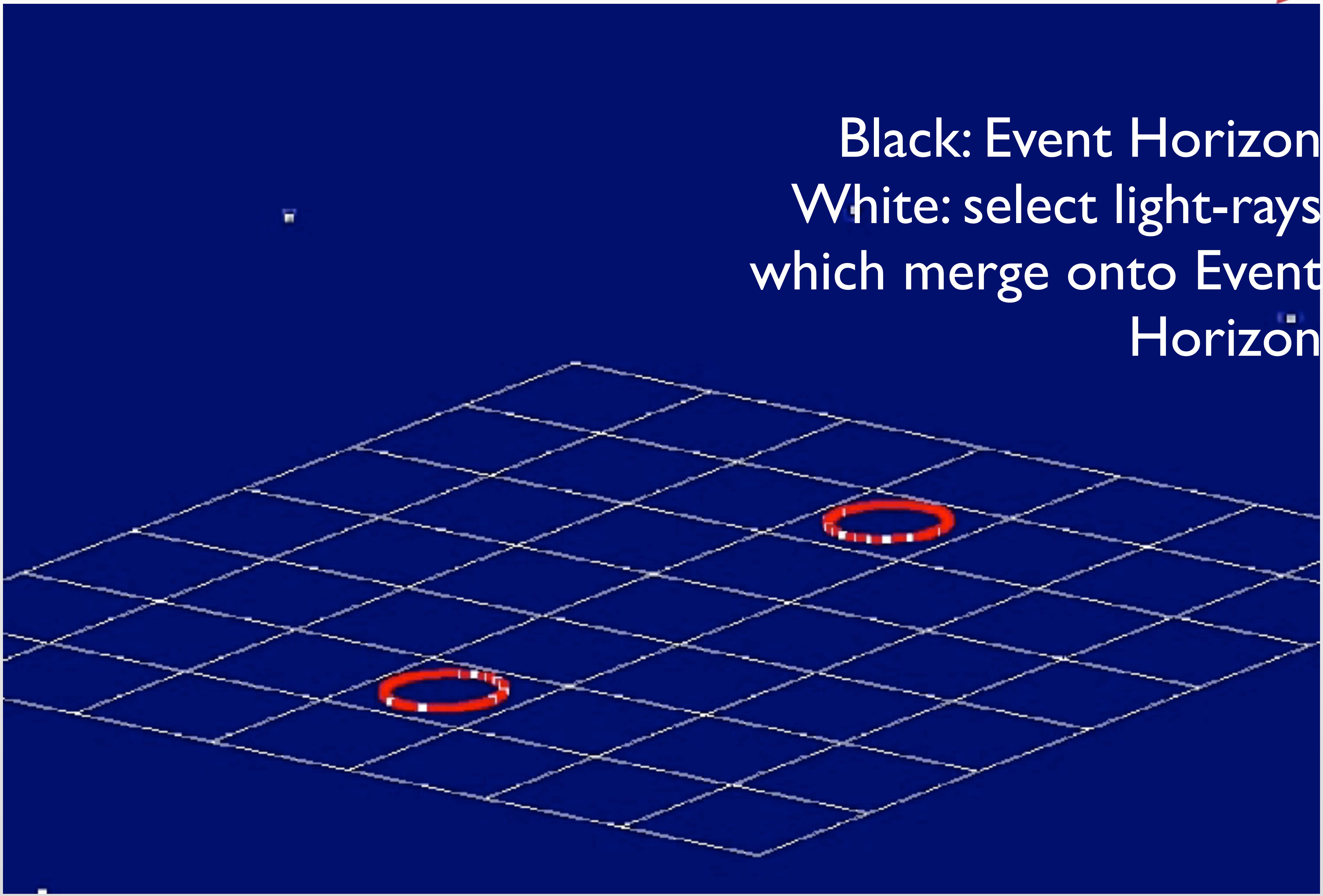


Light moving **away** from center of black hole

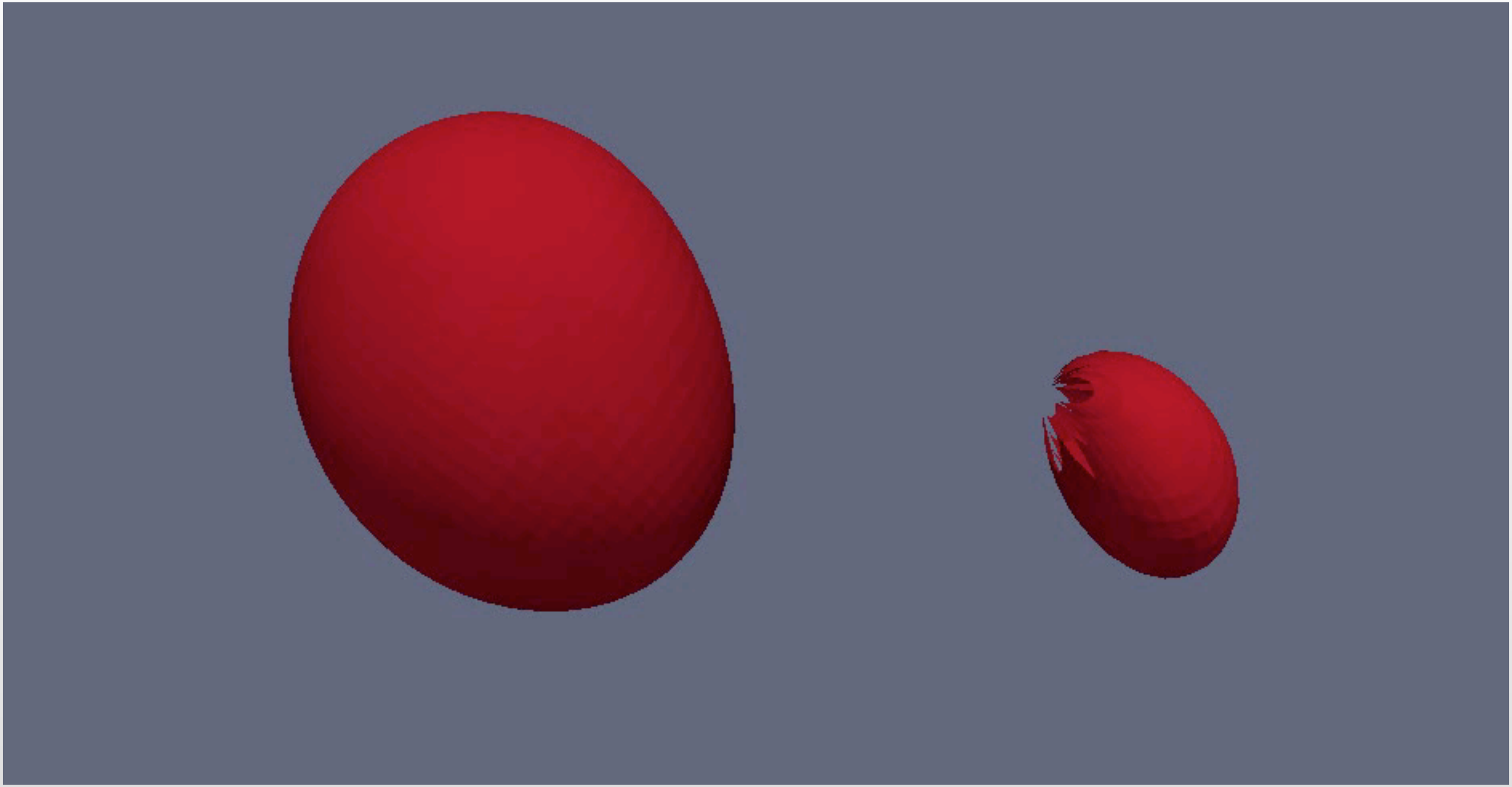
Event horizon for head-on colission



Space-time diagram



Event horizon of rotating, inspiraling BBH



Mike Cohen; and Szilagyi, Lindblom, Scheel

Apparent Horizons

❖ Surface where light ***instantaneously*** appears stationary
  - Outgoing null-rays have zero expansion

Tools of the trade

- Spectral Einstein Code
- Post-Newtonian expansions

Numerical Relativity - Basic idea

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

- Split spacetime into space *and* time

- Evolution equations

$$\partial_t g_{ij} = \ldots$$
$$\partial_t K_{ij} = \ldots$$

- Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij}K\right) = 0$$

cf. Maxwell's equations

$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

Numerical Relativity - Basic idea

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

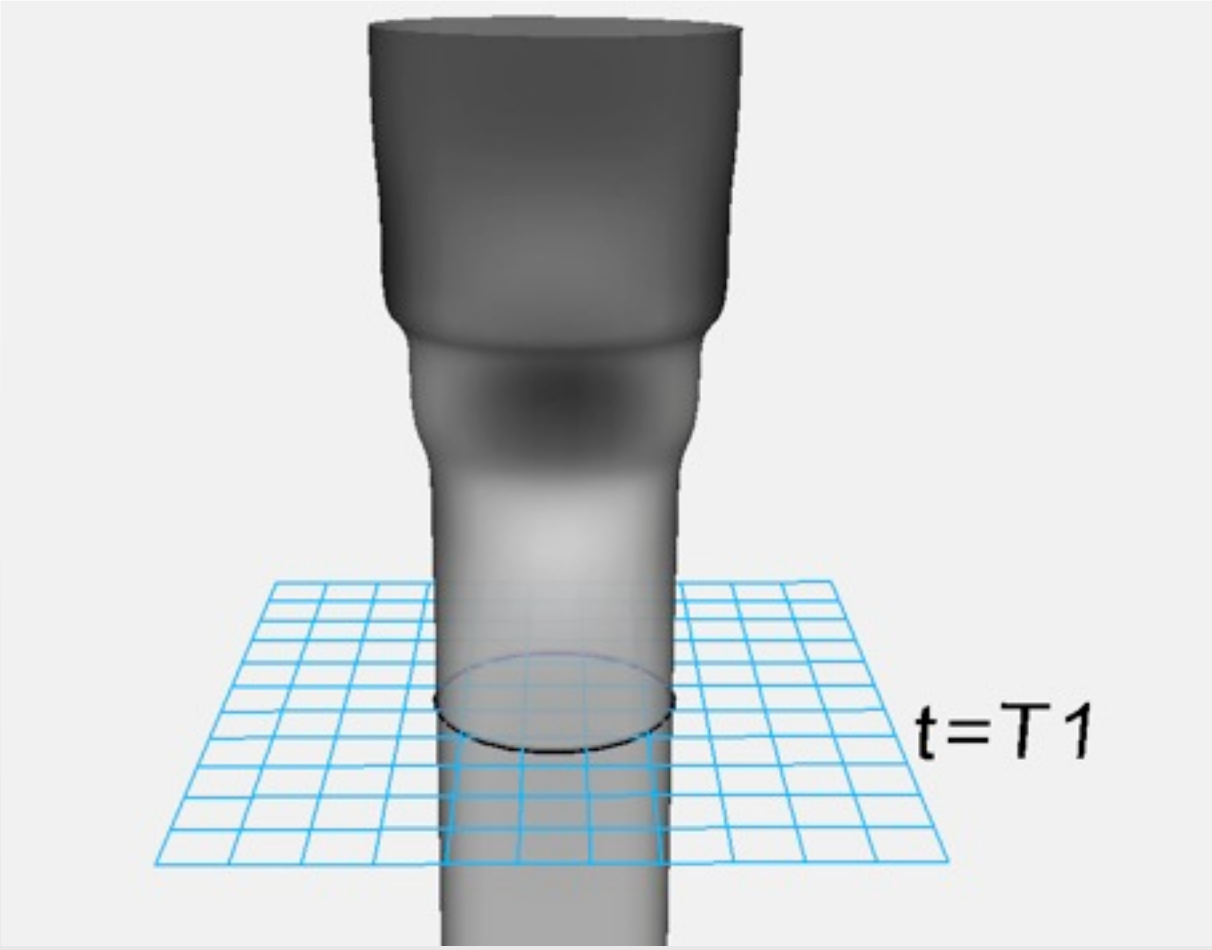
- Split spacetime into space *and* time

- Evolution equations

$$\partial_t g_{ij} = \dots$$
$$\partial_t K_{ij} = \dots$$

- Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij}K\right) = 0$$



cf. Maxwell's equations

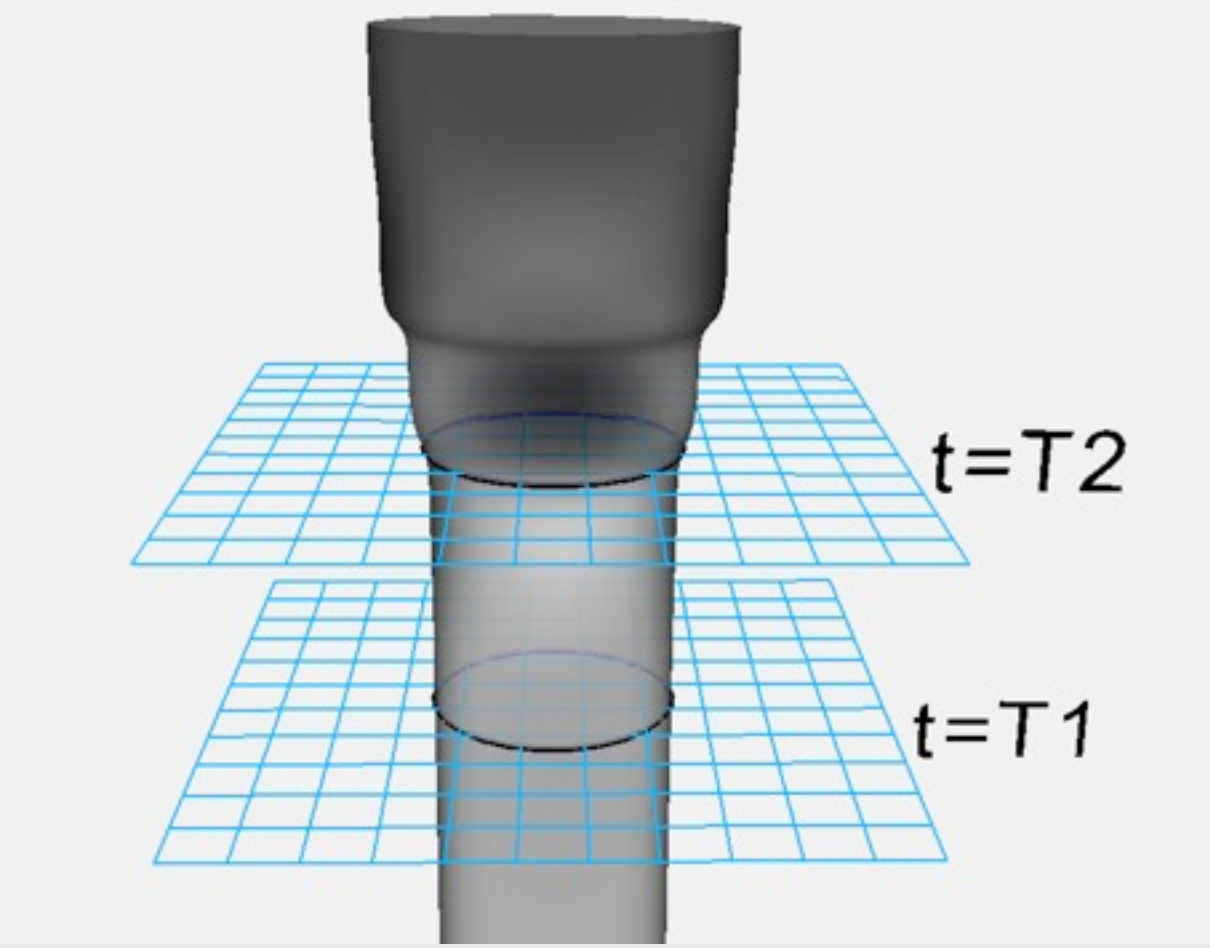
$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

Numerical Relativity - Basic idea

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

- Split spacetime into space *and* time



- Evolution equations

$$\partial_t g_{ij} = \dots$$
$$\partial_t K_{ij} = \dots$$

- Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j\left(K^{ij} - g^{ij}K\right) = 0$$

cf. Maxwell's equations

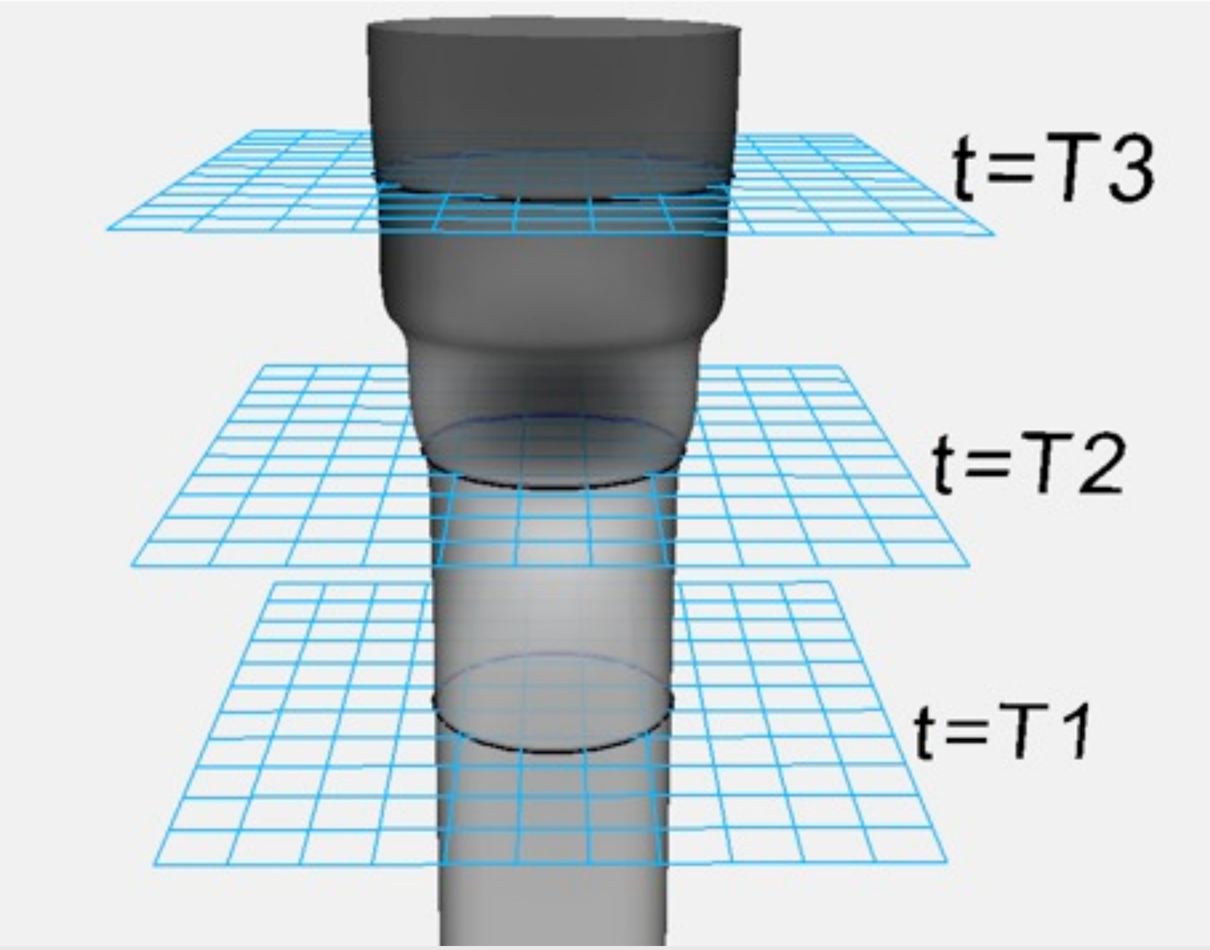
$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

Numerical Relativity - Basic idea

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

- Split spacetime into space *and* time



- Evolution equations

$$\partial_t g_{ij} = \ldots$$
$$\partial_t K_{ij} = \ldots$$

- Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij}K\right) = 0$$

cf. Maxwell's equations

$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

Numerics 1: Spectral methods

- Expand in basis-functions, solve for coefficients

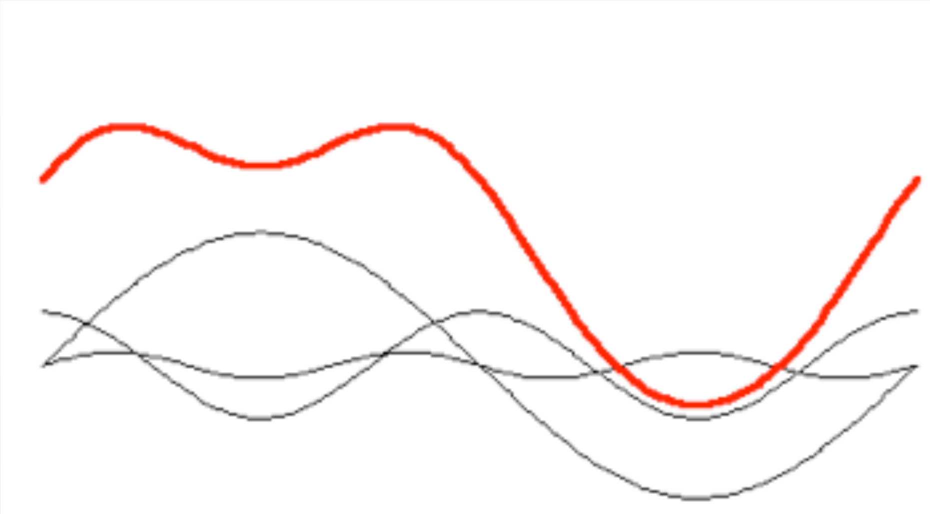
$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$

- Compute derivatives analytically

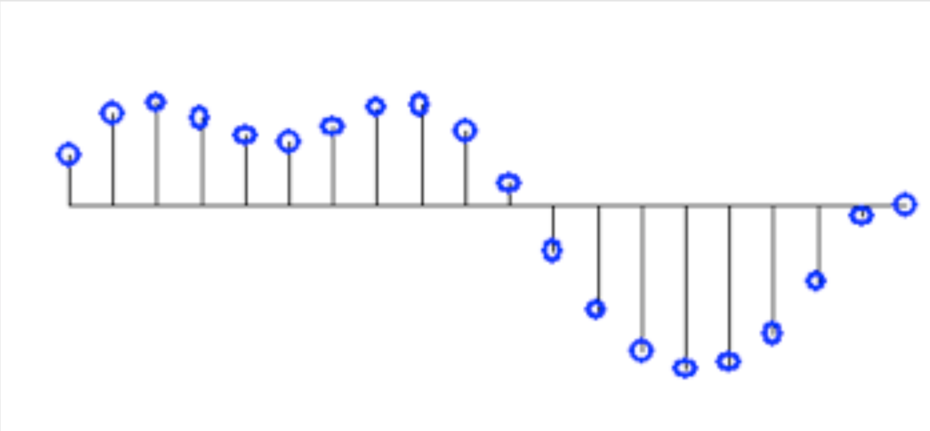
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$

- Compute nonlinearities in physical space

Spectral

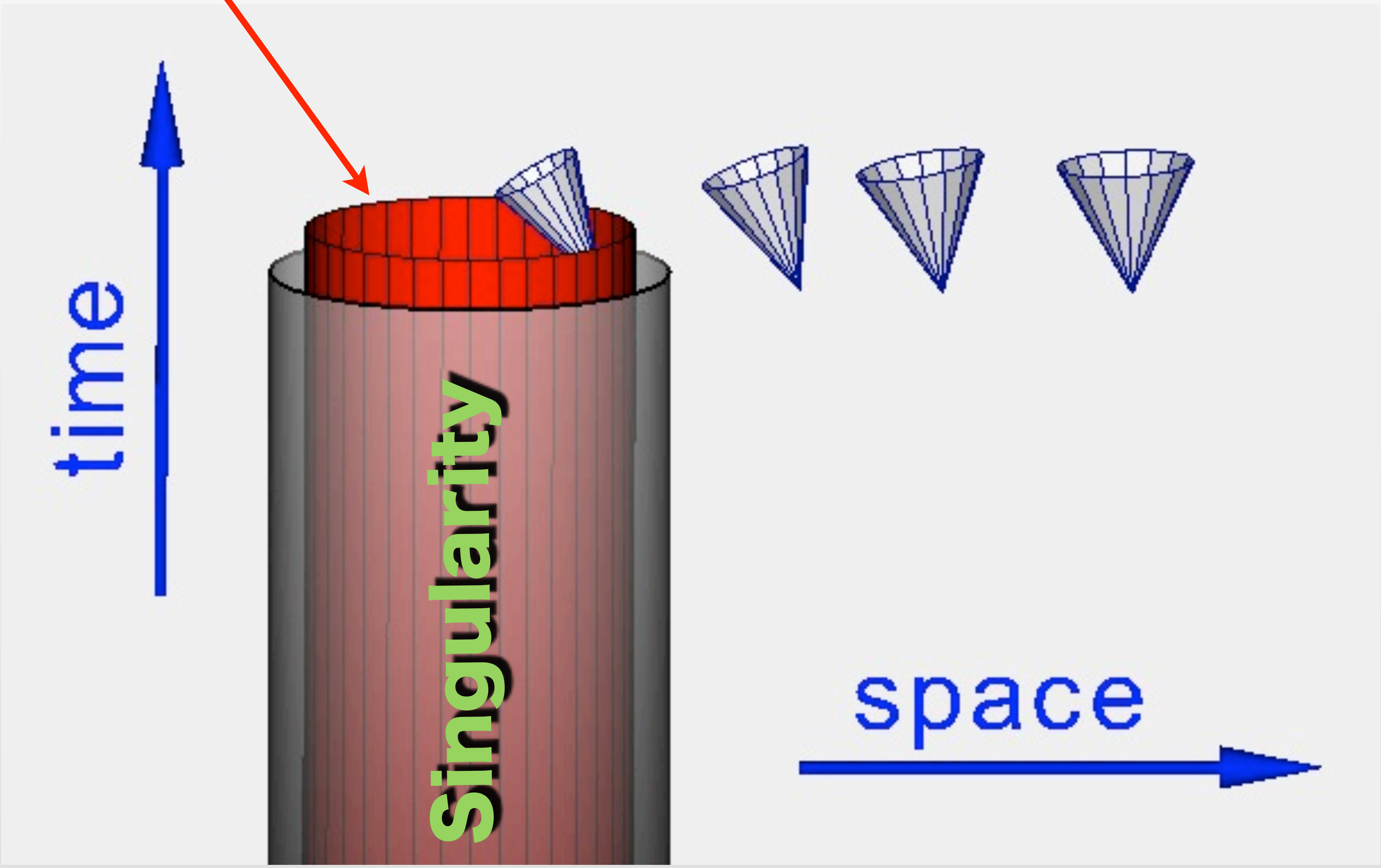


More widely used: Finite differences

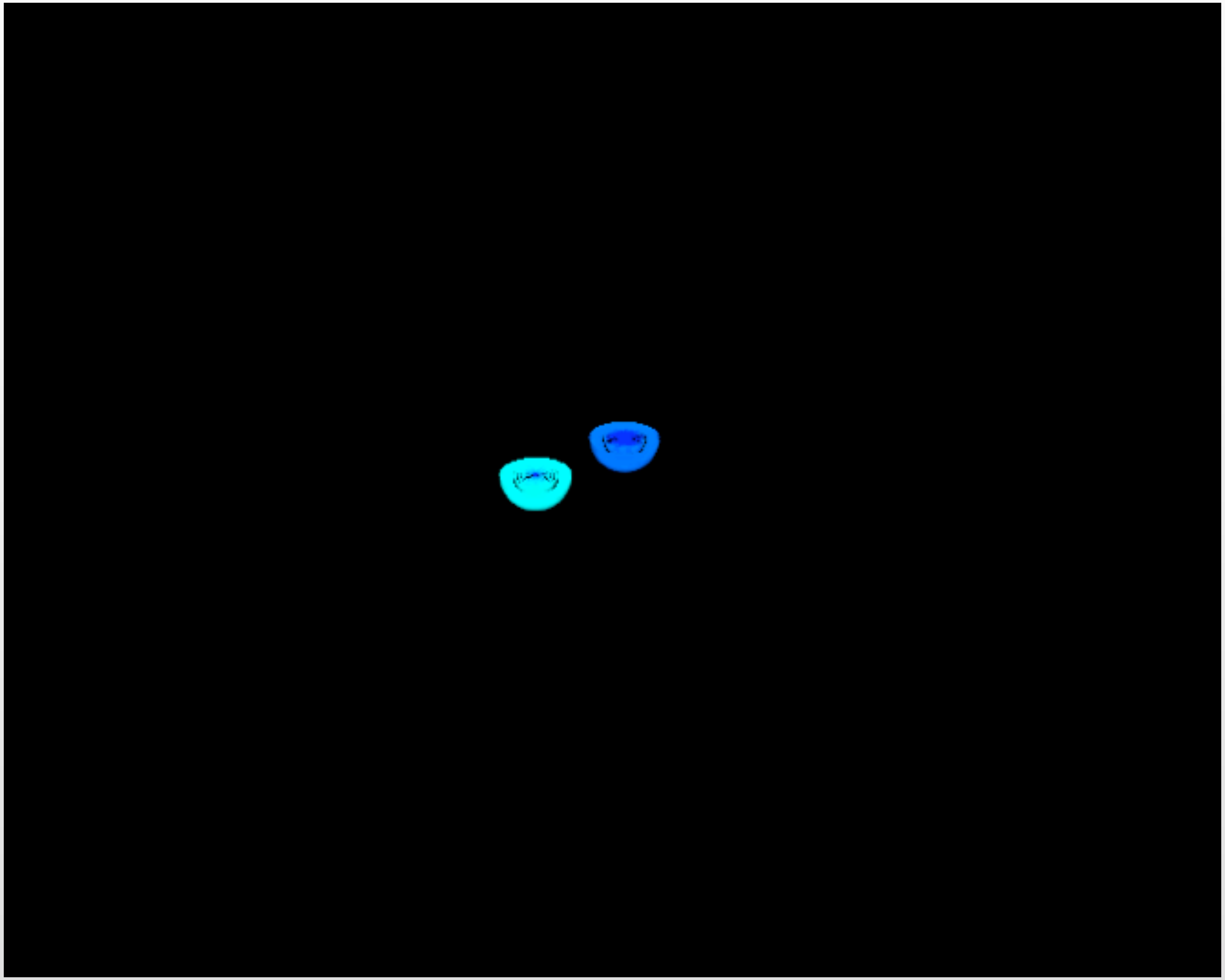


Numerics II: Black Hole Excision

Artificial boundary
inside horizon

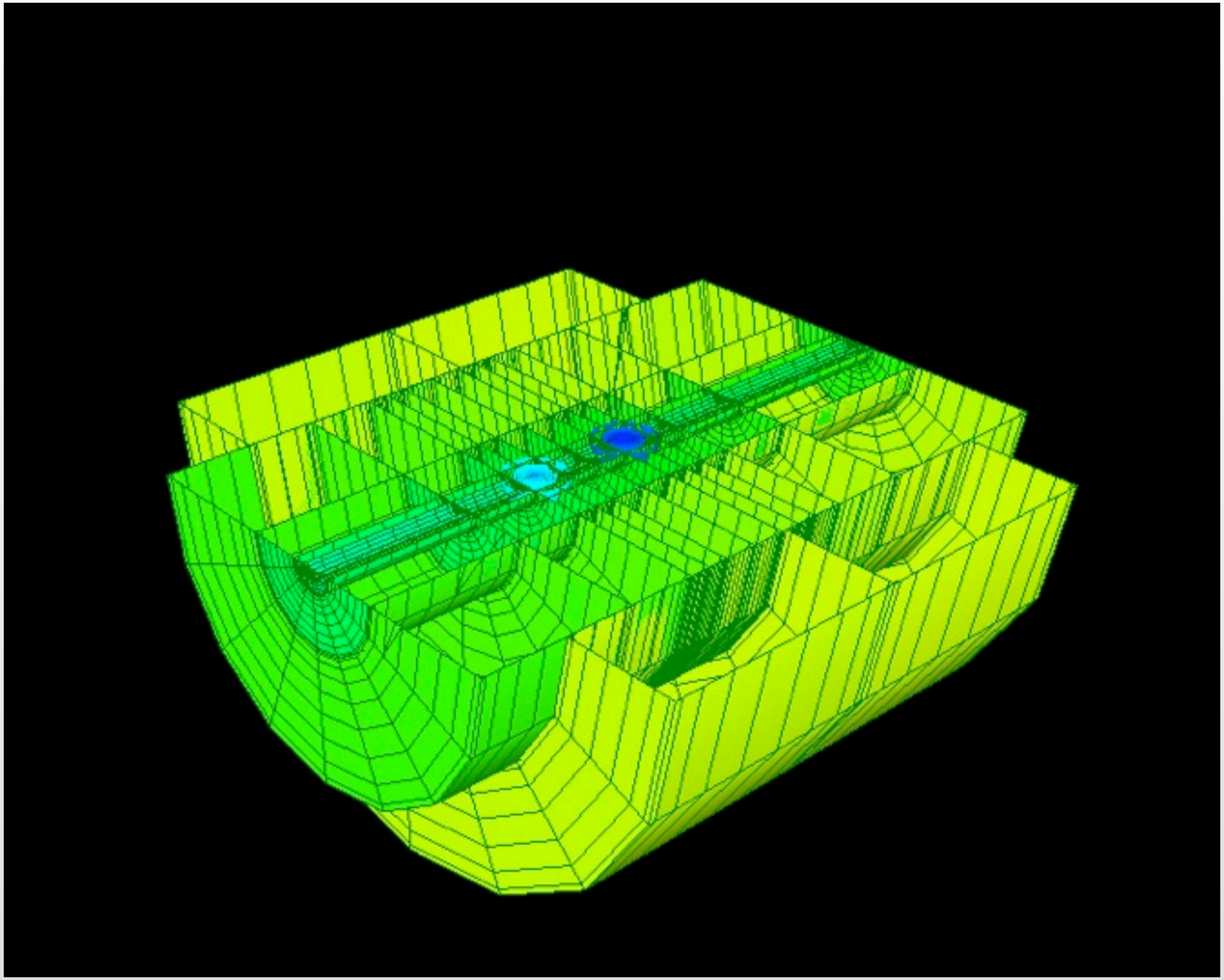


Numerics III: Domain-decomposition



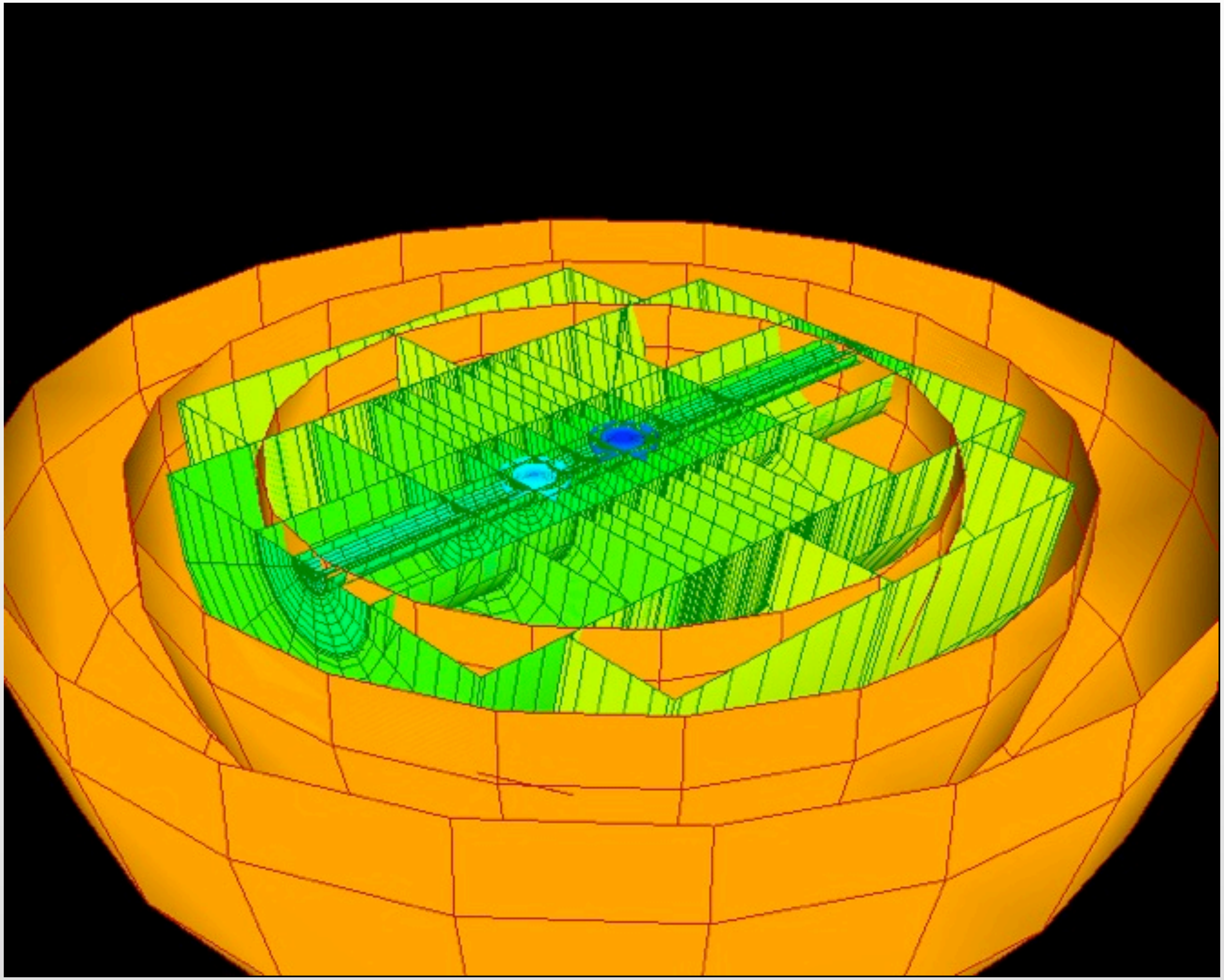
Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

Numerics III: Domain-decomposition



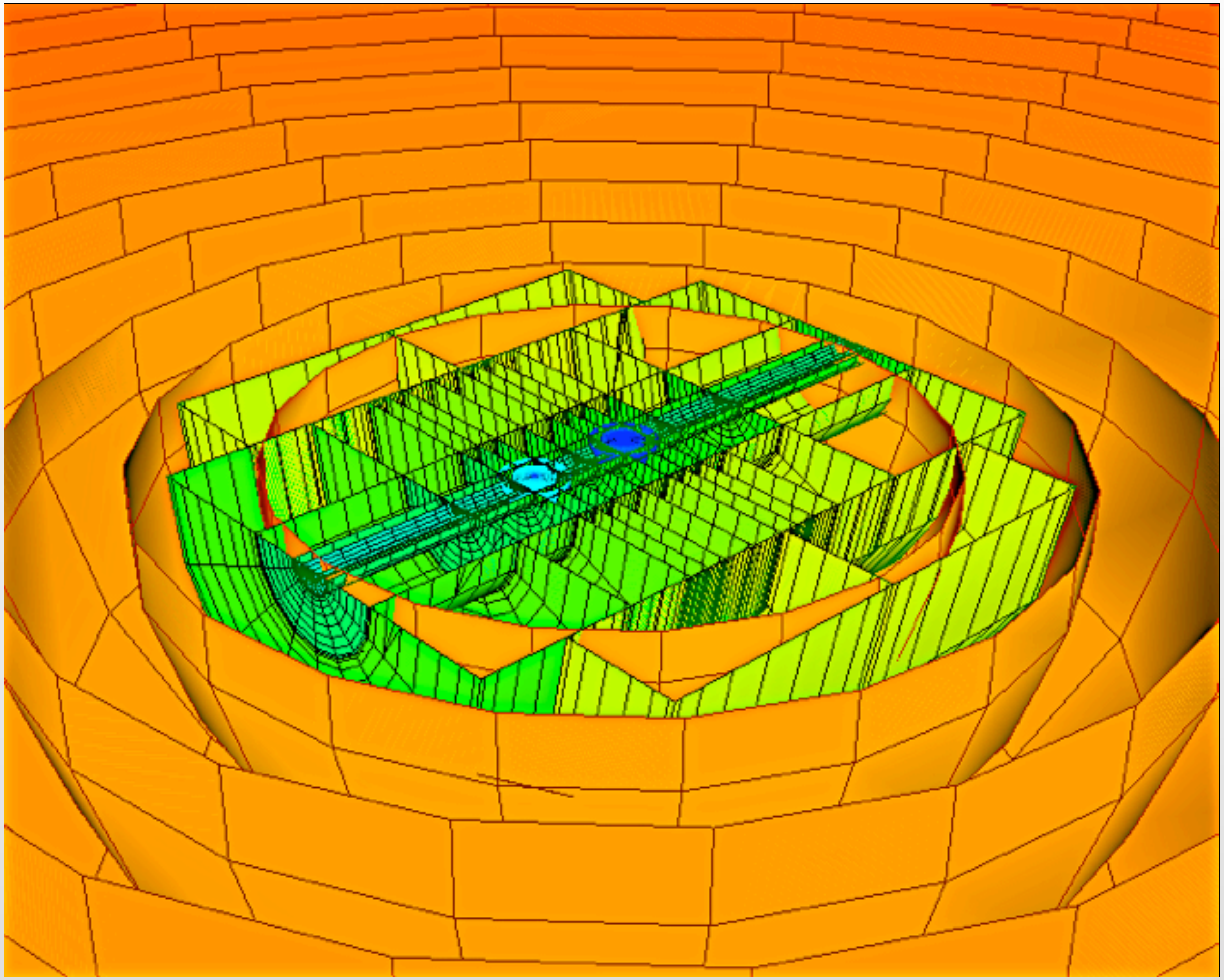
Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

Numerics III: Domain-decomposition



Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

Numerics III: Domain-decomposition



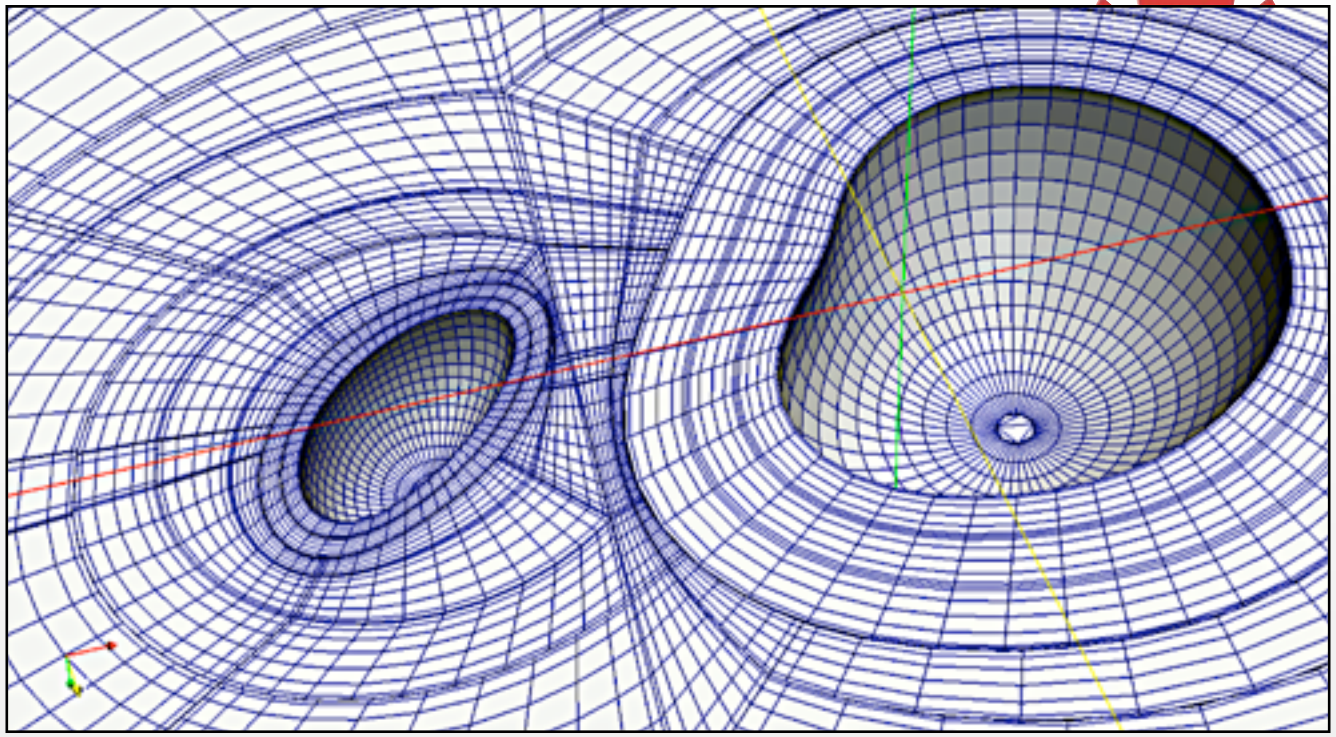
Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)

http://www.black-holes.org/SpEC.html

IV: Merger & Ringdown

❖ ***Mark Scheel, Bela Szilagyi***

Szilagyi, Lindblom, Scheel 08,
Hemberger ea, 13



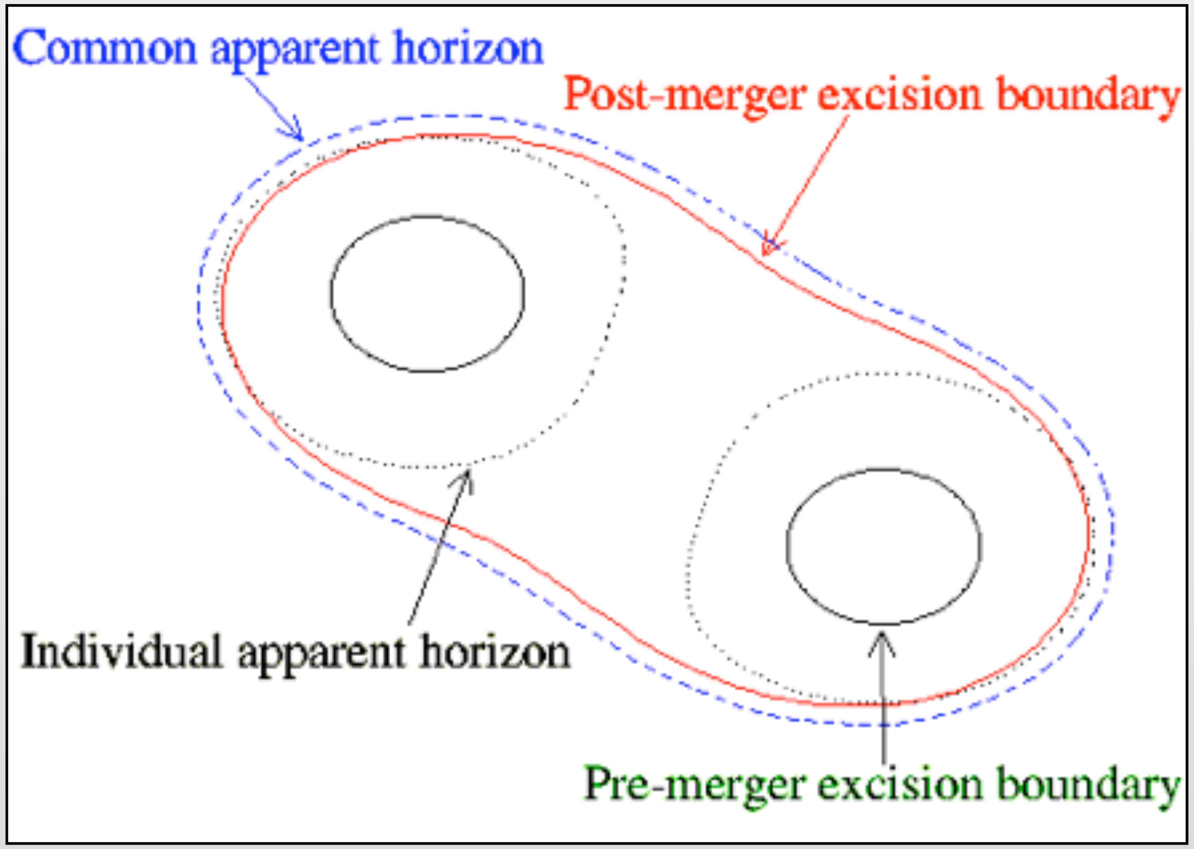
Bela Szilagyi

❖ Close to merger

- Switch domain-decomposition
- Active gauge conditions
- Adaptive Mesh Refinement

❖ After common horizon

- Switch to distorted concentric shells



Mark Scheel

V. Recent improvements

❖ Szilagyi: Adaptive Mesh Refinement

❖ Blackman & Szilagyi: Spectral Cauchy Characteristic Evolution

❖ Lovelace et al: Very high spin simulations

- S/M^2=0.97, 0.98, and going

❖ Ossokine, HP et al: Precessing binaries

❖ Ossokine, HP et al: More robust initial data

- higher spins, higher mass-ratio, lower eccentricity, less CPU time

Post-Newtonian Theory

Numerous workers over many decades

❖ Expand Einstein's equations in powers of $(v/c)^2$

- Newtonian like equations of motion

0-PN (Newtonian)

$$a_1^i = -\frac{G m_2 n_{12}^i}{r_{12}^2}$$

Post-Newtonian Theory

Numerous workers over many decades

❖ Expand Einstein's equations in powers of $(v/c)^2$

- Newtonian like equations of motion

0-PN (Newtonian)

1-PN (v/c)^2

$$a_1^i = -\frac{Gm_2 n_{12}^i}{r_{12}^2}$$
$$+\frac{1}{c^2}\left\{ \left[\frac{5G^2m_1m_2}{r_{12}^3} + \frac{4G^2m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1v_2) - 2v_2^2\right)\right] n_{12}^i \right.$$
$$\left. + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i \right\}$$

Post-Newtonian Theory

❖ Expand Einstein's equations in powers of $(v/c)^2$

- Newtonian like equations of motion

$$
\begin{aligned}
a_1^i = &-\frac{Gm_2 n_{12}^i}{r_{12}^2} \\
&+\frac{1}{c^2}\Bigg\{\left[\frac{5G^2 m_1 m_2}{r_{12}^3}+\frac{4G^2 m_2^2}{r_{12}^3}+\frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1v_2) - 2v_2^2\right)\right] n_{12}^i \\
&\qquad + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i\Bigg\} \\
&+\frac{1}{c^4}\Bigg\{\Bigg[-\frac{57G^3 m_1^2 m_2}{4r_{12}^4} - \frac{69G^3 m_1 m_2^2}{2r_{12}^4} - \frac{9G^3 m_2^3}{r_{12}^4} \\
&\qquad + \frac{Gm_2}{r_{12}^2}\Big(-\frac{15}{8}(n_{12}v_2)^4 + \frac{3}{2}(n_{12}v_2)^2 v_1^2 - 6(n_{12}v_2)^2(v_1v_2) - 2(v_1v_2)^2 + \frac{9}{2}(n_{12}v_2)^2 v_2^2 \\
&\qquad\qquad + 4(v_1v_2)v_2^2 - 2v_2^4\Big) \\
&\quad + \frac{G^2 m_1 m_2}{r_{12}^3}\left(\frac{39}{2}(n_{12}v_1)^2 - 39(n_{12}v_1)(n_{12}v_2) + \frac{17}{2}(n_{12}v_2)^2 - \frac{15}{4}v_1^2 - \frac{5}{2}(v_1v_2) + \frac{5}{4}v_2^2\right) \\
&\quad + \frac{G^2 m_2^2}{r_{12}^3}\left(2(n_{12}v_1)^2 - 4(n_{12}v_1)(n_{12}v_2) - 6(n_{12}v_2)^2 - 8(v_1v_2) + 4v_2^2\right)\Bigg] n_{12}^i \\
&\quad + \Bigg[\frac{G^2 m_2^2}{r_{12}^3}\left(-2(n_{12}v_1) - 2(n_{12}v_2)\right) + \frac{G^2 m_1 m_2}{r_{12}^3}\left(-\frac{63}{4}(n_{12}v_1) + \frac{55}{4}(n_{12}v_2)\right) \\
&\qquad + \frac{Gm_2}{r_{12}^2}\Big(-6(n_{12}v_1)(n_{12}v_2)^2 + \frac{9}{2}(n_{12}v_2)^3 + (n_{12}v_2)v_1^2 - 4(n_{12}v_1)(v_1v_2) \\
&\qquad\qquad + 4(n_{12}v_2)(v_1v_2) + 4(n_{12}v_1)v_2^2 - 5(n_{12}v_2)v_2^2\Big)\Bigg] v_{12}^i\Bigg\}
\end{aligned}
$$

2-PN (v/c)^4

Post-Newtonian Theory

- Expand Einstein's equations in powers of $(v/c)^2$
  - Newtonian like equations of motion:

2.5-PN and 3-PN

$$
\begin{aligned}
&+\frac{1}{c^5}\left\{\left[\frac{208G^3m_1m_2^2}{15r_{12}^4}(n_{12}v_{12})-\frac{24G^3m_1^2m_2}{5r_{12}^4}(n_{12}v_{12})+\frac{12G^2m_1m_2}{5r_{12}^3}(n_{12}v_{12})v_{12}^2\right]n_{12}^i\right.\\
&\qquad\left.+\left[\frac{8G^3m_1^2m_2}{5r_{12}^4}-\frac{32G^3m_1m_2^2}{5r_{12}^4}-\frac{4G^2m_1m_2}{5r_{12}^3}v_{12}^2\right]v_{12}^i\right\}\\
&+\frac{1}{c^6}\left\{\left[\frac{Gm_2}{r_{12}^2}\left(\frac{35}{16}(n_{12}v_2)^6-\frac{15}{8}(n_{12}v_2)^4v_1^2+\frac{15}{2}(n_{12}v_2)^4(v_1v_2)+3(n_{12}v_2)^2(v_1v_2)^2\right.\right.\right.\\
&\qquad-\frac{15}{2}(n_{12}v_2)^4v_2^2+\frac{3}{2}(n_{12}v_2)^2v_1^2v_2^2-12(n_{12}v_2)^2(v_1v_2)v_2^2-2(v_1v_2)^2v_2^2\\
&\qquad\left.+\frac{15}{2}(n_{12}v_2)^2v_2^4+4(v_1v_2)v_2^4-2v_2^6\right)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{171}{8}(n_{12}v_1)^4+\frac{171}{2}(n_{12}v_1)^3(n_{12}v_2)-\frac{723}{4}(n_{12}v_1)^2(n_{12}v_2)^2\right.\\
&\qquad+\frac{383}{2}(n_{12}v_1)(n_{12}v_2)^3-\frac{455}{8}(n_{12}v_2)^4+\frac{229}{4}(n_{12}v_1)^2v_1^2\\
&\qquad-\frac{205}{2}(n_{12}v_1)(n_{12}v_2)v_1^2+\frac{191}{4}(n_{12}v_2)^2v_1^2-\frac{91}{8}v_1^4-\frac{229}{2}(n_{12}v_1)^2(v_1v_2)\\
&\qquad+244(n_{12}v_1)(n_{12}v_2)(v_1v_2)-\frac{225}{2}(n_{12}v_2)^2(v_1v_2)+\frac{91}{2}v_1^2(v_1v_2)\\
&\qquad-\frac{177}{4}(v_1v_2)^2+\frac{229}{4}(n_{12}v_1)^2v_2^2-\frac{283}{2}(n_{12}v_1)(n_{12}v_2)v_2^2\\
&\qquad\left.+\frac{259}{4}(n_{12}v_2)^2v_2^2-\frac{91}{4}v_1^2v_2^2+43(v_1v_2)v_2^2-\frac{81}{8}v_2^4\right)\\
&\quad+\frac{G^2m_2^2}{r_{12}^3}\left(-6(n_{12}v_1)^2(n_{12}v_2)^2+12(n_{12}v_1)(n_{12}v_2)^3+6(n_{12}v_2)^4\right.\\
&\qquad+4(n_{12}v_1)(n_{12}v_2)(v_1v_2)+12(n_{12}v_2)^2(v_1v_2)+4(v_1v_2)^2\\
&\qquad\left.-4(n_{12}v_1)(n_{12}v_2)v_2^2-12(n_{12}v_2)^2v_2^2-8(v_1v_2)v_2^2+4v_2^4\right)\\
&\quad+\frac{G^3m_2^3}{r_{12}^4}\left(-(n_{12}v_1)^2+2(n_{12}v_1)(n_{12}v_2)+\frac{43}{2}(n_{12}v_2)^2+18(v_1v_2)-9v_2^2\right)\\
&\quad+\frac{G^3m_1m_2^2}{r_{12}^4}\left(\frac{415}{8}(n_{12}v_1)^2-\frac{375}{4}(n_{12}v_1)(n_{12}v_2)+\frac{1113}{8}(n_{12}v_2)^2-\frac{615}{64}(n_{12}v_{12})^2\pi^2\right.\\
&\qquad\left.+18v_1^2+\frac{123}{64}\pi^2v_{12}^2+33(v_1v_2)-\frac{33}{2}v_2^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{G^3m_1^2m_2}{r_{12}^4}\left(-\frac{45887}{168}(n_{12}v_1)^2+\frac{24025}{42}(n_{12}v_1)(n_{12}v_2)-\frac{10469}{42}(n_{12}v_2)^2+\frac{48197}{840}v_1^2\right.\\
&\qquad\left.-\frac{36227}{420}(v_1v_2)+\frac{36227}{840}v_2^2+110(n_{12}v_{12})^2\ln\left(\frac{r_{12}}{r_1'}\right)-22v_{12}^2\ln\left(\frac{r_{12}}{r_1'}\right)\right)\\
&+\frac{16G^4m_2^4}{r_{12}^5}+\frac{G^4m_1^2m_2^2}{r_{12}^5}\left(175-\frac{41}{16}\pi^2\right)+\frac{G^4m_1^3m_2}{r_{12}^5}\left(-\frac{3187}{1260}+\frac{44}{3}\ln\left(\frac{r_{12}}{r_1'}\right)\right)\\
&+\frac{G^4m_1m_2^3}{r_{12}^5}\left(\frac{110741}{630}-\frac{41}{16}\pi^2-\frac{44}{3}\ln\left(\frac{r_{12}}{r_2'}\right)\right)\Bigg]n_{12}^i\\
&+\left[\frac{Gm_2}{r_{12}^2}\left(\frac{15}{2}(n_{12}v_1)(n_{12}v_2)^4-\frac{45}{8}(n_{12}v_2)^5-\frac{3}{2}(n_{12}v_2)^3v_1^2+6(n_{12}v_1)(n_{12}v_2)^2(v_1v_2)\right.\right.\\
&\qquad-6(n_{12}v_2)^3(v_1v_2)-2(n_{12}v_2)(v_1v_2)^2-12(n_{12}v_1)(n_{12}v_2)^2v_2^2+12(n_{12}v_2)^3v_2^2\\
&\qquad+(n_{12}v_2)v_1^2v_2^2-4(n_{12}v_1)(v_1v_2)v_2^2+8(n_{12}v_2)(v_1v_2)v_2^2+4(n_{12}v_1)v_2^4\\
&\qquad\left.-7(n_{12}v_2)v_2^4\right)\\
&\quad+\frac{G^2m_2^2}{r_{12}^3}\left(-2(n_{12}v_1)^2(n_{12}v_2)+8(n_{12}v_1)(n_{12}v_2)^2+2(n_{12}v_2)^3+2(n_{12}v_1)(v_1v_2)\right.\\
&\qquad\left.+4(n_{12}v_2)(v_1v_2)-2(n_{12}v_1)v_2^2-4(n_{12}v_2)v_2^2\right)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{243}{4}(n_{12}v_1)^3+\frac{565}{4}(n_{12}v_1)^2(n_{12}v_2)-\frac{269}{4}(n_{12}v_1)(n_{12}v_2)^2\right.\\
&\qquad-\frac{95}{12}(n_{12}v_2)^3+\frac{207}{8}(n_{12}v_1)v_1^2-\frac{137}{8}(n_{12}v_2)v_1^2-36(n_{12}v_1)(v_1v_2)\\
&\qquad\left.+\frac{27}{4}(n_{12}v_2)(v_1v_2)+\frac{81}{8}(n_{12}v_1)v_2^2+\frac{83}{8}(n_{12}v_2)v_2^2\right)\\
&\quad+\frac{G^3m_2^3}{r_{12}^4}\left(4(n_{12}v_1)+5(n_{12}v_2)\right)\\
&\quad+\frac{G^3m_1m_2^2}{r_{12}^4}\left(-\frac{307}{8}(n_{12}v_1)+\frac{479}{8}(n_{12}v_2)+\frac{123}{32}(n_{12}v_{12})\pi^2\right)\\
&\quad\left.\left.+\frac{G^3m_1^2m_2}{r_{12}^4}\left(\frac{31397}{420}(n_{12}v_1)-\frac{36227}{420}(n_{12}v_2)-44(n_{12}v_{12})\ln\left(\frac{r_{12}}{r_1'}\right)\right)\right]v_{12}^i\right\}
\end{aligned}
$$

Post-Newtonian Theory

- Newtonian-like equations known to (v/c)^7
  - GW emission appears at (v/c)^5; fractional accuracy "only" (v/c)^2
- For *circular orbits*, one can compute directly GW energy flux

$$\mathcal{L} = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \Big[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)$$
$$+ \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\Big] x^3$$
$$+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}.$$

1995
1996
2002
2002

- Inspiral-rate w/ fractional accuracy (v/c)^7
- Spin contributions work in progress, as are the (v/c)^8 terms

Periastron advance

A.H. Mroue, HP, L.E. Kidder, S.A. Teukolsky 2009

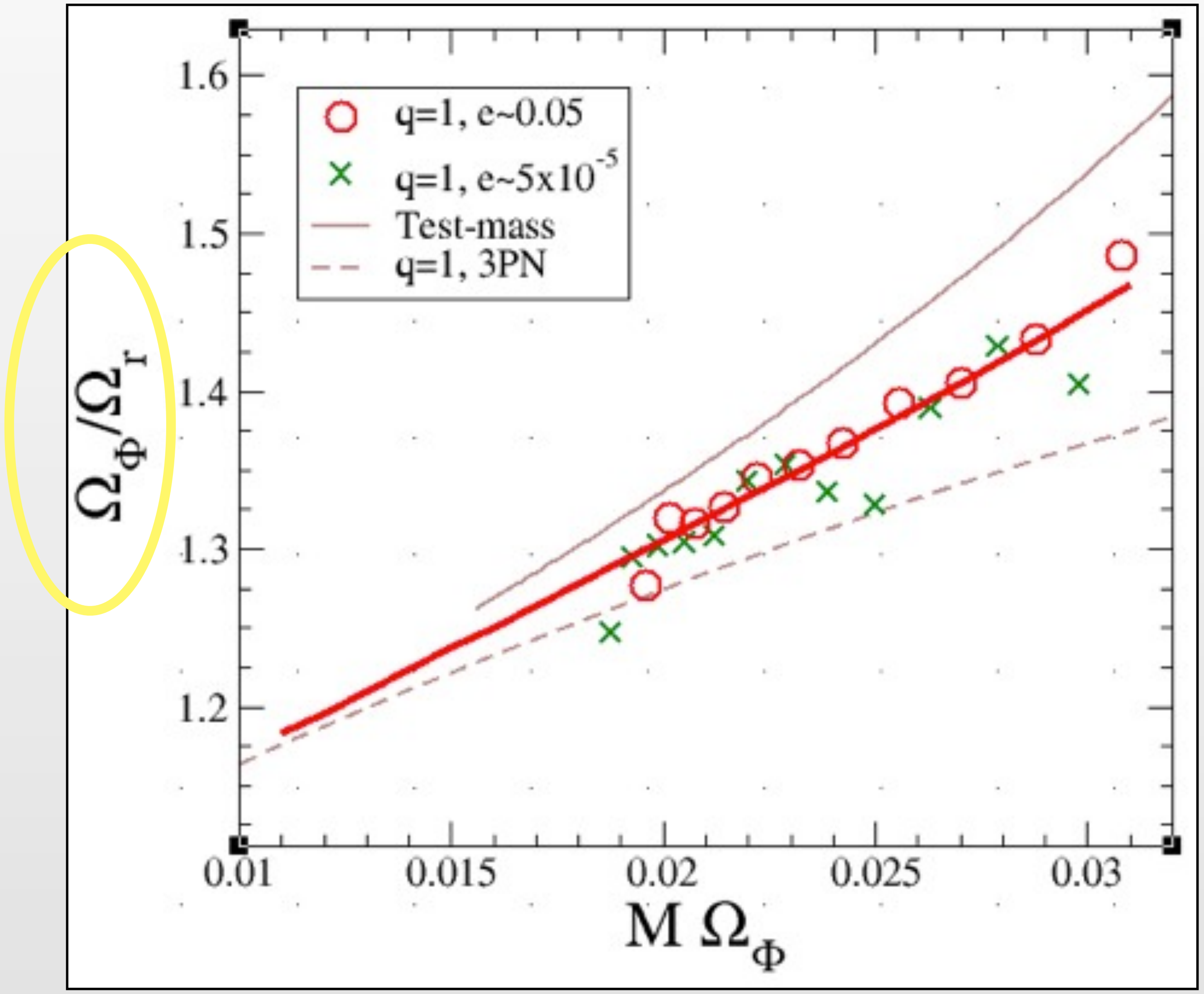
A. Le Tiec, A.H. Mroue, L. Barack, A. Buonanno, HP, N. Sago, A. Taracchini, 2011

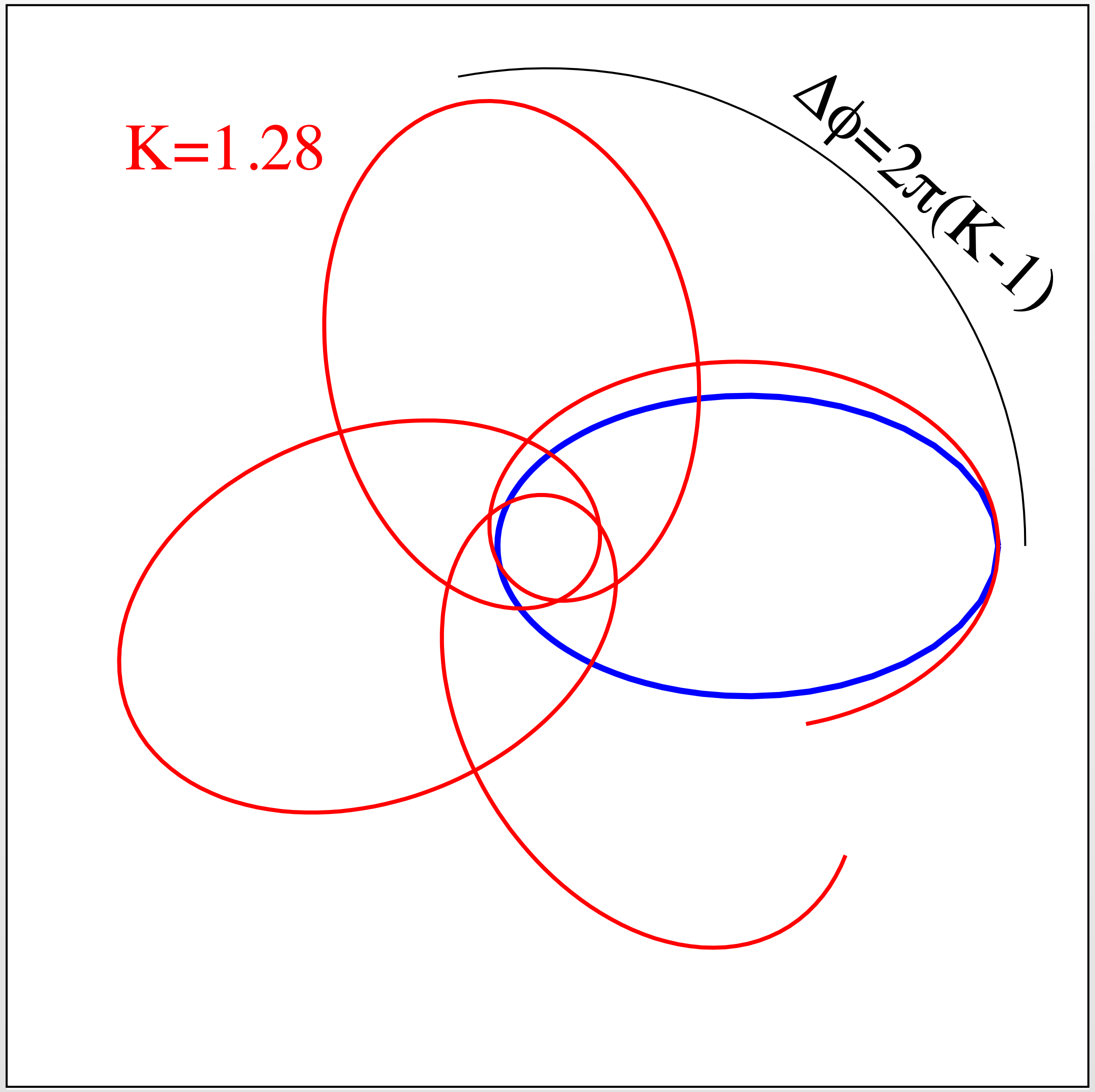
A. Le Tiec + UMD + SXS, 1309.0541

T. Hinderer + UMD + SXS, 1309.0544

Periastron advance

Equal mass BBH

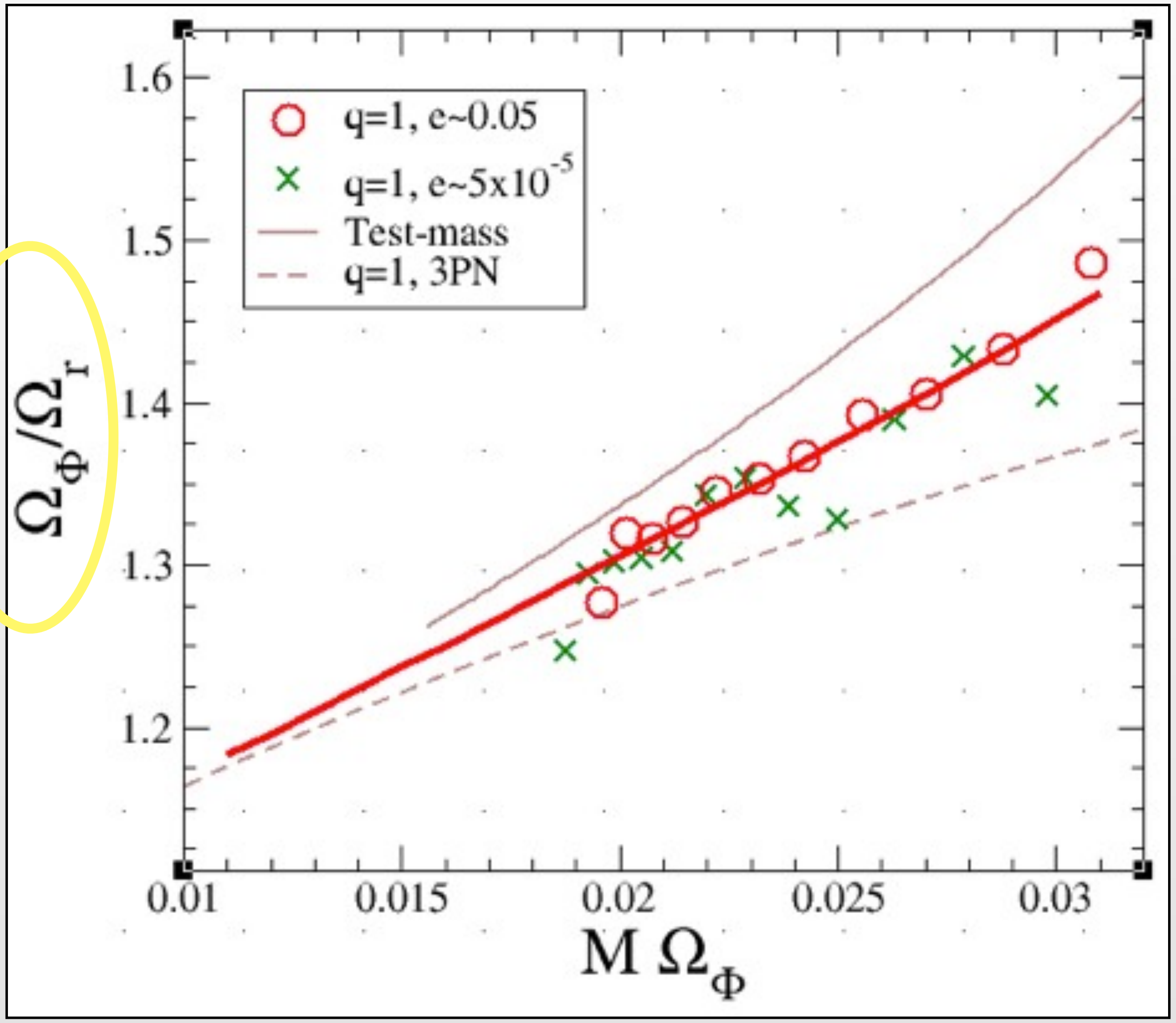




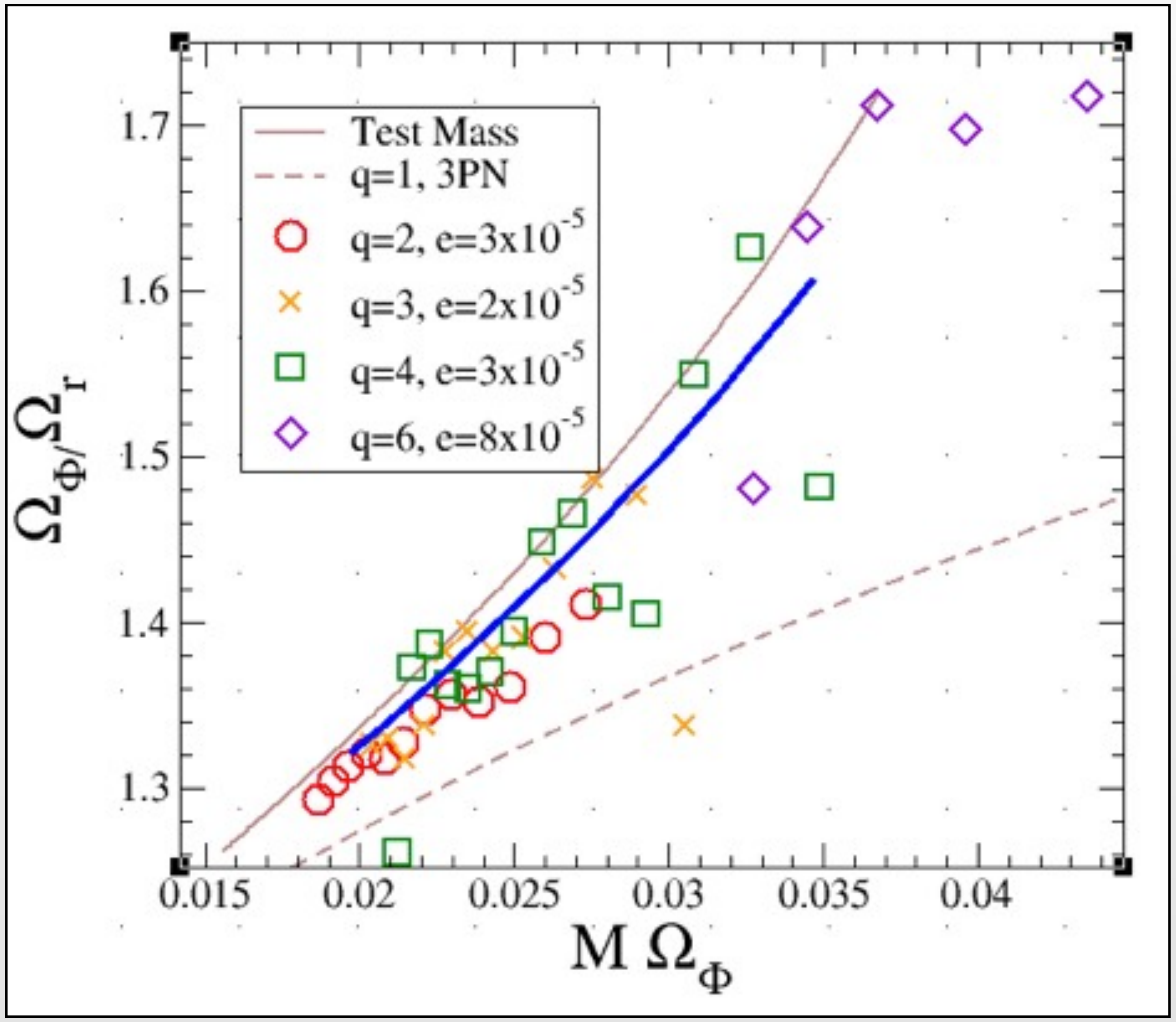
Mroue, et al 2009, Le Tiec et al 2011

Periastron advance

Equal mass BBH



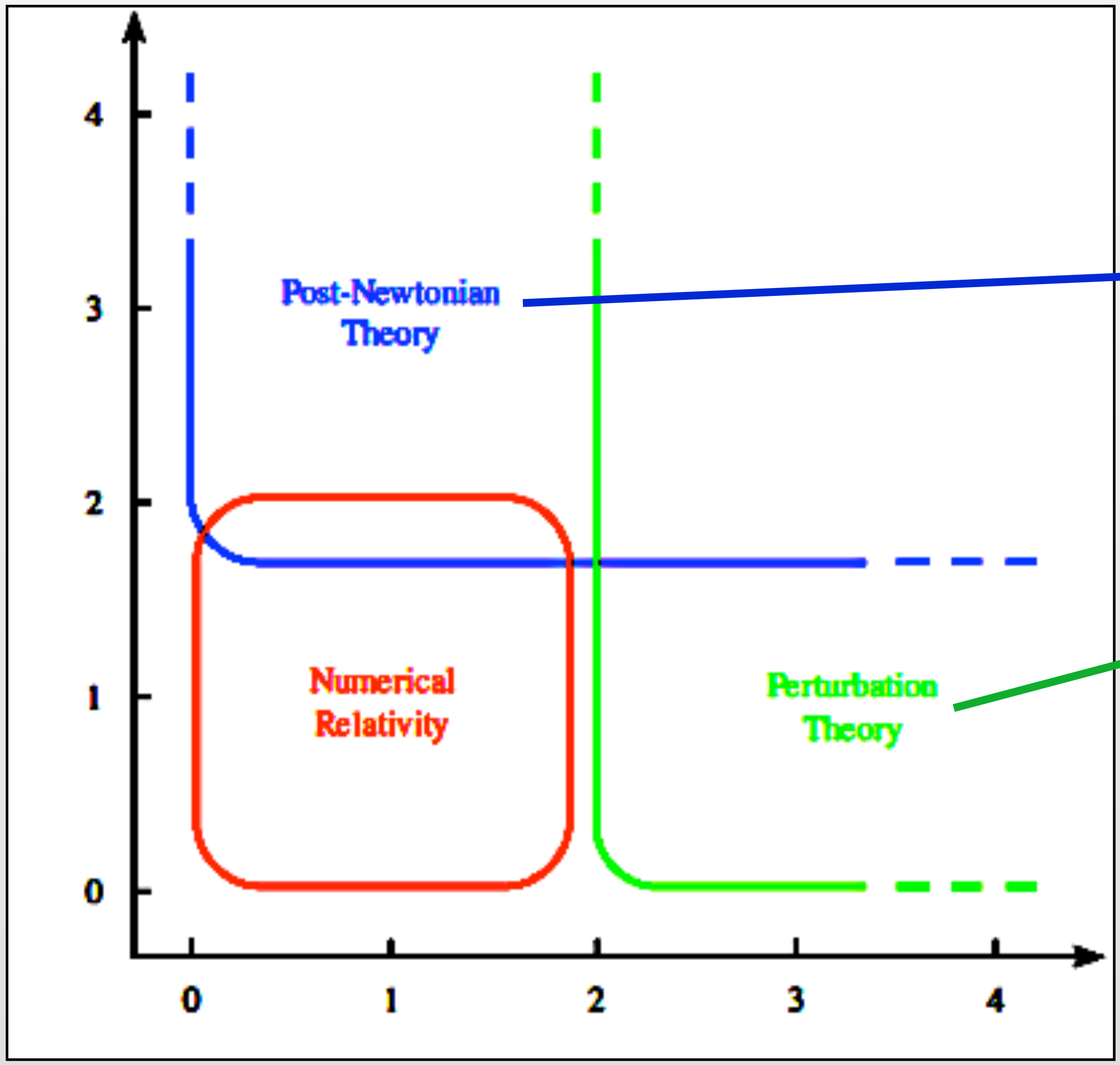
Mass-ratio q=1/8



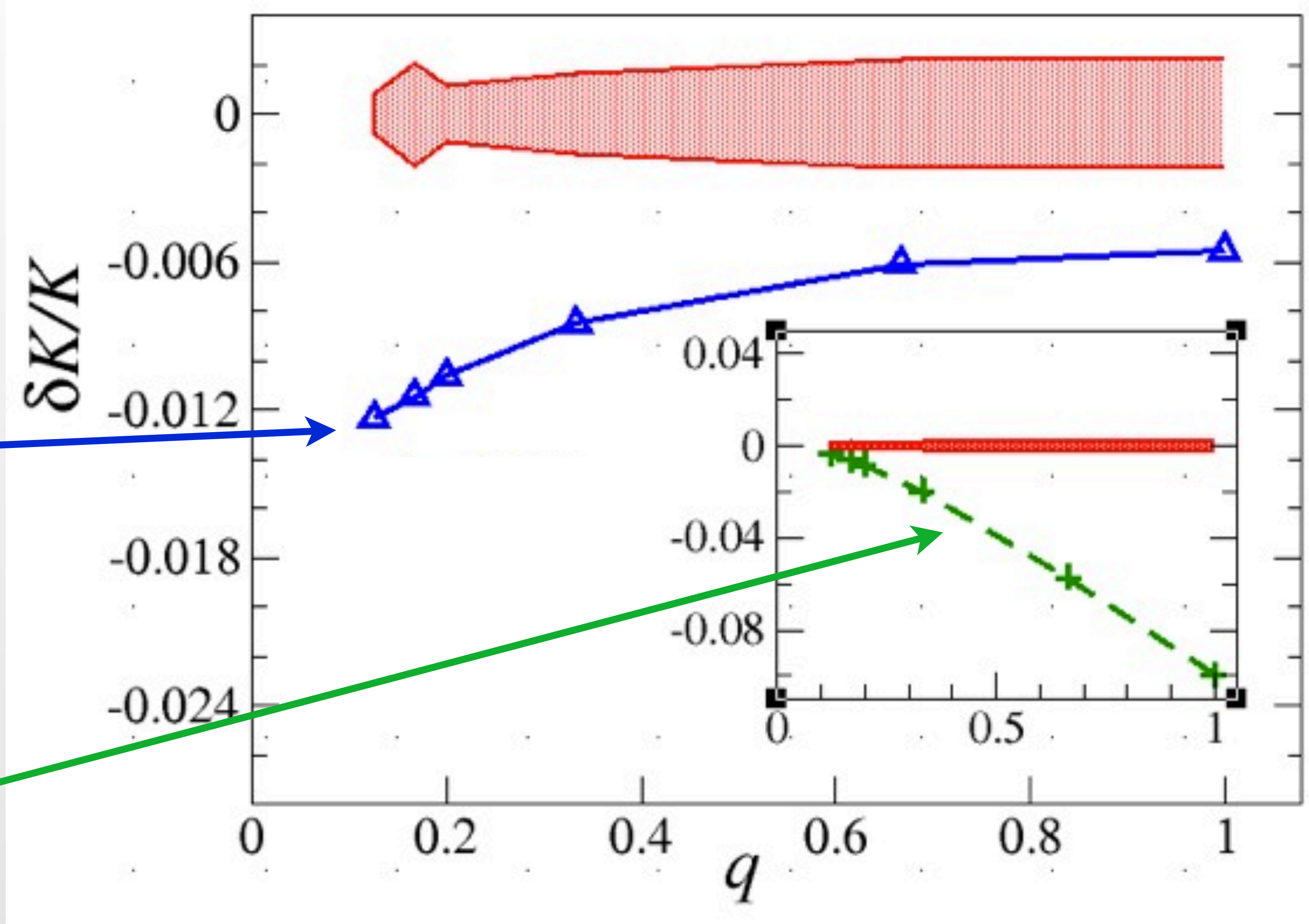
Mroue, et al 2009, Le Tiec et al 2011

Comparison w/ analytical calc's

log(r/M)



log(M1/M2)

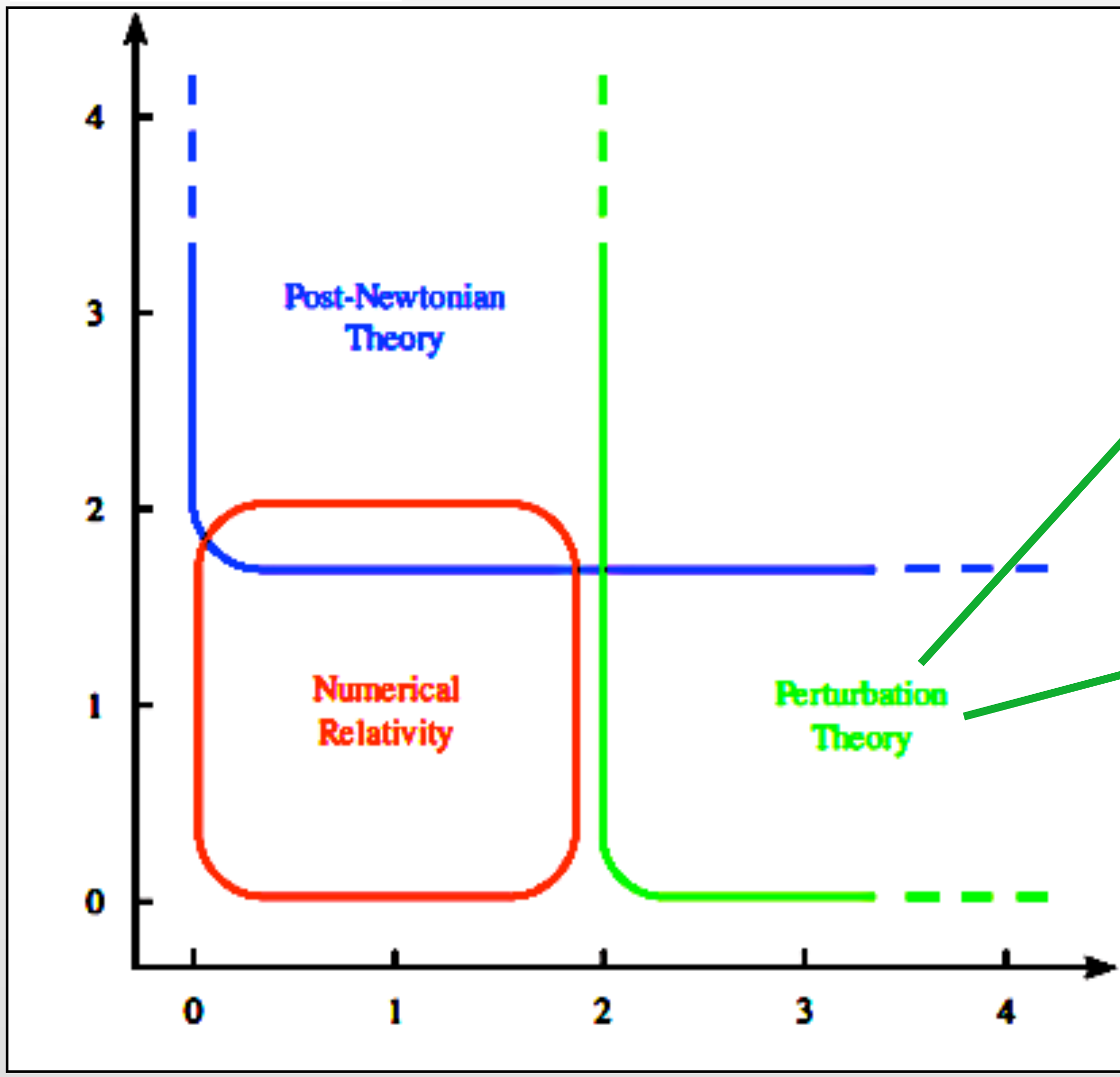


Le Tiec et al 2011

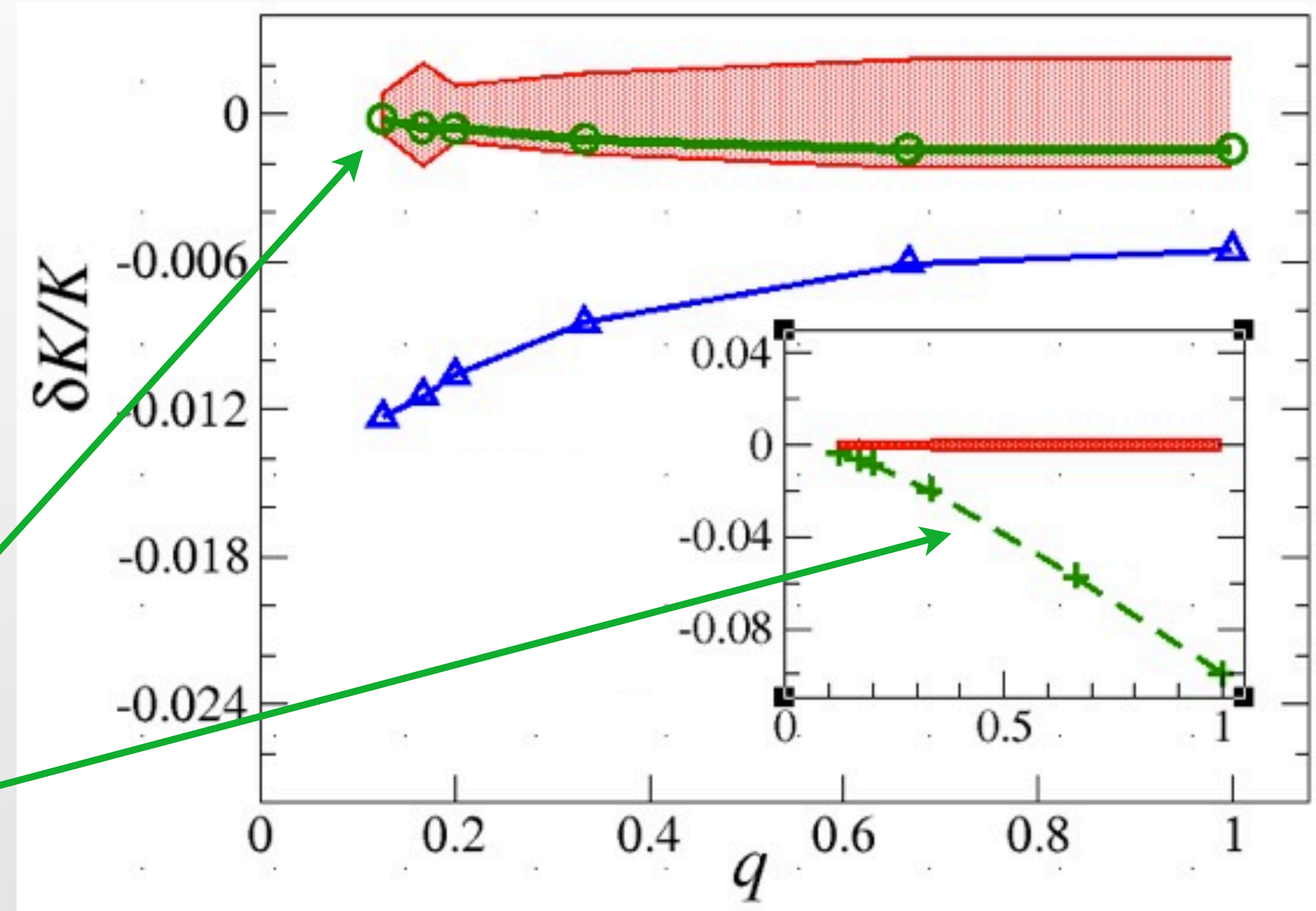
Blanchet et al, arXiv:1007.2614

Comparison w/ analytical calc's

log(r/M)



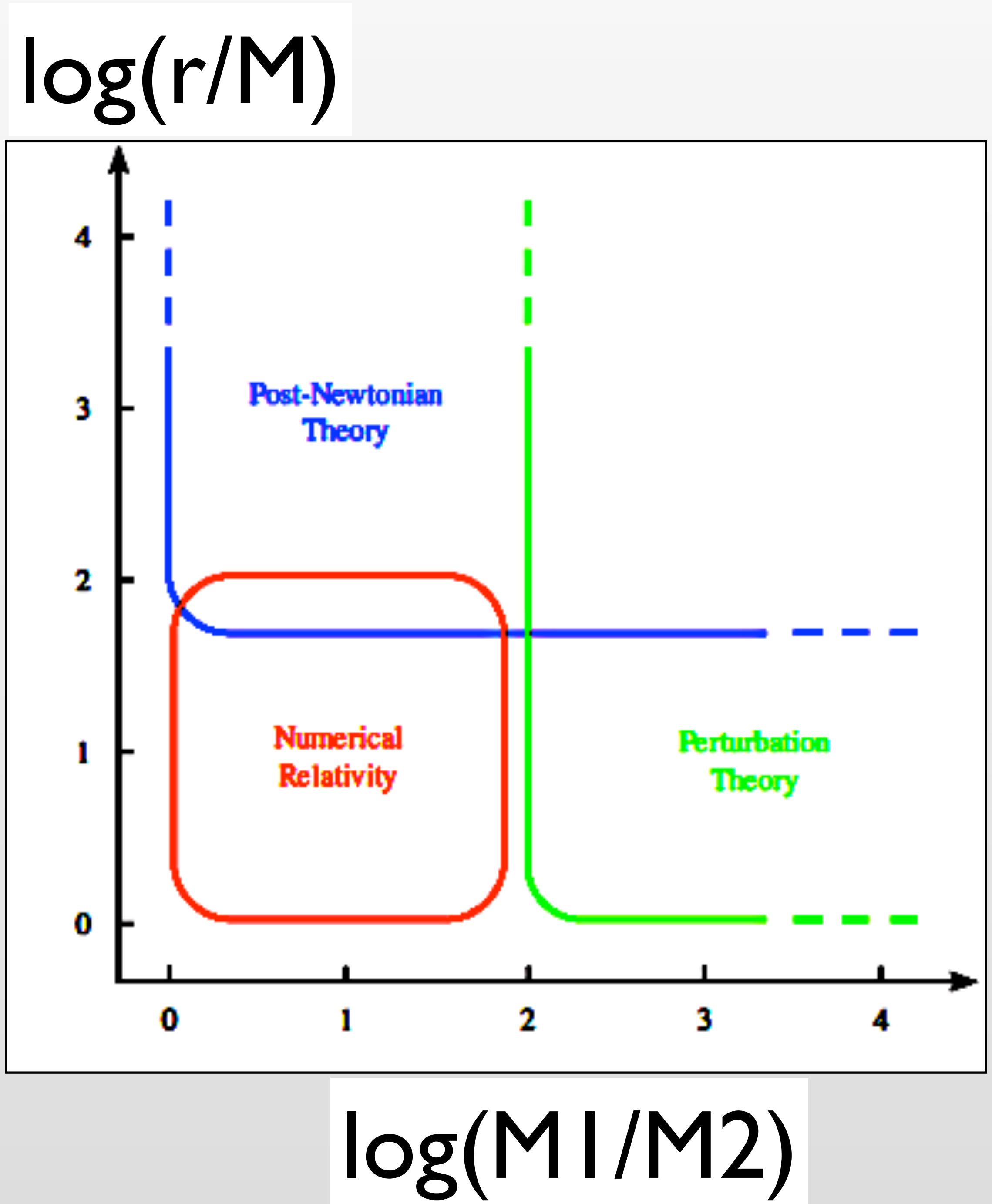
log(M1/M2)



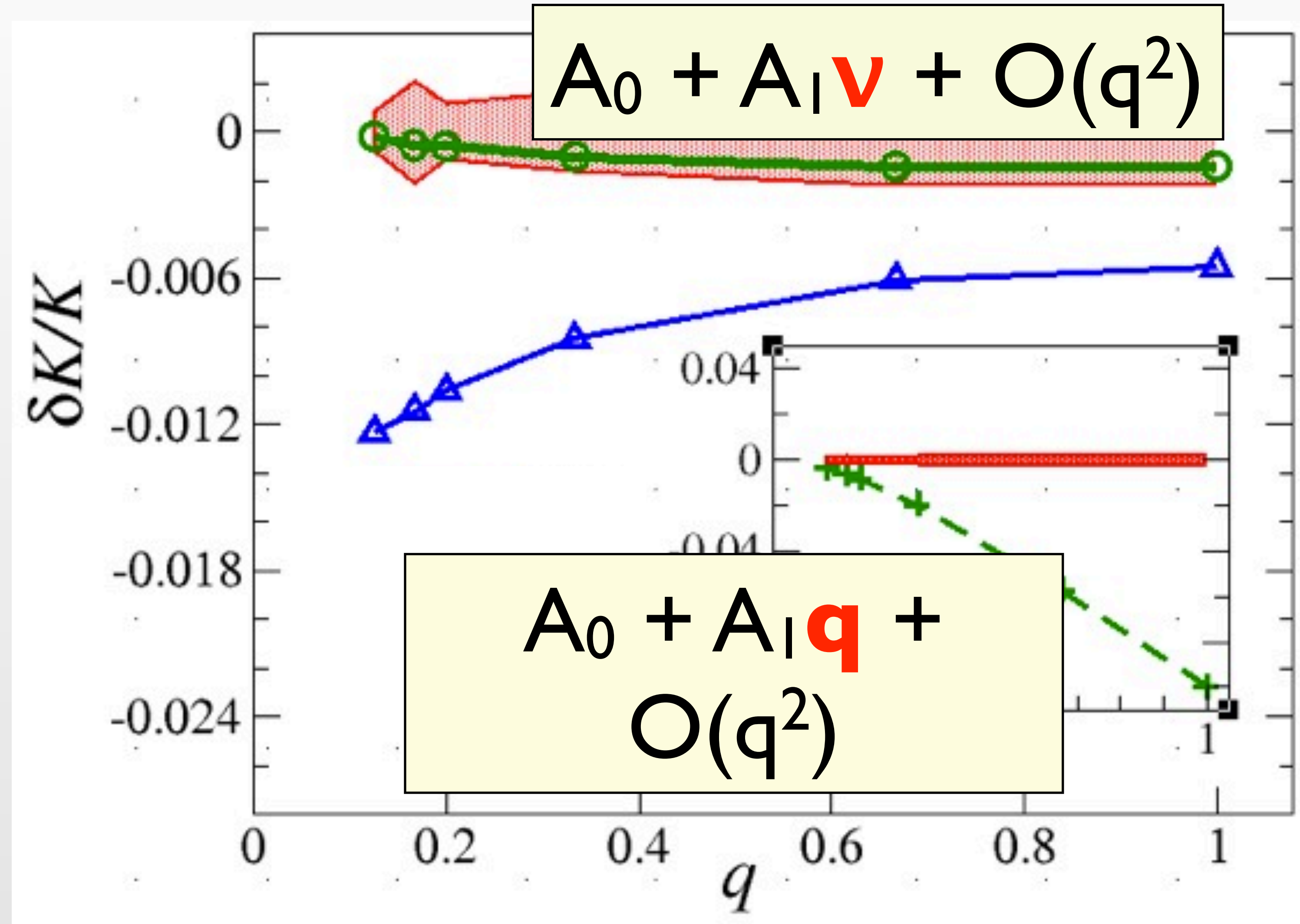
Le Tiec et al 2011

Blanchet et al, arXiv:1007.2614

Comparison w/ analytical calc's



Blanchet et al, arXiv:1007.2614



Expressed in symmetric mass-ratio $\nu=q/(1+q)^2$, perturbation theory works at equal masses!

Le Tiec et al 2011

Periastron Advance for Spinning Primary

- ❖ No results for self-force calculations on Kerr background
- ❖ Can we *measure* the self-force term *from simulations*?

Le Tiec ea 1309.0541

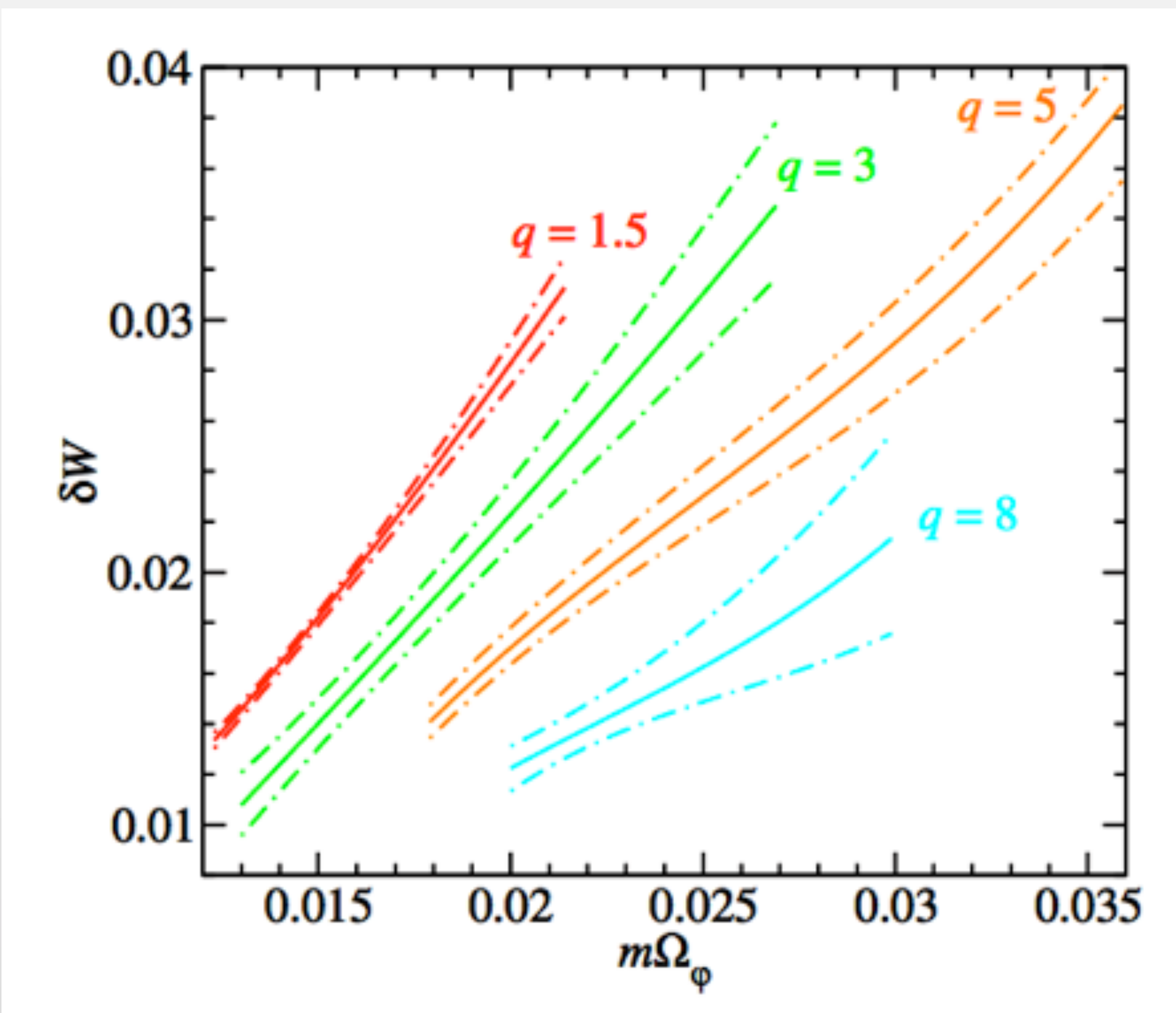
- ❖ Available information:
  - Kerr geodesics
  - post-Newtonian expansions
  - Combine and incorporate symmetry under exchange of BH labels:

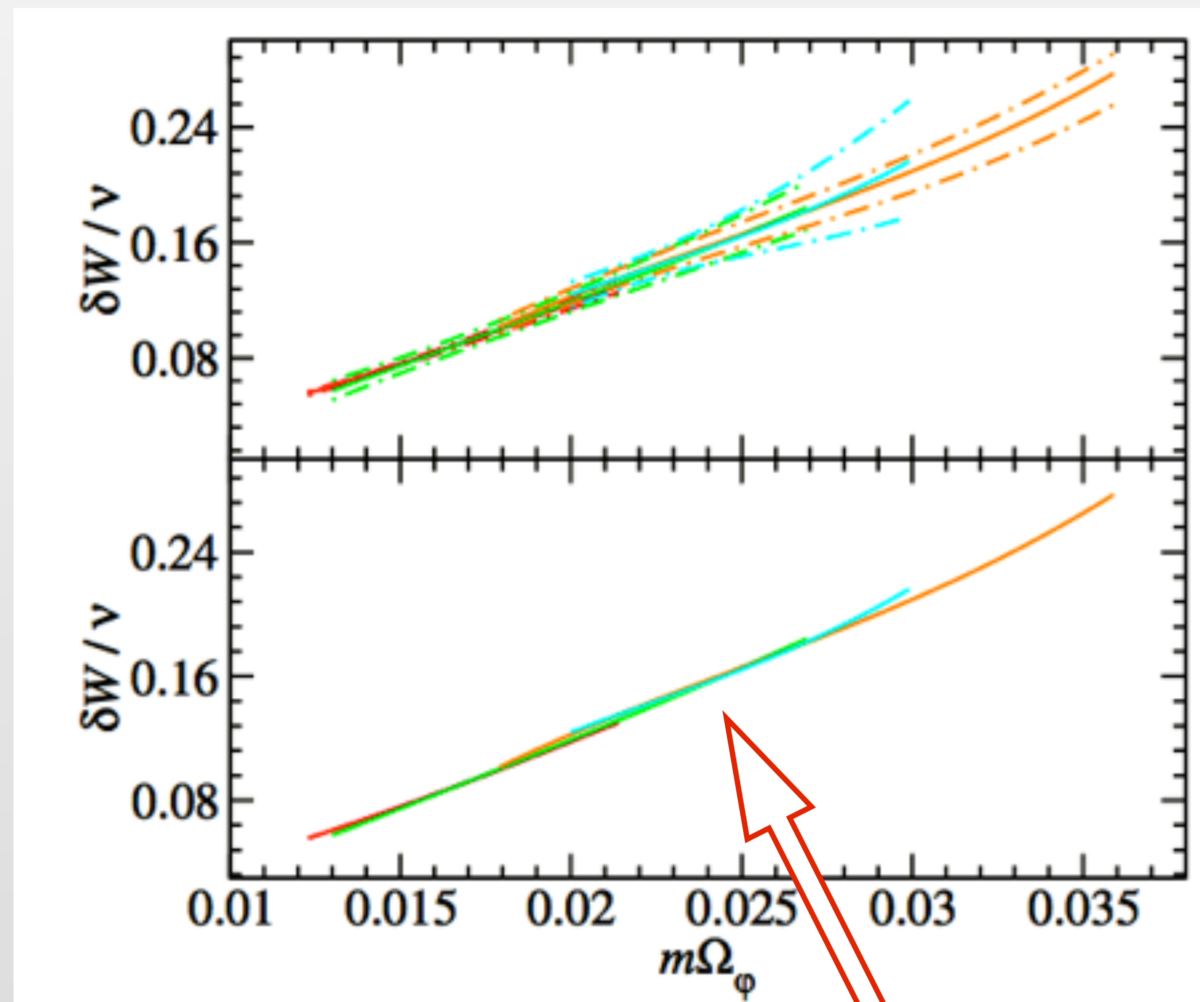
$$W_{\rm SB} = 1 - 6x + \left[(4+4\Delta-2\nu)\chi_1 + (4-4\Delta-2\nu)\chi_2\right]x^{3/2} + \left[\left(-\frac{3}{2}-\frac{3}{2}\Delta+3\nu\right)\chi_1^2 - 6\nu\chi_1\chi_2 + \left(-\frac{3}{2}+\frac{3}{2}\Delta+3\nu\right)\chi_2^2\right]x^2$$
$$- \left[\left(2+2\Delta+\frac{45}{2}\nu+\frac{17}{2}\Delta\nu\right)\chi_1 + \left(2-2\Delta+\frac{45}{2}\nu-\frac{17}{2}\Delta\nu\right)\chi_2\right]x^{5/2} + \left[\left(4+4\Delta+\frac{15}{2}\nu+\frac{31}{2}\Delta\nu-11\nu^2\right)\chi_1^2\right.$$
$$\left. + (36+22\nu)\nu\chi_1\chi_2 + \left(4-4\Delta+\frac{15}{2}\nu-\frac{31}{2}\Delta\nu-11\nu^2\right)\chi_2^2\right]x^3 + \mathcal{O}(x^{7/2}). \quad (34)$$

Measuring self-force for spinning BBH

- ❖ The full periastron advance is $$W = W_{\rm SB} + \sum_{n=1}^{\infty} \nu^n W_n \,,$$

- ❖ Consider difference $$\delta W \equiv W_{\rm NR} - W_{\rm SB} = W_1 \nu + \mathcal{O}(\nu^2)$$





Le Tiec ea 1309.0541

$$W_1^{\rm fit} = 14x^2 \, \frac{1 + c_1 x}{1 + c_2 x + c_3 x^2}$$

Measuring self-force for spinning BBH

- The gravitational self-force contribution is the entire term proportional to the mass-ratio $\bar{q} = m_{\rm small}/m_{\rm big} \leq 1$

$$W = W_{\rm Kerr}(x;\chi) + \bar{q}\, W_{\rm GSF}(x;\chi) + \mathcal{O}(\bar{q}^2)$$

$$W_{\rm GSF} = W_1 - 10\chi v^3 + 6\chi^2 v^4 - 27\chi v^5 + 25\chi^2 v^6 + (\gamma - 4)\,\chi^3 v^7 \,.$$

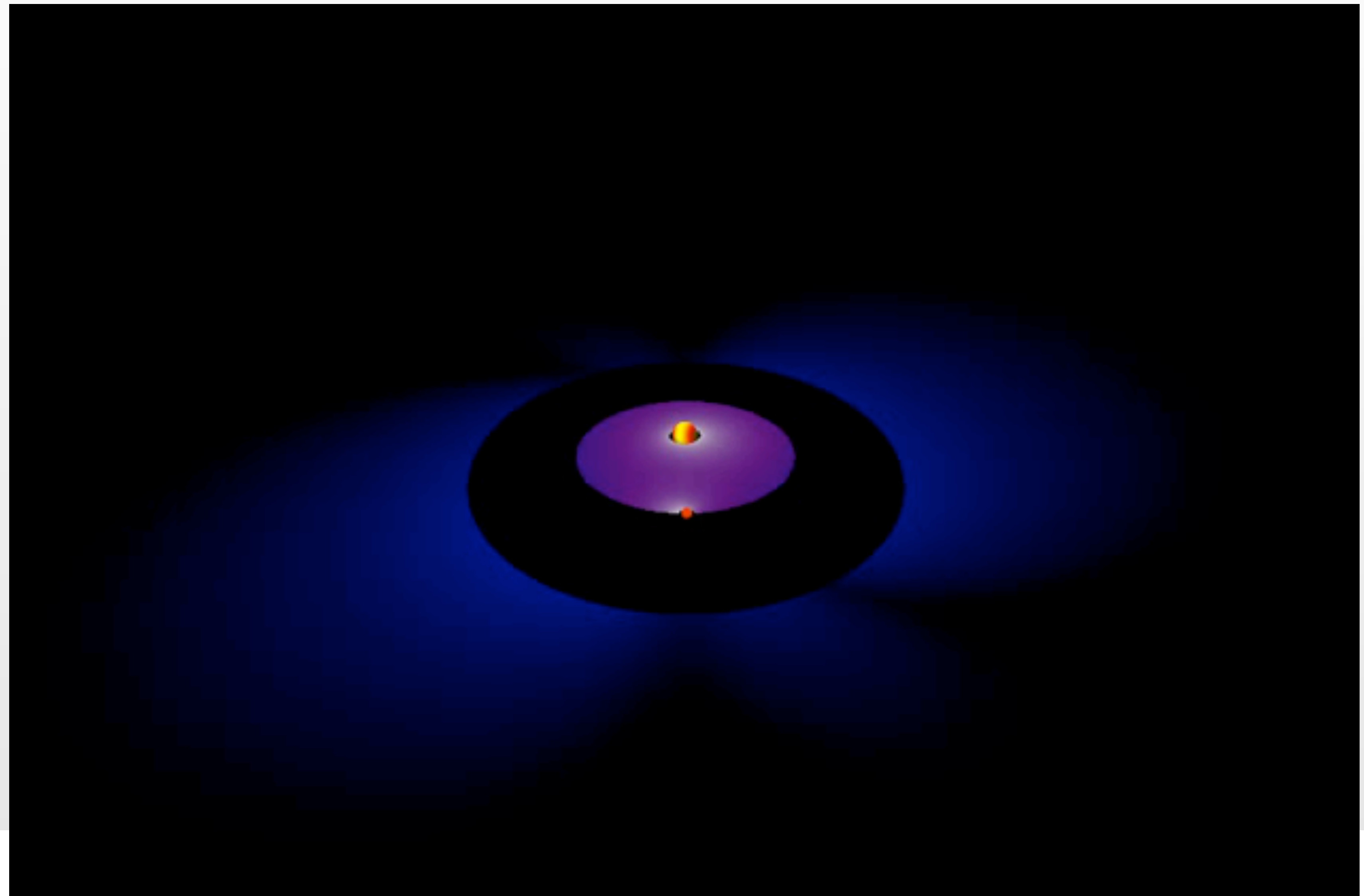
$$W_1^{\rm fit} = 14x^2 \frac{1 + c_1 x}{1 + c_2 x + c_3 x^2} \,,$$

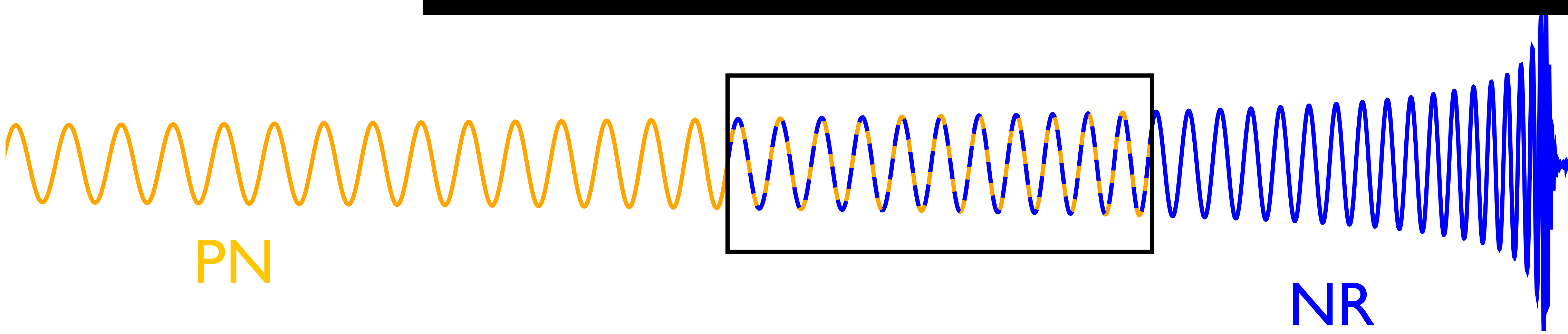
- Have computed self-force result from NR simulations at mass-ratios 1,..., 1/8 !

$$c_1^{\rm down} = 1.1973 \,,$$
$$c_2^{\rm down} = -6.88457 \,,$$
$$c_3^{\rm down} = 37.3406 \,.$$

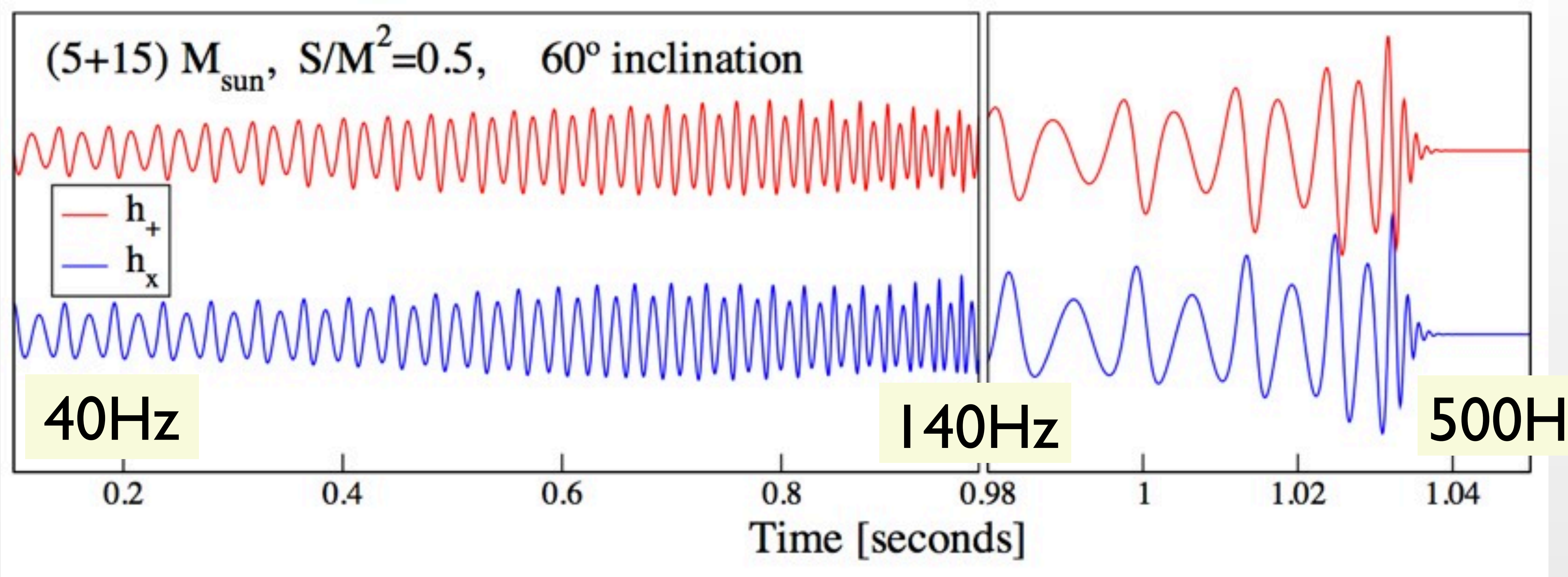
Le Tiec ea 1309.0541

Gravitational Waves

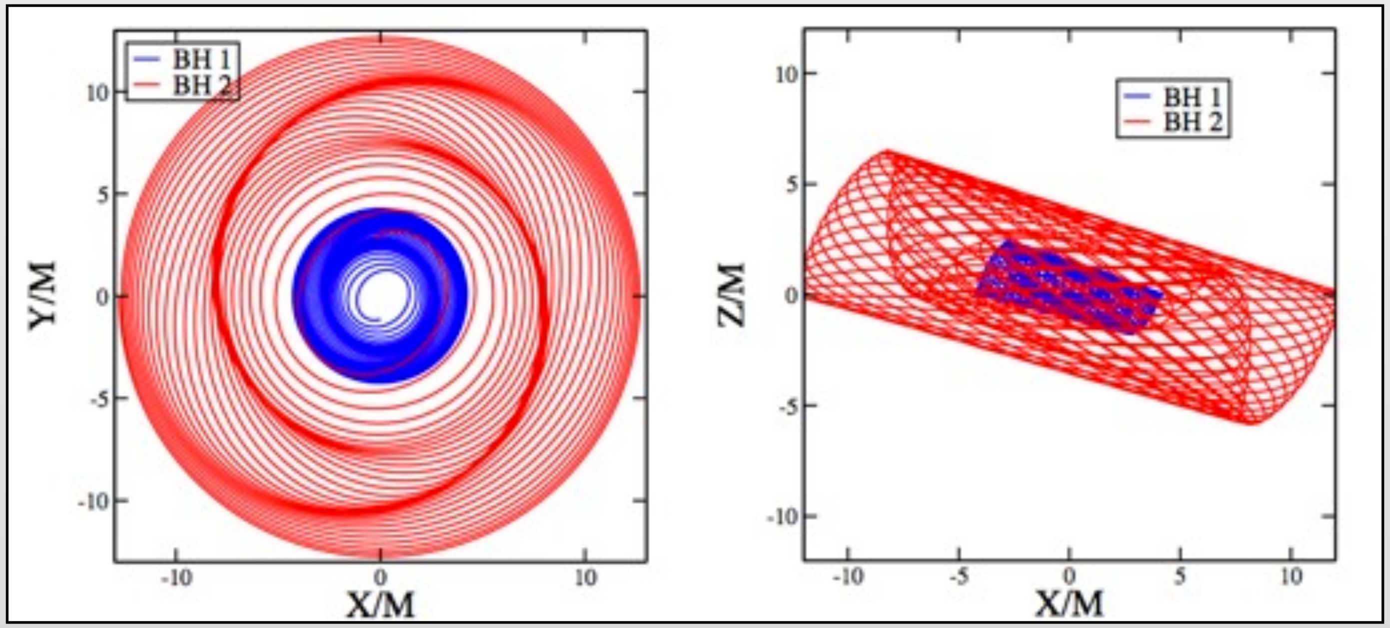




Precessing BH-BH



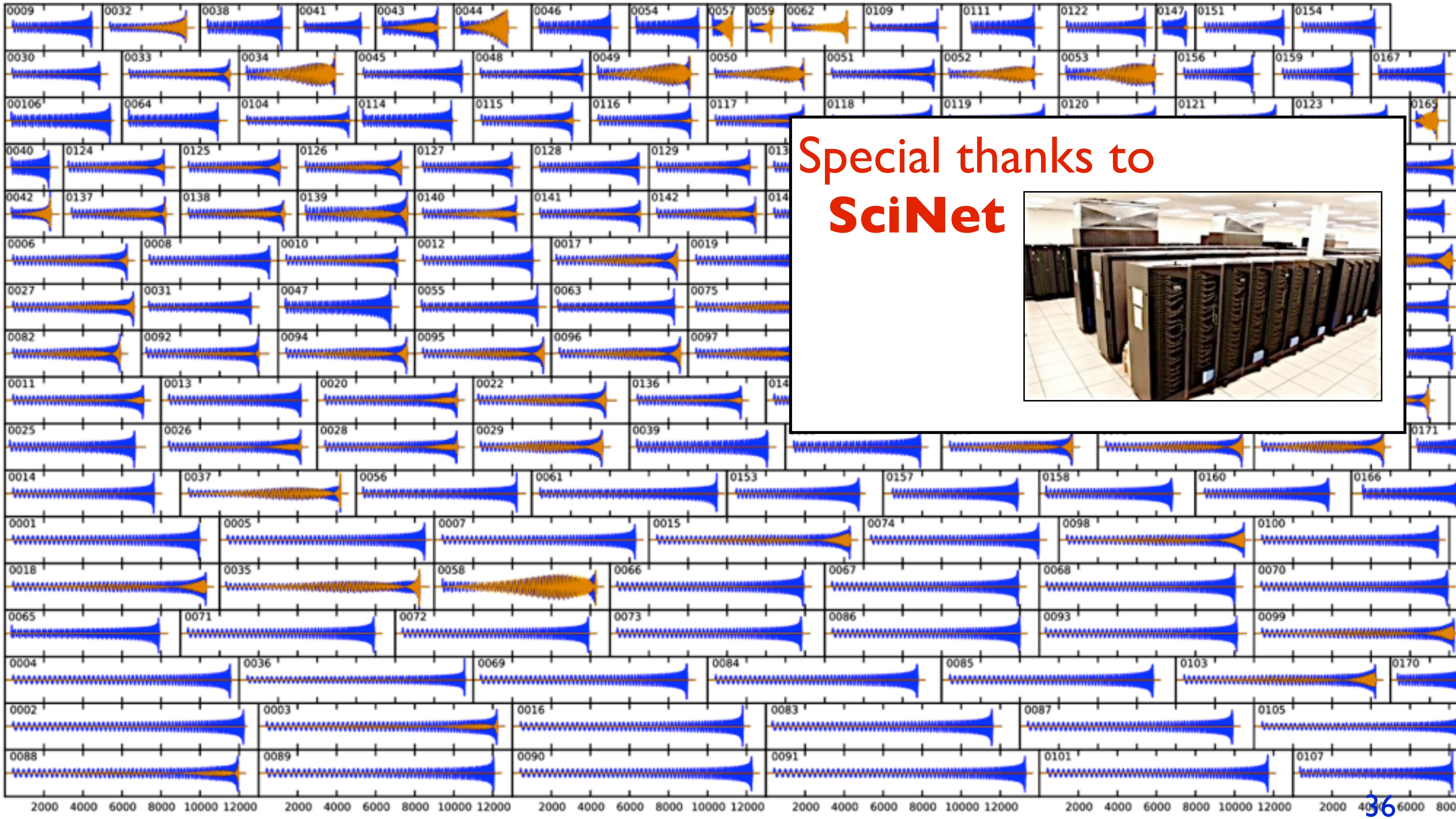
- Modulated amplitude
- Temporal harmonics
- Dependence on inclination
- Modified phasing



SXS numerical waveform catalog

A. Mroue, M.Scheel, B.Szilagyi, HP et al, 1304.6077, PRL 2013

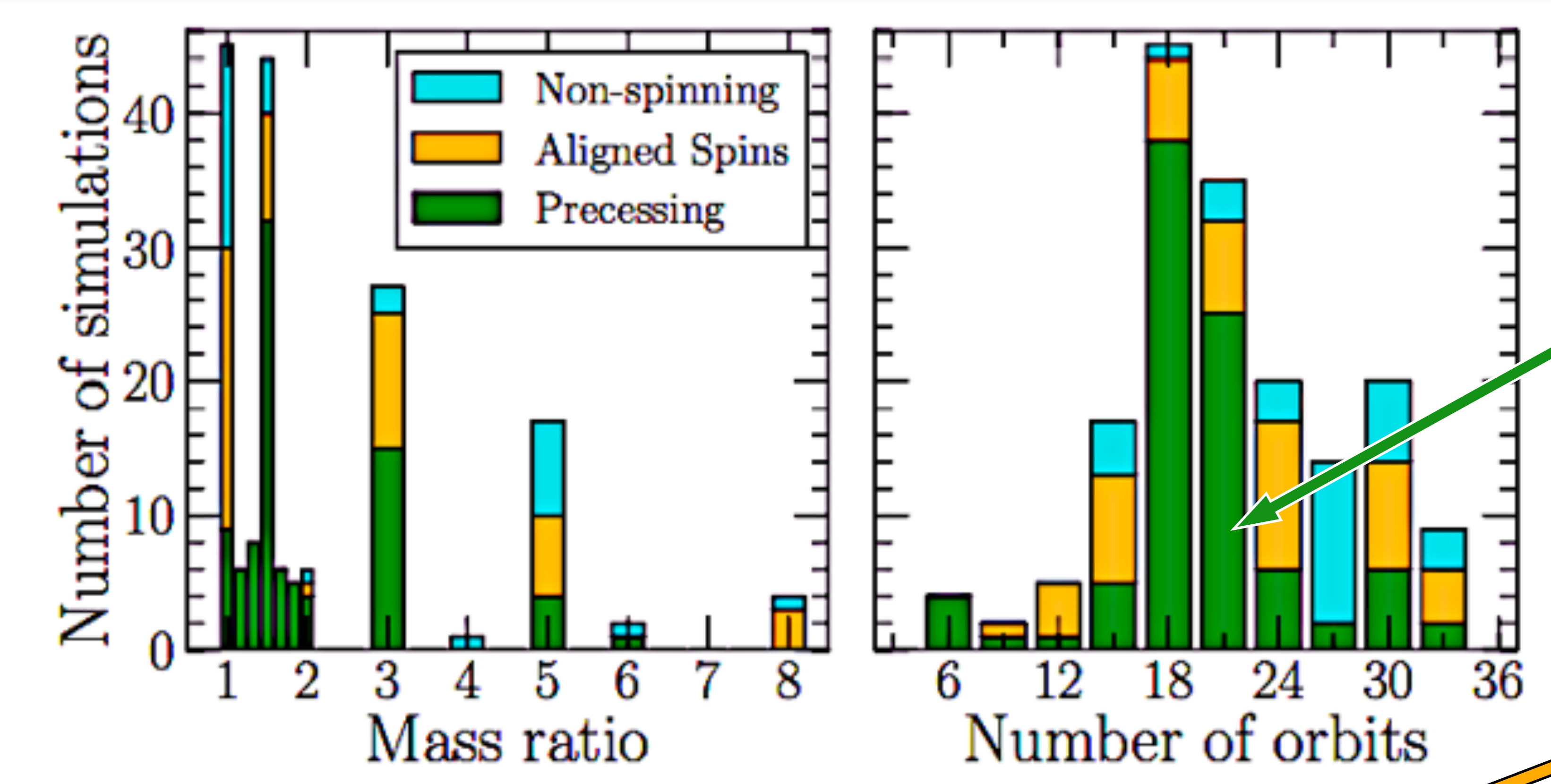
Data publicly available www.black-holes.org/waveforms



Special thanks to

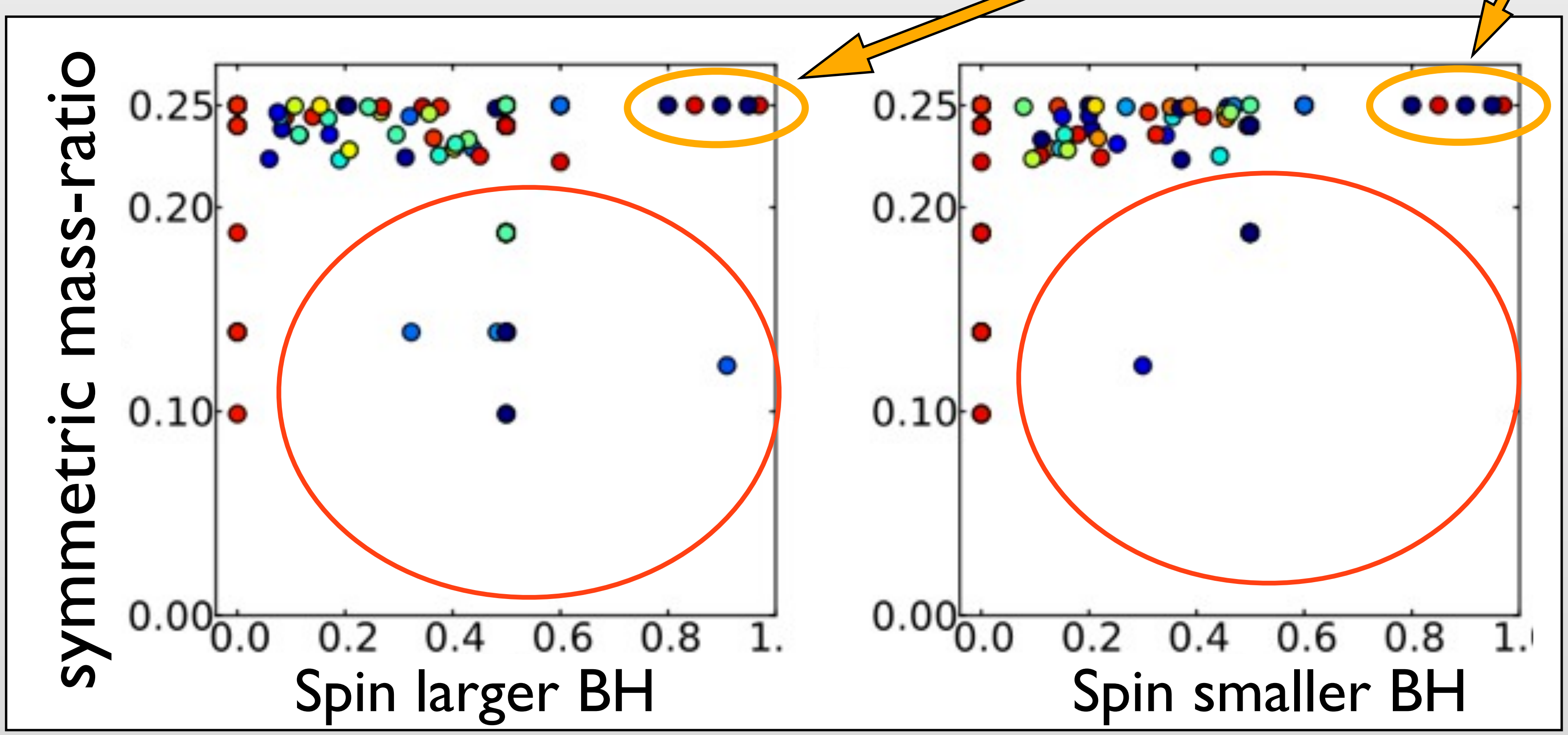
SciNet

SXS catalog: parameter space coverage



91 precessing binaries

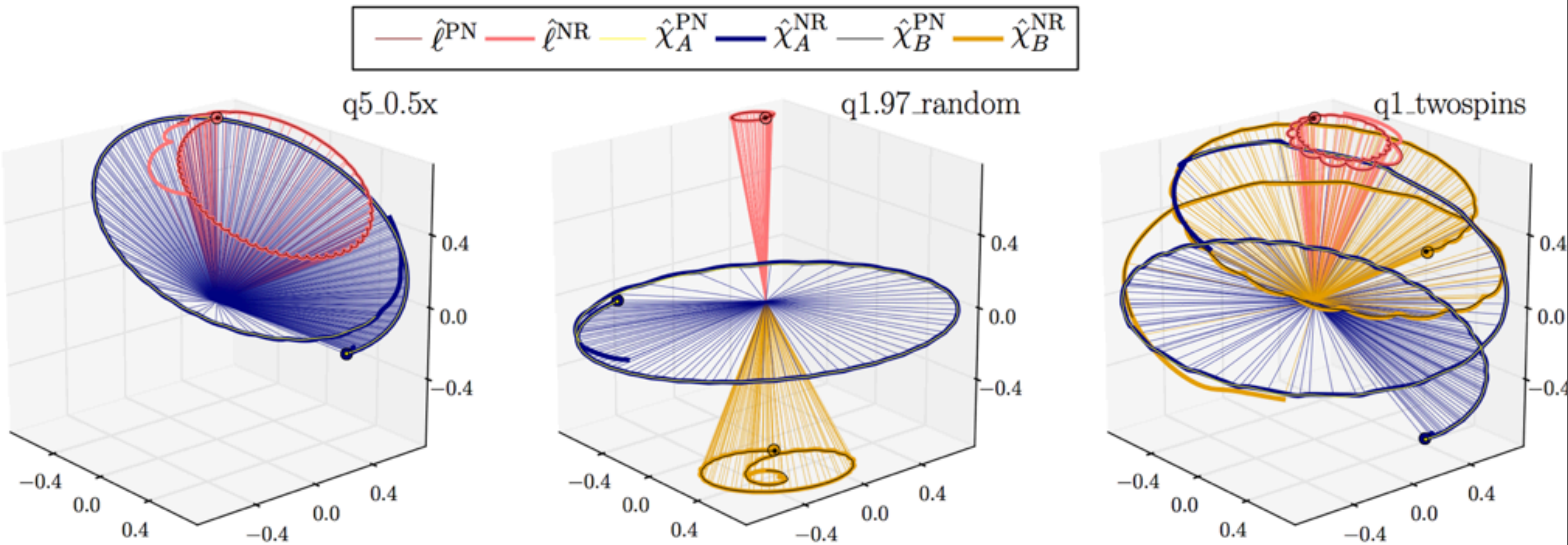
Max aligned spin 0.98



Few unequal-mass systems with spin

Investigate precession dynamics

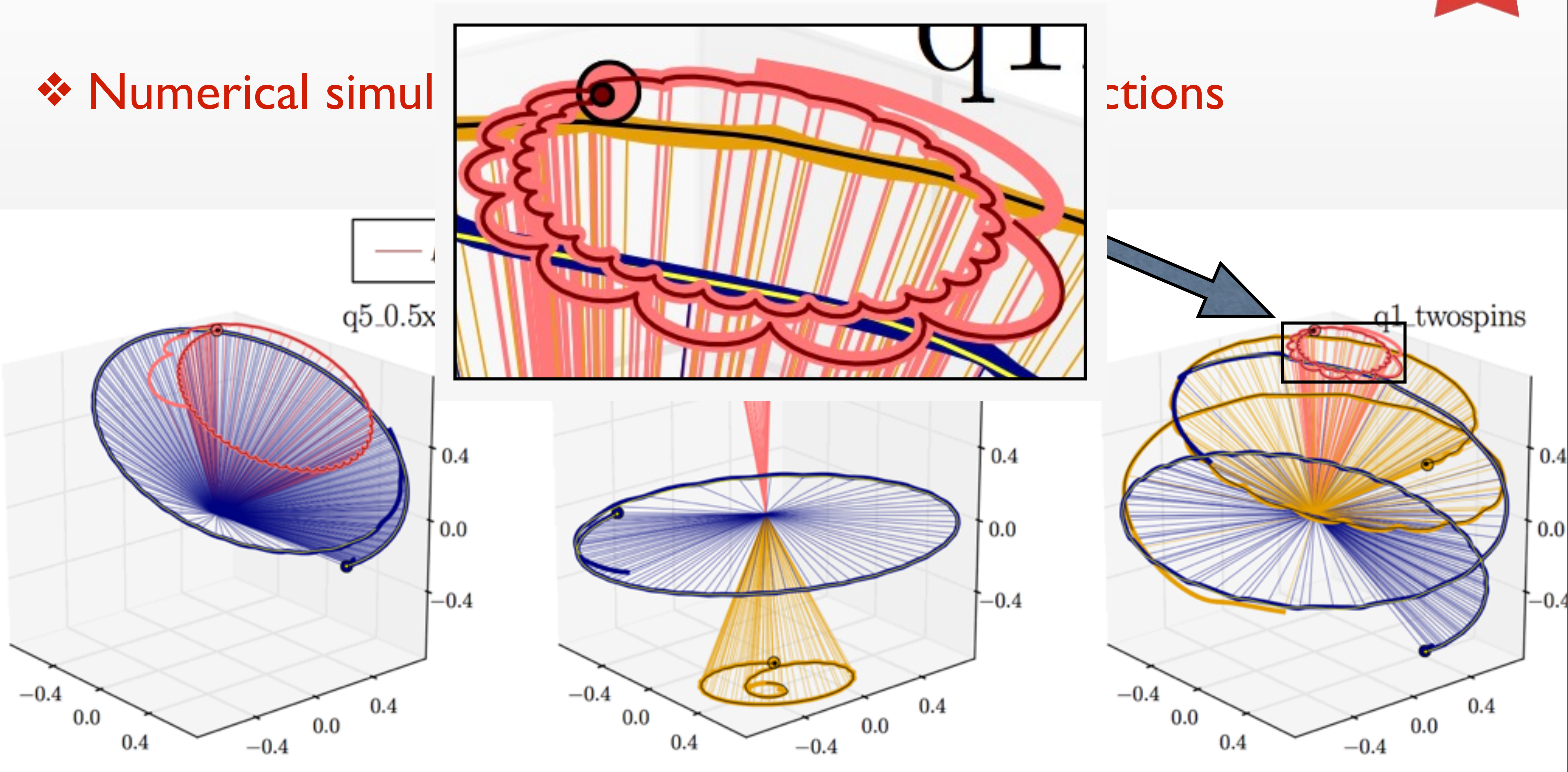
❖ Numerical simulations & post-Newtonian predictions



Ossokine ea, in prep

Numerics (red) agree w/ post-Newtonian (black)

❖ Numerical simul [...] ctions

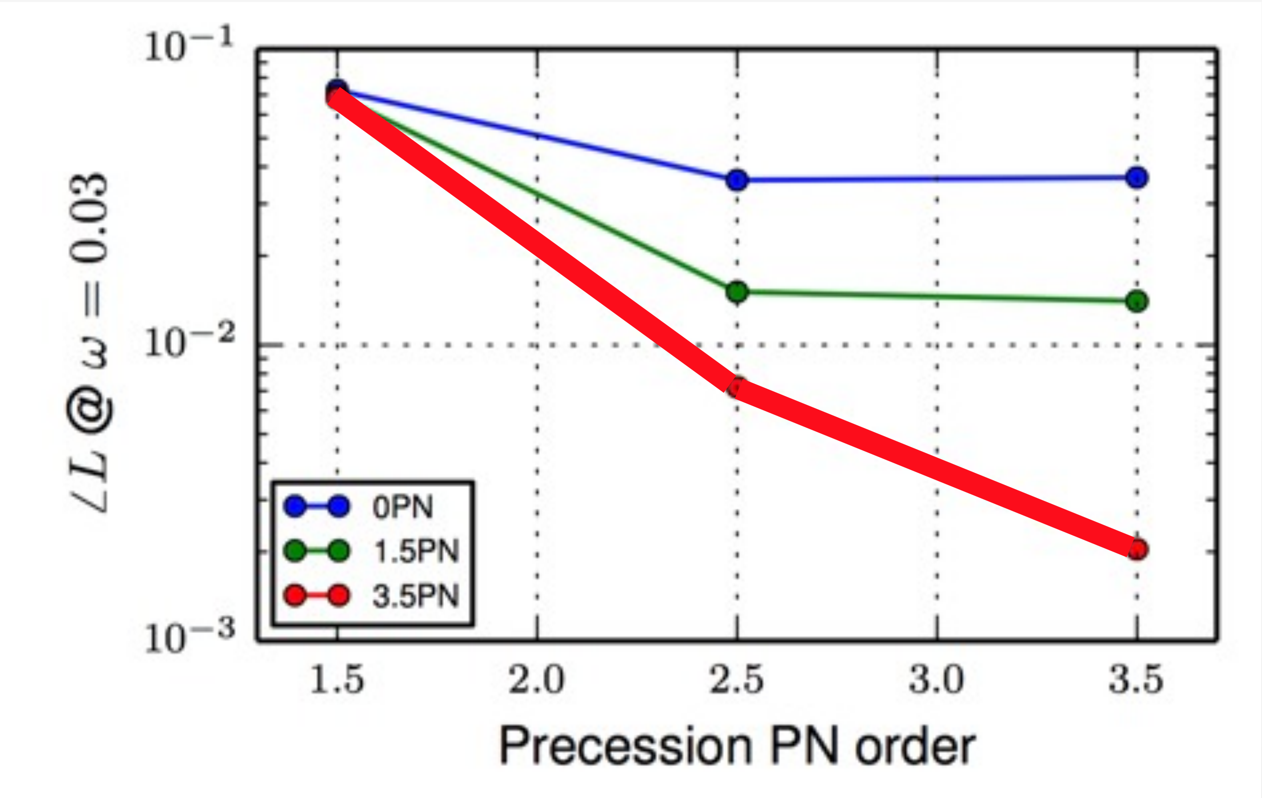


Ossokine ea, in prep

Convergence of precessing PN

orbital plane precession

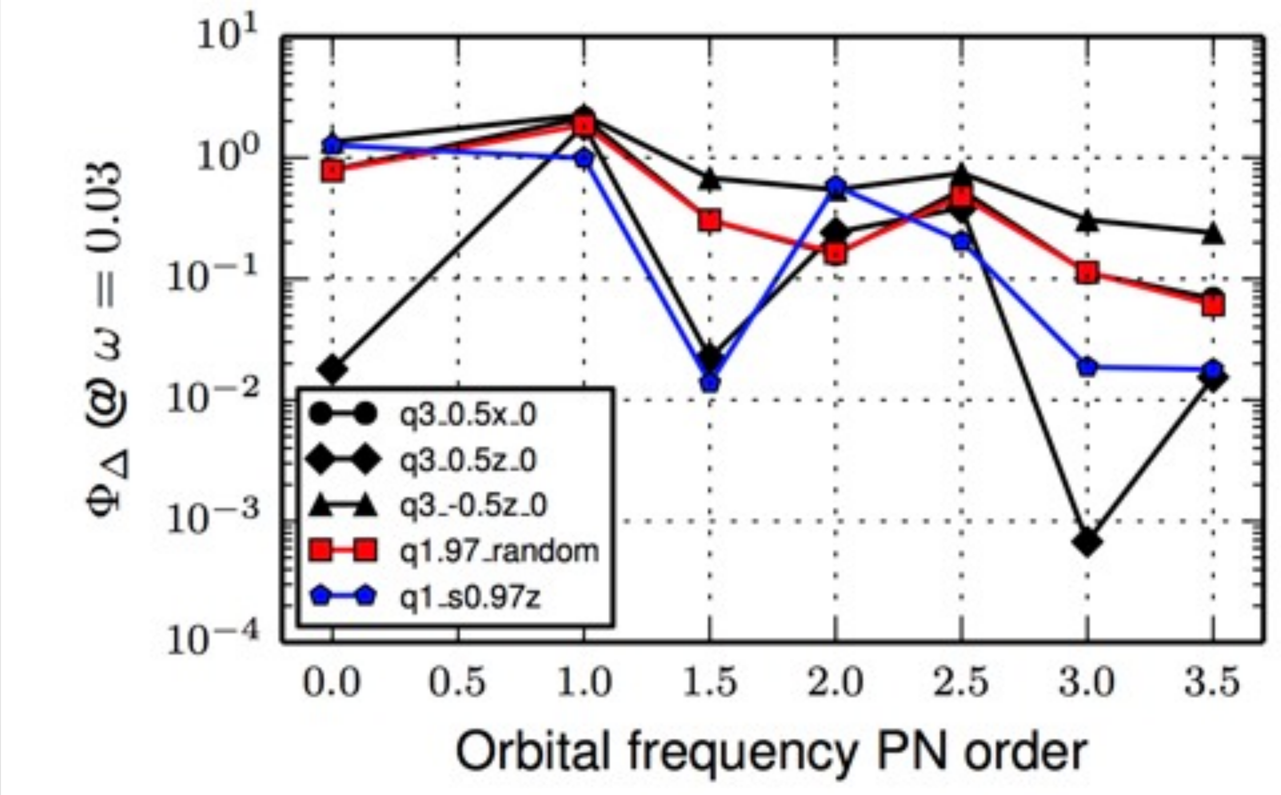
quick, monotonic convergence



Ossokine ea, in prep

orbital phase

slow, erratic convergence



As bad as non-precessing PN requires many-orbit NR & careful modeling

Effective-one-body models

- ❖ Buonanno, Damour 1999; many papers since
- ❖ Effective Hamiltonian to capture conservative dynamics

$$H = \mu\sqrt{p_r^2 + A(r)\left[1 + \frac{p_r^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]}, \qquad A(r) = \sum_{k=0}^{4}\frac{a_k(\nu)}{r^k} + \frac{a_5(\nu)}{r^5}$$

- Radiation reaction terms

$$\frac{dp_r}{dt} = -\frac{\partial H}{\partial p_r} + a^r_{\rm RR}\frac{\dot r}{r^2\Omega}\widehat{\mathcal{F}}_\phi$$

$$\frac{dp_\varphi}{dt} = = 0 - \frac{v_\Omega^3}{\nu V_\phi^6}F_4^4(V_\phi;\nu, v_{\rm pole}), \quad \text{using 4-PN term } \mathcal{F}_{8,\nu=0} + \nu A_8$$

- Attach BH ringdown modes

★ Fit free parameters to NR simulations

Advantages of EOB

- EOB Hamiltonian provides complete inspiral dynamics
  - from equal masses to extreme mass-ratio
  - non-adiabatic inspiral/plunge features
  - BH Trajectories, Spin-evolution, waveforms
- Well-identified free functions, specifically A(ν)
  - Can use any of these aspects to improve inspiral model
    - post-Newtonian determines low powers in velocity
    - Kerr geodesic limit determines A(ν=0)
    - Self-force calculations feed into O(ν)-terms
    - Numerical relativity feeds into comparable mass-ratio contributions
- It works!

Disadvantages of EOB

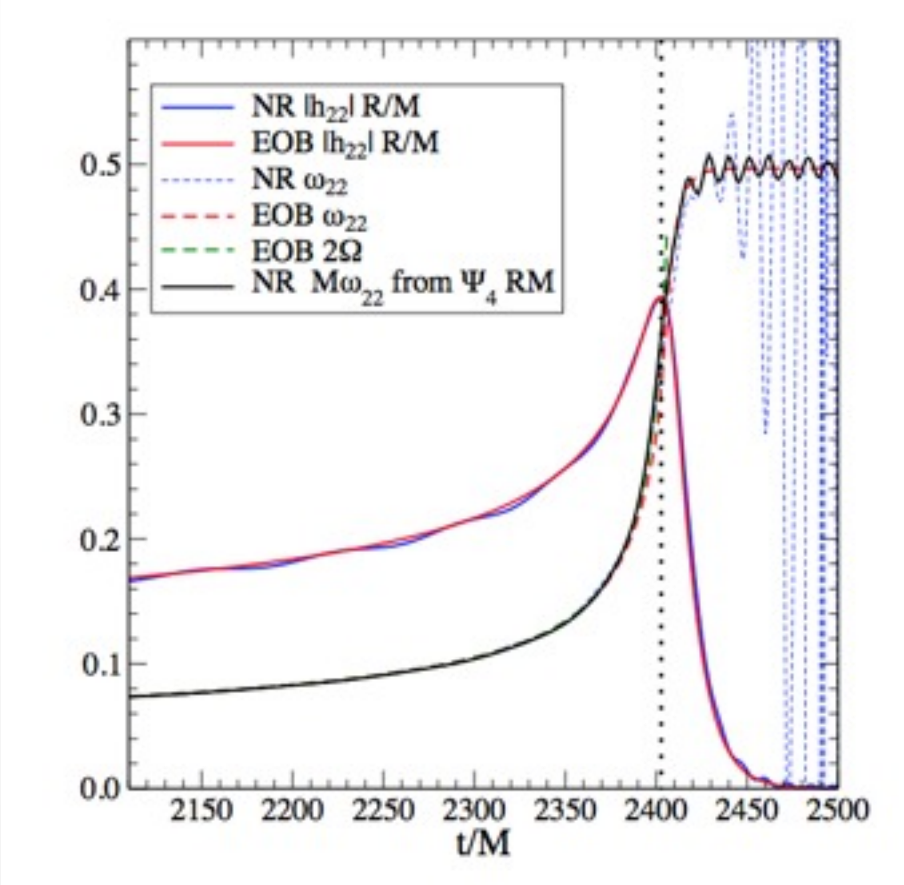
- ❖ Very complicated, many "knobs":
  - higher order PN terms
  - non-adiabatic corrections
  - waveform non-quasi-circular corrections
  - Pade resummation (less emphasized recently)
- ❖ Ever evolving: Many slightly different versions
  - Continued improvements, both physically motivated and to improve agreement with NR
  - Difficult to distinguish prediction from postdiction
- ❖ Difficult to identify ***why*** EOB works well:
  - Deep physical insight?
  - Sheer number of data used from elsewhere (NR, PN, Self-force, ...)?

Disadvantages II: Inspiral → Ringdown

- EOB-inspiral until $A_{GW,EOB}$~max($A_{GW,NR}$)
- Attach BH perturbation ringdown modes with ***comb-matching***

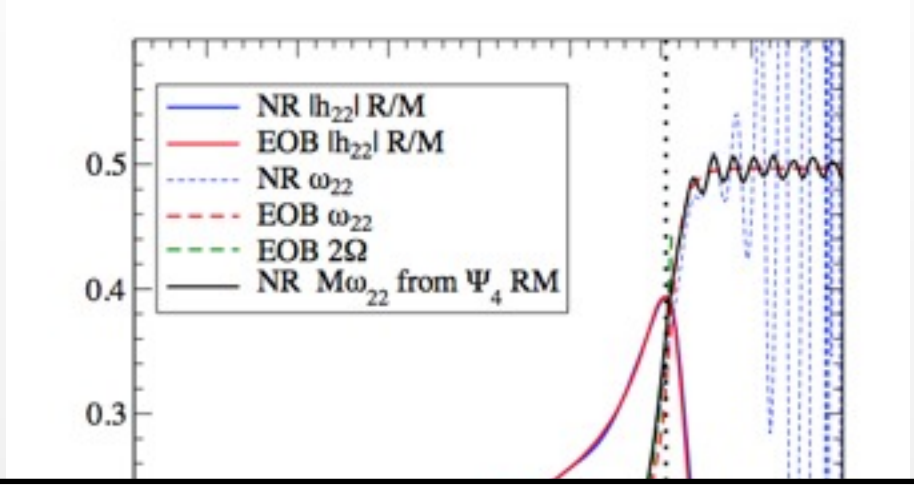
$$h_{\ell m}^{\rm insp-plunge}(t_{\rm match}^{\ell m} + \frac{2k-N+1}{2N-2}\Delta t_{\rm match}^{\ell m}) = h_{\ell m}^{\rm merger-RD}(t_{\rm match}^{\ell m} + \frac{2k-N+1}{2N-2}\Delta t_{\rm match}^{\ell m}), \quad (k=0,1,2,\cdots,N-1). \quad (21)$$

Buonnano ea, PRD 79, 124028



Each (lm) mode matched at different time, with different comb-spacing. It all matters.

Disadvantages II: Inspiral → Ringdown



- EOB-inspiral until $A_{GW,EOB}$~max($A_{GW,NR}$)
- Attach BH perturbation ringdown modes with ***comb-matching***

$$h_{\ell m}^{\rm insp-plunge}\left(t_{\rm match}^{\ell m}+\frac{2k-N+1}{2N-2}\Delta t_{\rm match}^{\ell m}\right)$$
$$=h_{\ell m}^{\rm merger-RD}\left(t_{\rm match}^{\ell m}+\frac{2k-N+1}{2N-2}\Delta t_{\rm match}^{\ell m}\right),$$
$$(k=0,1,2,\cdots,N-1)\,. \quad (21)$$

Buonnano ea, PRD 79, 124028

$$t_{\rm match}^{\ell m}=t_{\rm match}^{22,\rm Cal}-10M(1-|\kappa_{LJ}(t_{\Omega\rm peak}^{\rm EOB})|), \quad (23)$$

$$\Delta t_{\rm match}^{\ell m}=\Delta t_{\rm match}^{22,\rm Cal}(10-9|\kappa_{LJ}(t_{\Omega\rm peak}^{\rm EOB})|), \quad (24)$$

where

$$t_{\rm match}^{22,\rm Cal}=t_{\Omega\rm peak}^{EOB}-\begin{cases}2.5M & \chi\le 0\\ 2.5M+1.77M(\frac{\chi}{0.437})^4 & \chi>0\end{cases},$$
$$\Delta t_{\rm match}^{22,\rm Cal}=7.5M \quad (25)$$

are the calibrated values of the (2,2) mode in Ref. [13],

$$\chi=\chi_S+\chi_A\frac{\sqrt{1-4\nu}}{1-2\nu} \quad (26)$$

is a linear combination of initial spin projections on L_N, and

$$\kappa_{LJ}(t_{\Omega\rm peak}^{\rm EOB})=\hat{L}(t_{\Omega\rm peak}^{\rm EOB})\cdot\hat{J}(t_{\Omega\rm peak}^{\rm EOB}) \quad (27)$$

Even more ad-hoc matching parameters for precessing systems

Pan ea, PRD 89, 084006

Analytical waveform modeling

❖ Effective one body

- Buonanno, Damour 1999; many papers since
- Effective Hamiltonian to capture conservative dynamics

$$H = \mu\sqrt{p_r^2 + A(r)\left[1 + \frac{p_r^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]}, \qquad A(r) = \sum_{k=0}^{4}\frac{a_k(\nu)}{r^k} + \frac{a_5(\nu)}{r^5}$$

- Radiation reaction terms

$$\frac{dp_r}{dt} = -\frac{\partial H}{\partial p_r} + a^r_{\rm RR}\frac{\dot r}{r^2\Omega}\widehat{\mathcal{F}}_\phi$$

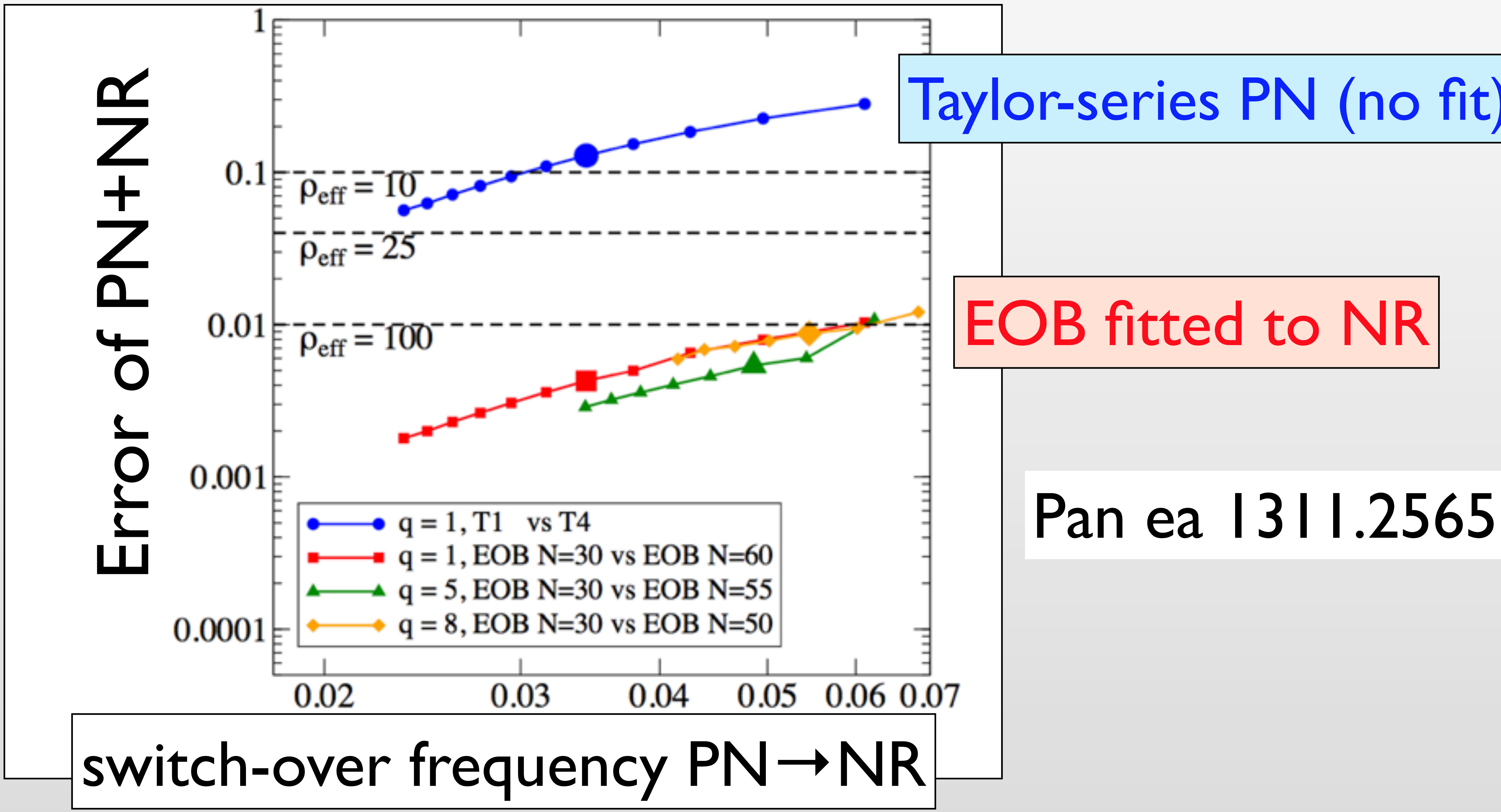
$$\frac{dp_\varphi}{dt} = = 0 - \frac{v_\Omega^3}{\nu V_\phi^6}F_4^4(V_\phi;\nu, v_{\rm pole}), \quad \text{using 4-PN term } \mathcal{F}_{8,\nu=0} + \nu A_8$$

- Attach BH ringdown modes

★ Fit free parameters to NR simulations

EOB progress (1)

❖ Non-spinning case: ***Error-estimate*** of EOB fit



Pan ea 1311.2565

EOB progress (2)

❖ Aligned-spin case: Spin-magnitudes ***up to extremal***

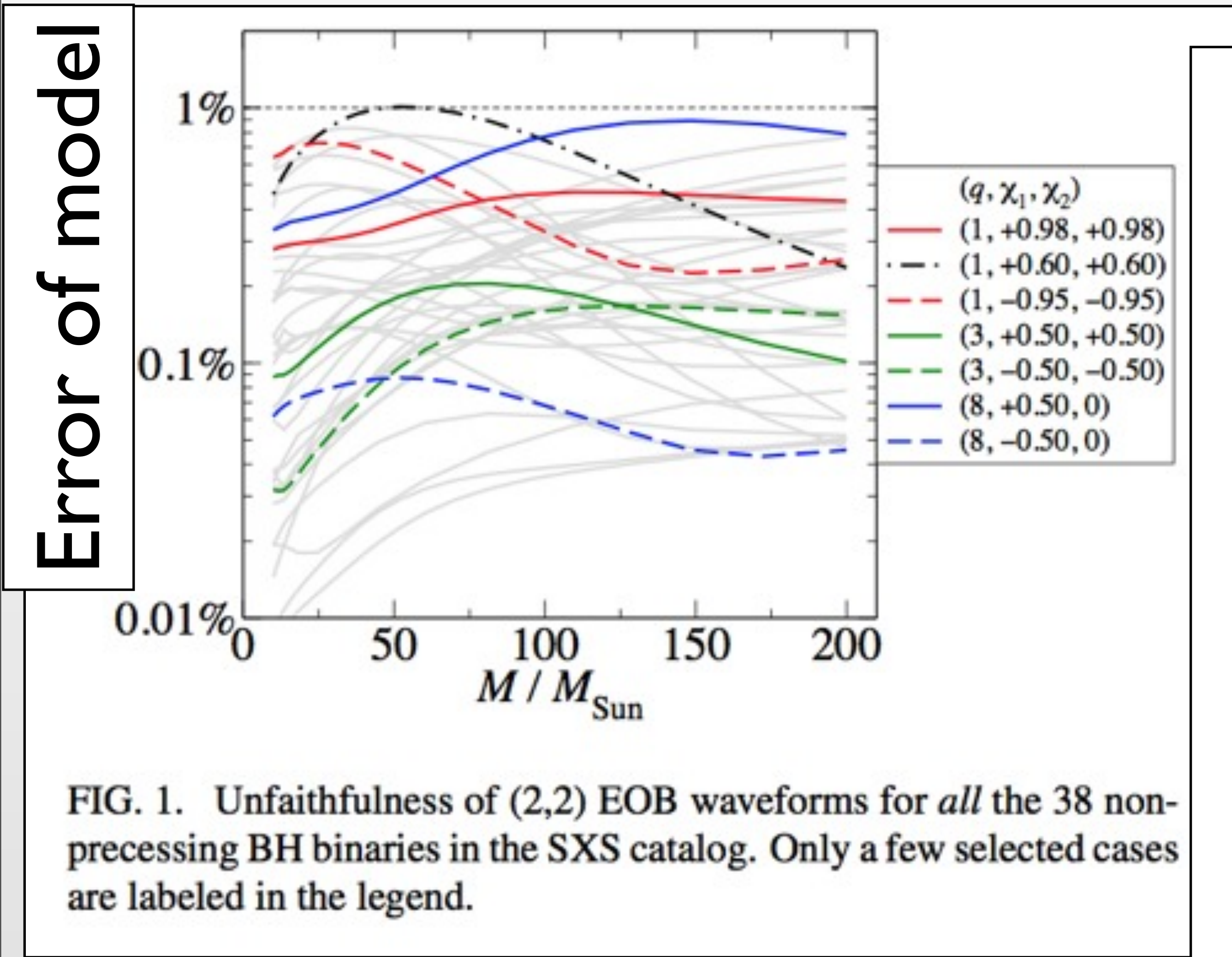
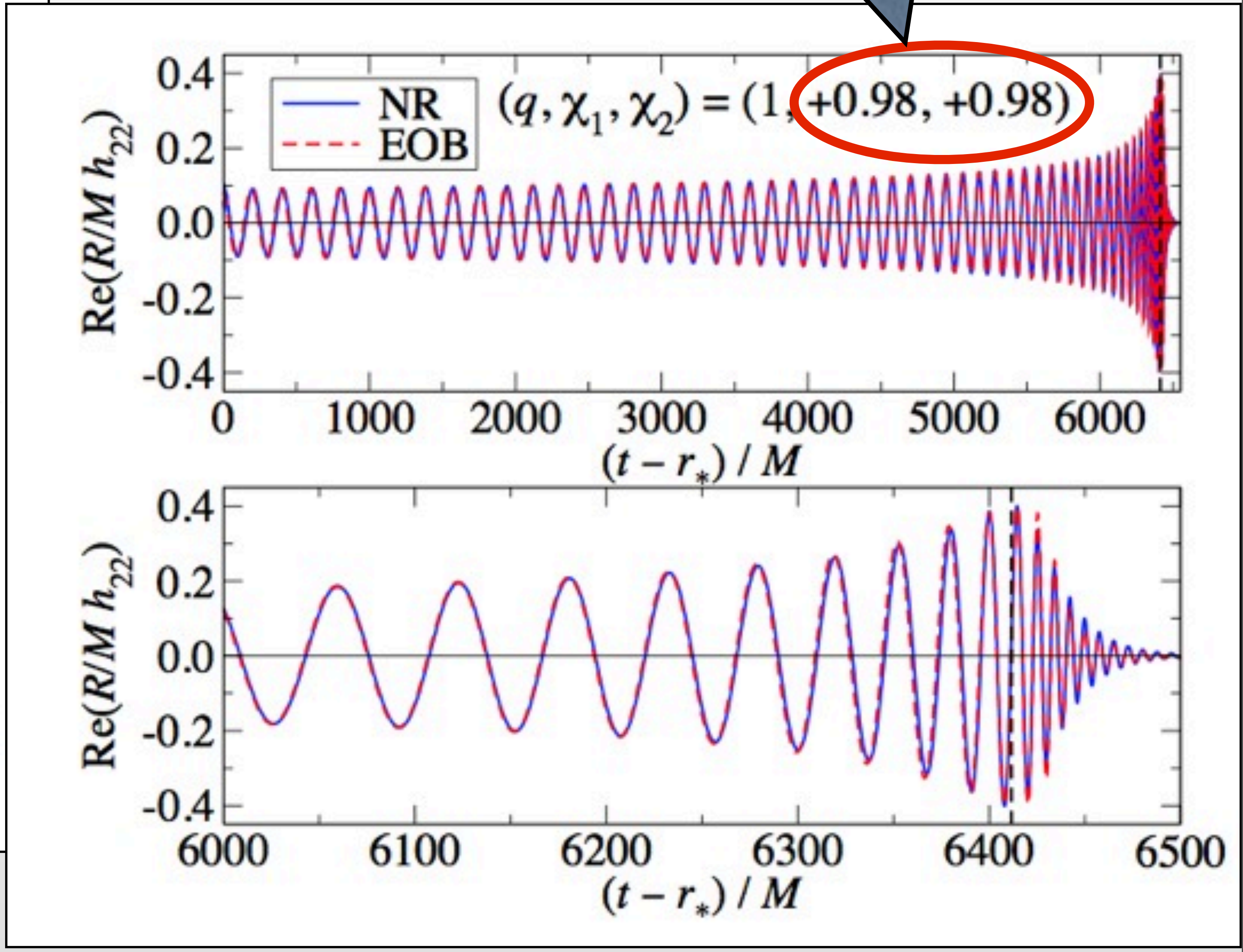


FIG. 1. Unfaithfulness of (2,2) EOB waveforms for *all* the 38 non-precessing BH binaries in the SXS catalog. Only a few selected cases are labeled in the legend.

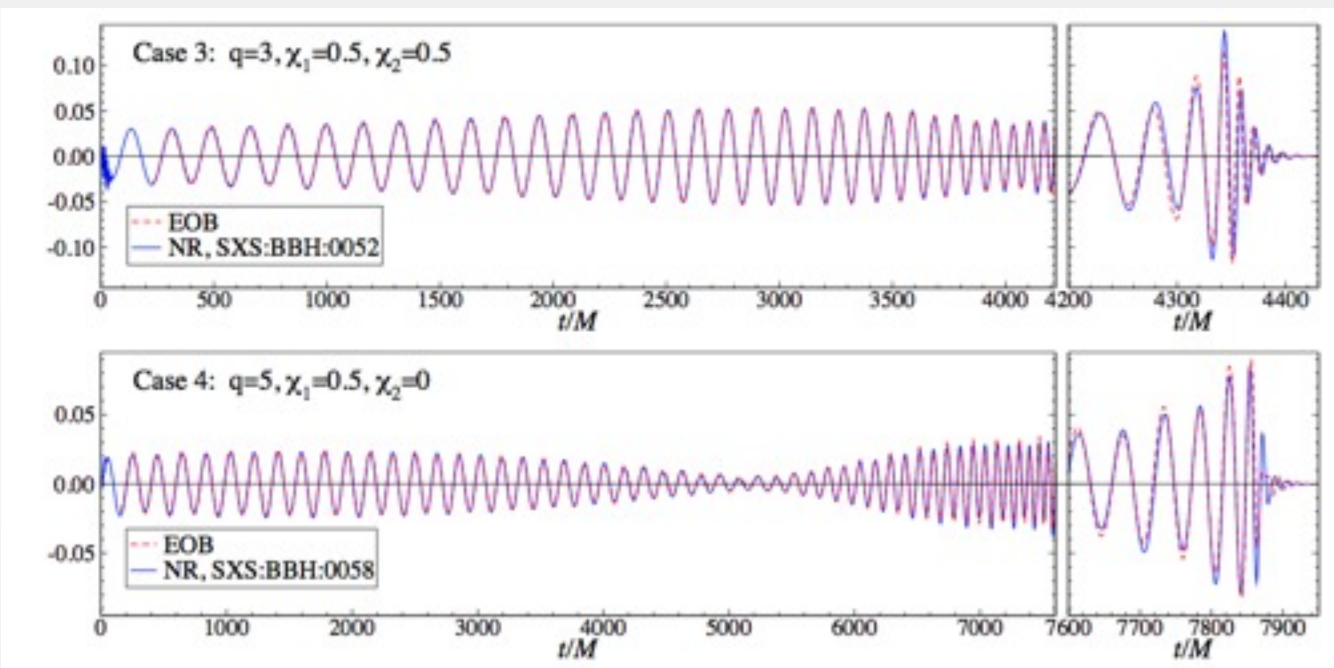


Taracchini ea, 1311.2544

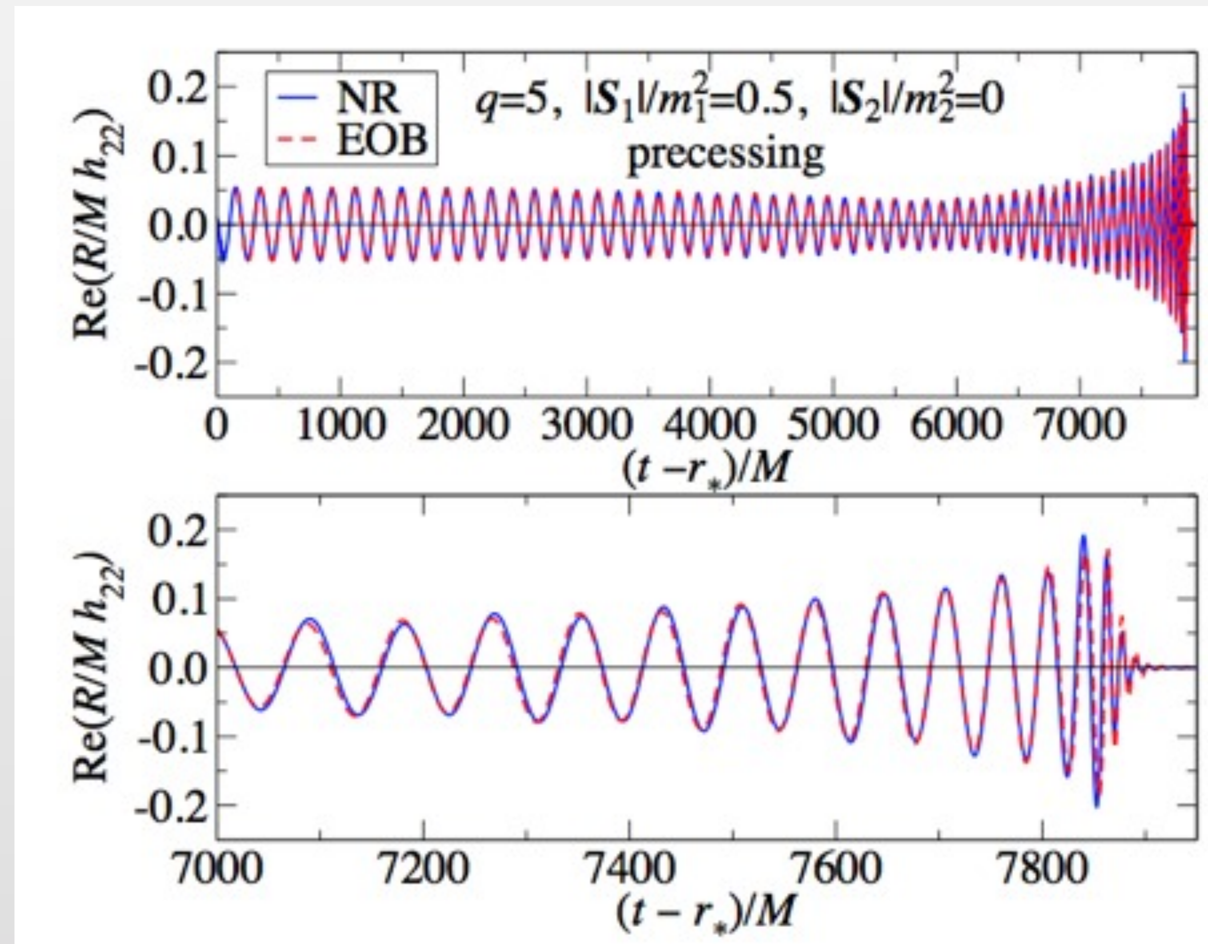
EOB progress (3)

❖ Precessing case: ***First generic, precessing EOB models***

- Generic spin EOB Hamiltonian (Buonanno ea 2005, Hannam ea 1308.3271)
- Aligned-spin waveforms, rotated into precessing frame



Pan ea, 1307.6232

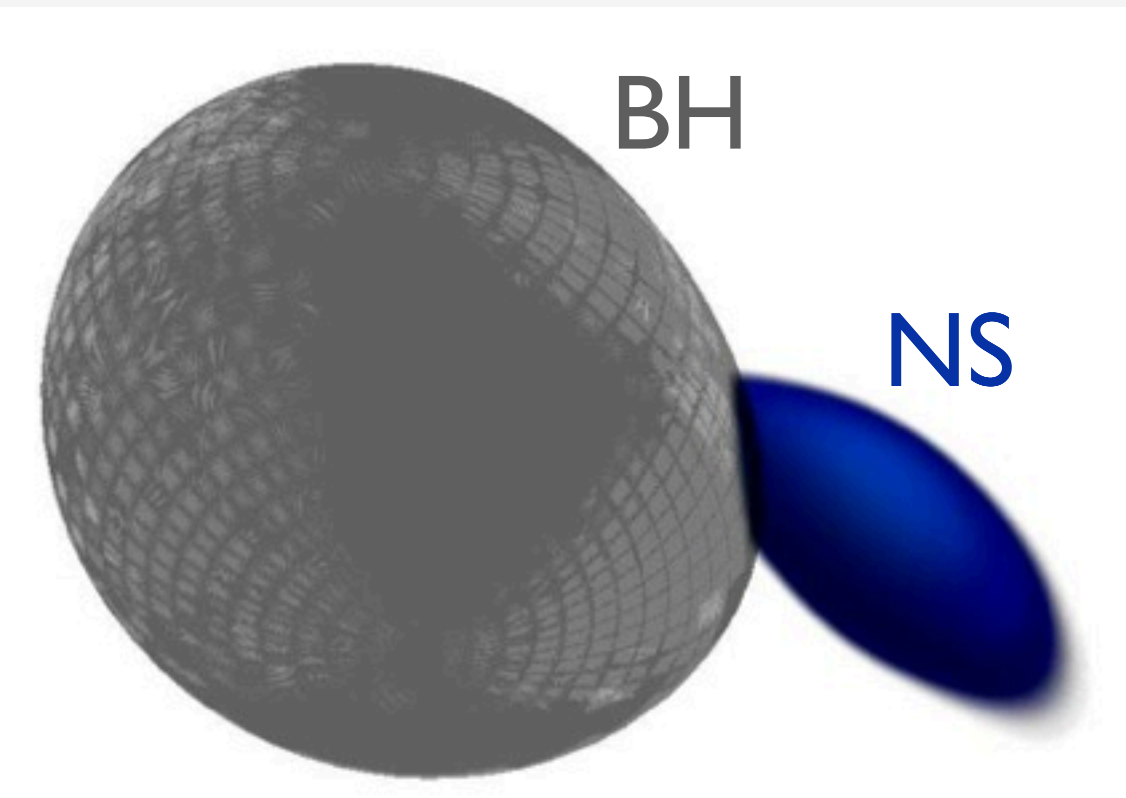


Taracchini ea, 1311.2544

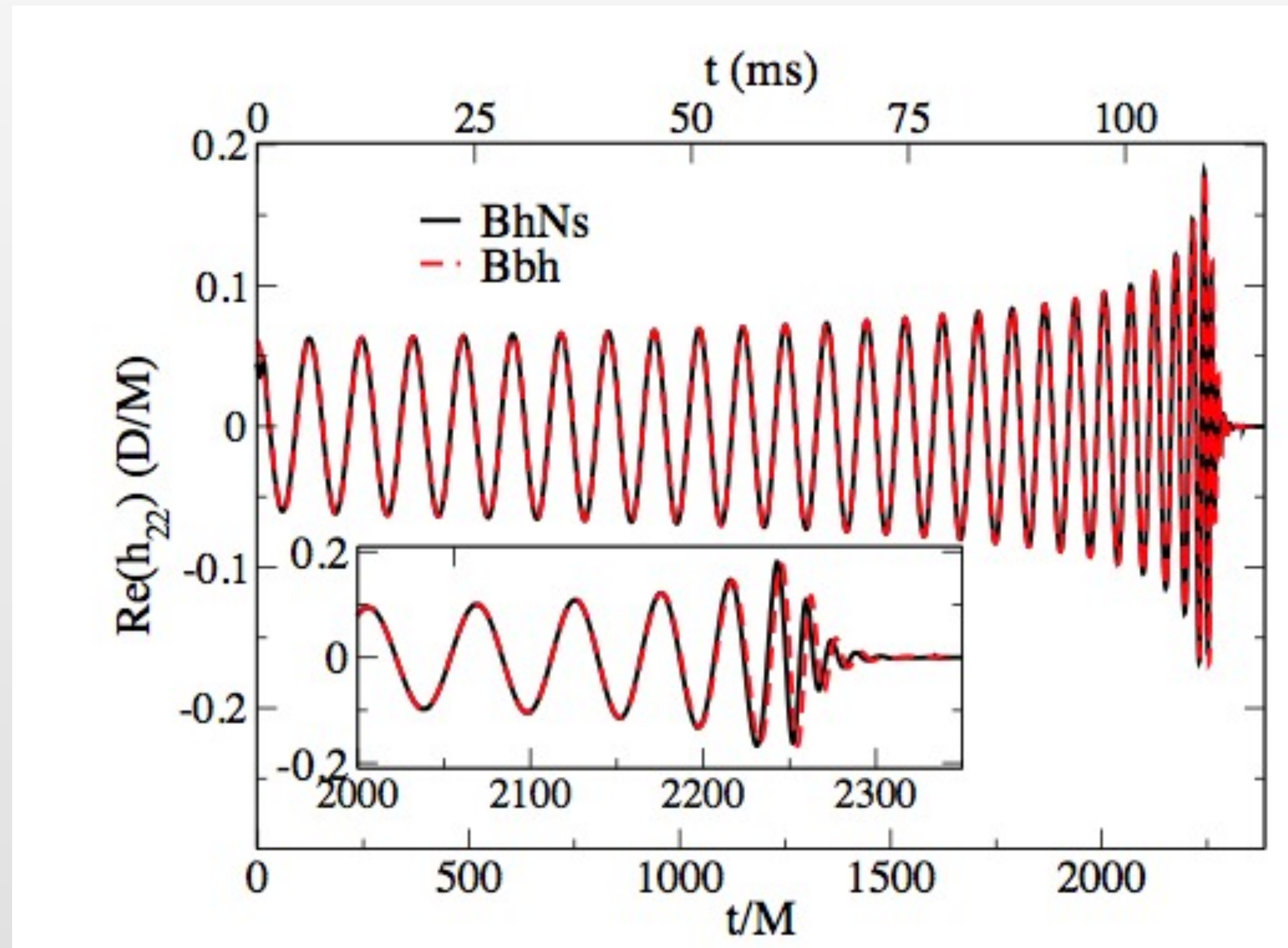
Mixed BH-NS binaries

❖ For high mass-ratio, low-spin: ***BH-NS ≡ BH=BH***

- NS eaten by BH in one piece, no disruption

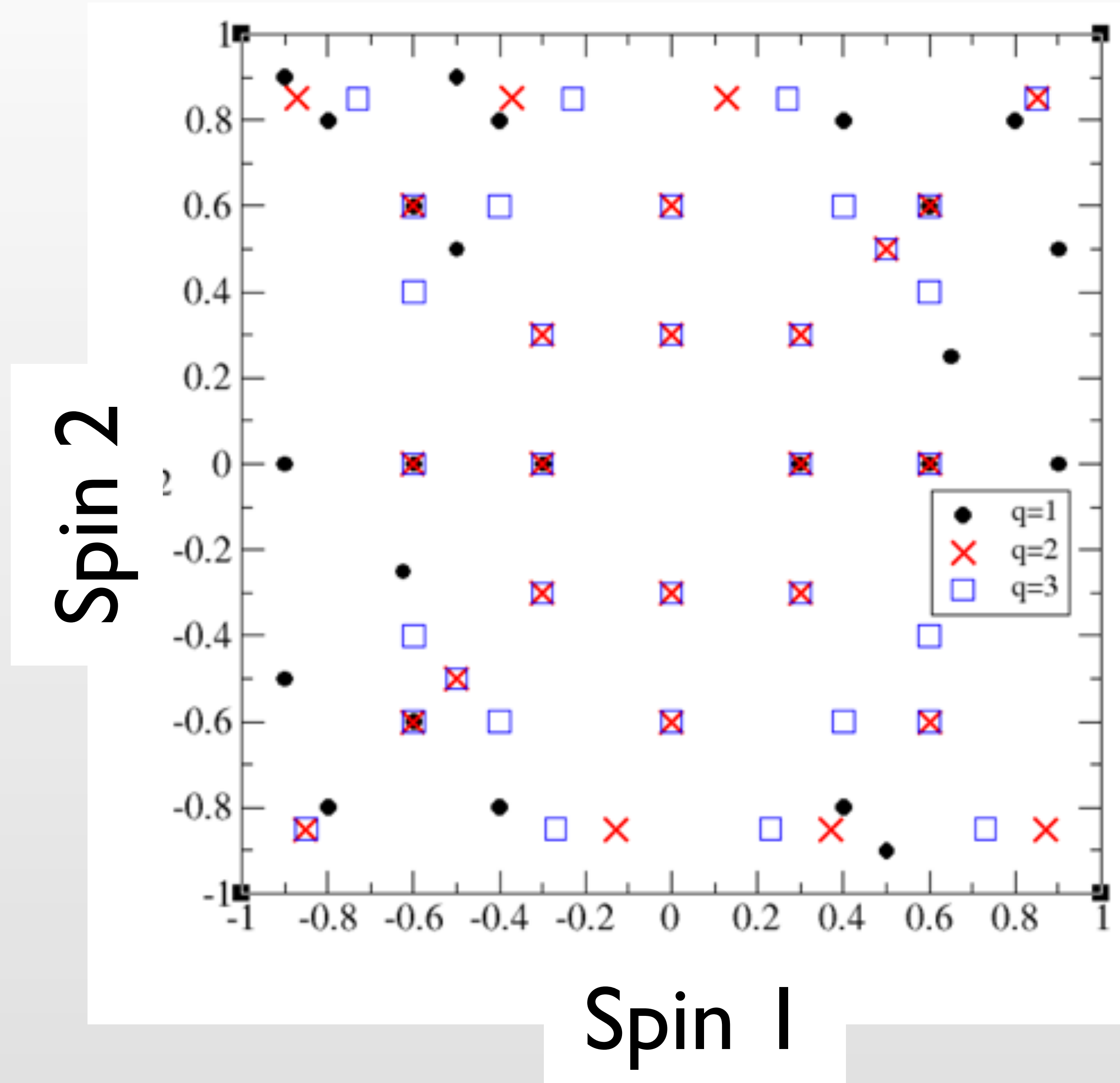


Foucart ea, 1311.2544



Near future

- Large sample of aligned spin BBH GW
  - q=1,2,3
  - $-0.9 <= S_{1/2}/M^2 <= 0.9$
- Independent test of EOB and other GW models
- Independent test of BBH GW detection pipelines



Chu ea, in prep