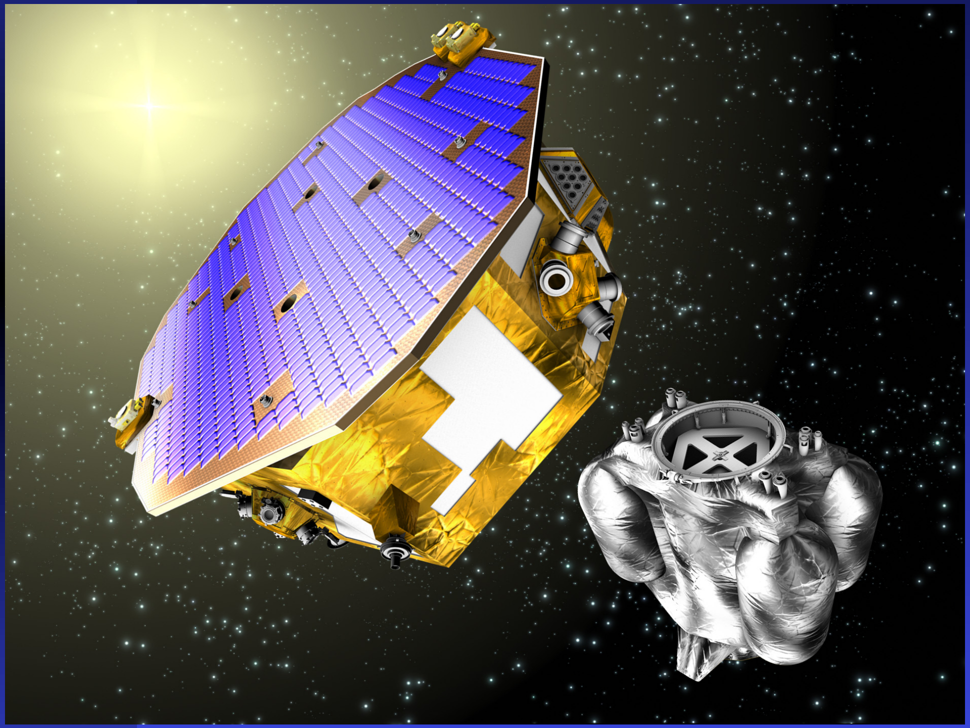
MATTERS OF GRAVITY:

The conflict "Dark Matter vs MOND" in the solar system



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Based on
Bekenstein & Magueijo, **PRD73 (2006) 103513;**
Bevis, Magueijo, Trenkel, Kemble, **CQG 27 (2010) 215014**
Trenkel, Kemble, Bevis, Magueijo, arXiv:**1001.1303**

Magueijo & Mozaffari
PRD85 (2012) 043527;
and arXiv:1204.6663.

The schism "dark matter vs modified gravity" is an old story

- LeVerrier prediction of Neptune from Uranus orbital anomalies.

VERSUS

- Attempts to explain the anomalous precession of the perihelion of Mercury with "Vulcan"

The matter has become very topical

- Everything outside the Solar system refuses to follow the laws of General Relativity/ Newtonian gravity
- Either gravity is fine, but there is an extra source we can't see: the dark matter (dark energy).
- Or the observations are telling us to modify gravity: MOND.

We need a *direct* detection!

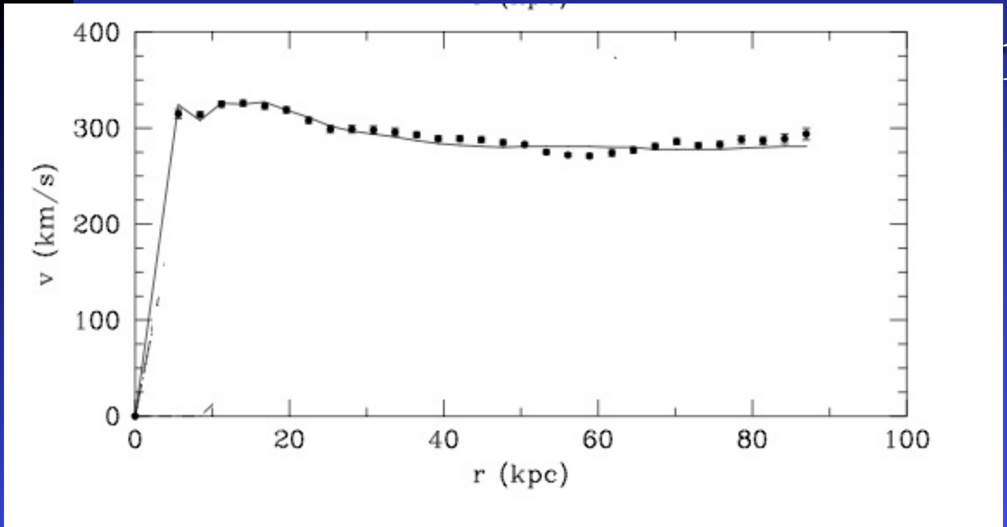
- Dark matter searches: the game is over if a dark matter particle is detected!
- The equivalent "backyard" detection for MOND is solar system gravity testing.

(Spiral) Galaxy rotation curves:

- Flattening of rotation curves

$$v \longrightarrow v_\infty$$

$$\frac{GM}{r^2} = \frac{v^2}{r} \Rightarrow v \propto \frac{1}{\sqrt{r}}$$



The dark matter solution:

- A DM halo:

$$M \propto r$$

$$\rho \propto \frac{1}{r^2}$$

$$\frac{GM}{r^2} = \frac{v^2}{r} \Rightarrow v \rightarrow const$$

- Mysteries remain

$$a_0 \approx 10^{-10} ms^{-2}$$

$$v_\infty^4 \propto L \propto M$$

- Halo not stable under its own gravity

Take these facts as "Kepler's laws" for a new theory of gravity

- Milgrom's insight:

$$a \le a_0$$

$$F = m\frac{a^2}{a_0} \Rightarrow \frac{GM}{r^2} = \frac{v^4}{a_0 r^2}$$

- A better formulation:

$$F_N = |\nabla\varphi| \le a_0 \Rightarrow F = \sqrt{F_N}$$

(often quoted as a rule of thumb)

In fact a consistent theory must be more complicated:

- A modified Poisson equation

$$\nabla \cdot [\tilde{\mu}(|\nabla \Phi|/a_0)\nabla \Phi] = 4\pi G\tilde{\rho}, \qquad \tilde{\mu}(x) = x(1+x)^{-1}$$

- (there are several relativistic, Lagrangian formulations leading to this)

$$\nabla \cdot [\mu(kl^2(\nabla \phi)^2)\nabla \phi] = kG\tilde{\rho}$$

Tensor Vector Scalar theory

Bekenstein, Phys. Rev. D 70, 083509 (2004); JHEP PoS (jhw2004) 012

TeVeS

Gravitation $$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta} (-g)^{1/2} d^4x$$

Matter $$S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha{}_{|\mu}, \cdots)(-\tilde{g})^{1/2} d^4x$$

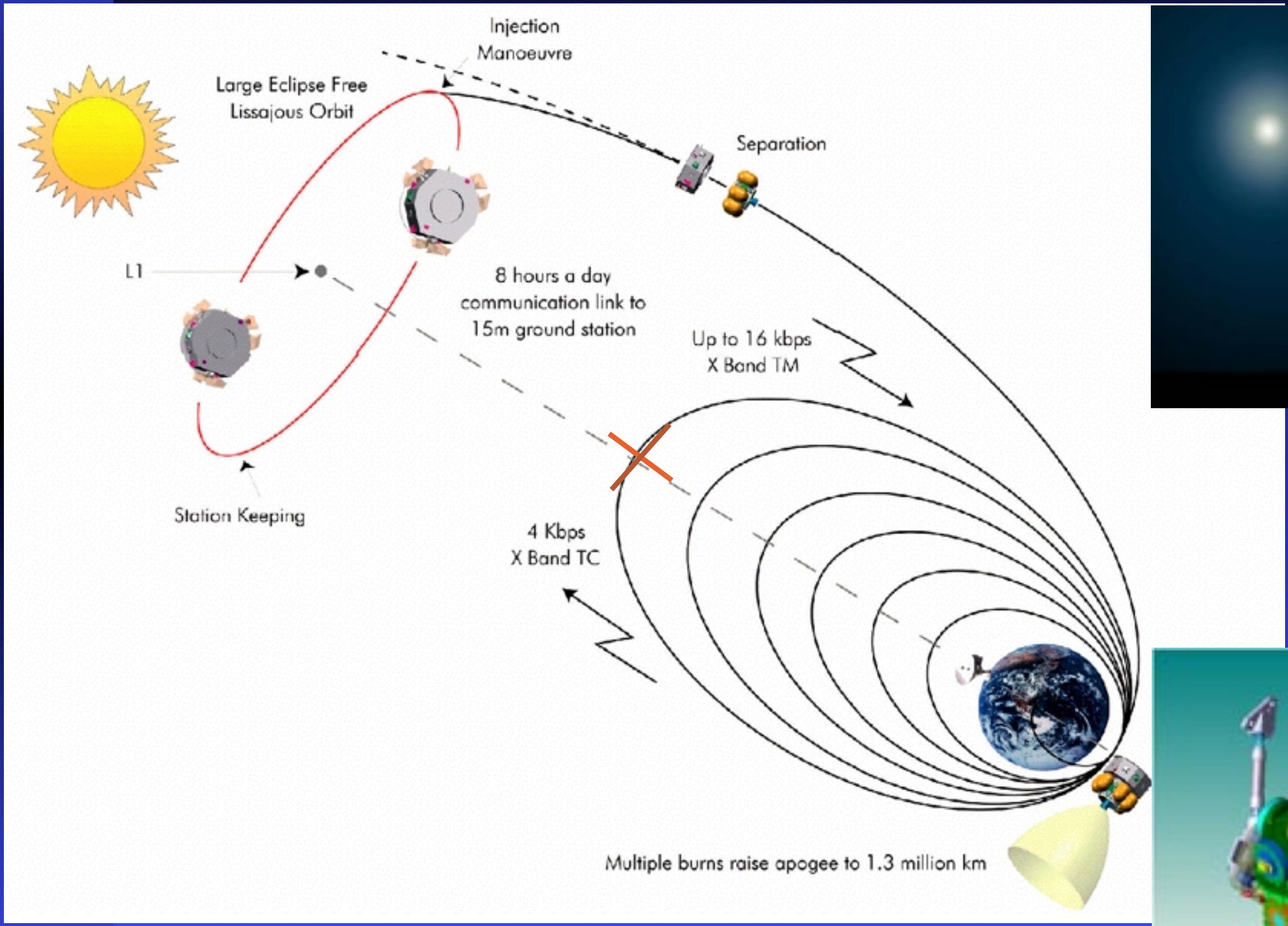
Vector $$S_v = -\frac{K}{32\pi G} \int \left(g^{\alpha\beta} g^{\mu\nu} U_{[\alpha,\mu]} U_{[\beta,\nu]}\right)(-g)^{1/2} d^4x$$

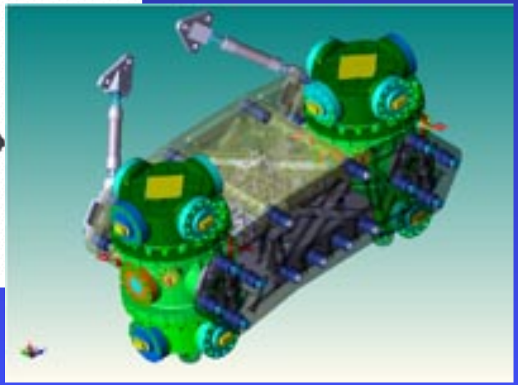
Constraint $$S_c = \frac{1}{16\pi G} \int \lambda\left(g^{\mu\nu} U_\mu U_\nu + 1\right)(-g)^{1/2} d^4x$$

Scalar $$S_s = -\frac{1}{2} \int \left[\sigma^2 g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{G\sigma^4}{2\ell^2} F(kG\sigma^2)\right](-g)^{1/2} d^4x$$

Interaction $$S_i = \frac{1}{2} \int \sigma^2 \left(g^{\mu\nu} U_\mu \phi_{,\nu}\right)^2 (-g)^{1/2} d^4x$$

LISA Pathfinder mission





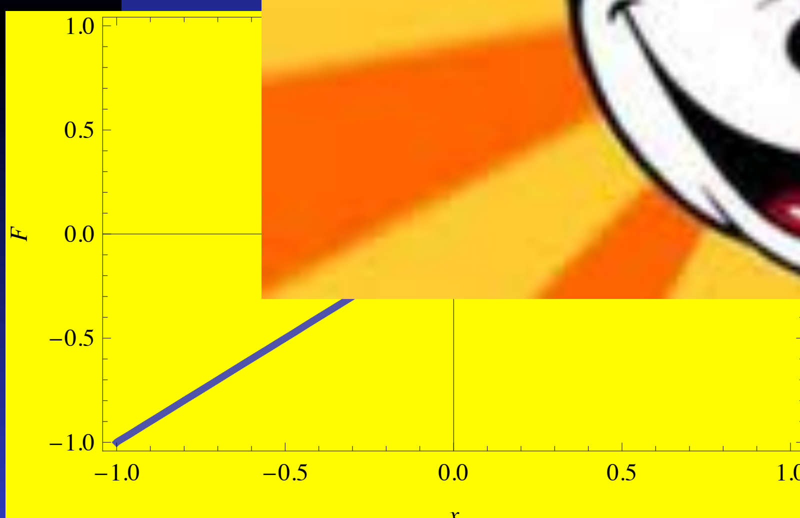
LISA Pathfinder as Gravitational Laboratory

- **LISA Pathfinder and its Payload will offer the following (see ESA-SCI(2007)1):**
  - **Differential Force Measurement Sensitivity:**
    $\approx 1.3\times10^{-14}$N / $\sqrt{\text{Hz}}$ around 1mHz
  - **Drag-Free Platform Stability:**
    Platform Free-Fall Quality of
    $\approx 10^{-13}$ms^{-2}/ $\sqrt{\text{Hz}}$ around 1mHz
    $\approx 10^{-9}$ms^{-2} at DC
  - **Gravity Gradiometer Sensitivity:**
    $\approx 1.5\times10^{-14}$s^{-2}/ $\sqrt{\text{Hz}}$ around 1mHz
  - **… and more…**

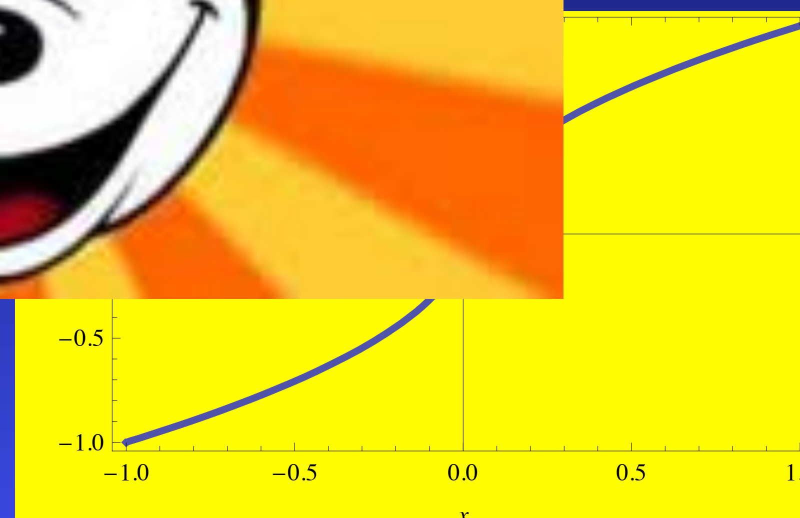
We want to exploit this!

The intuition behind the basic pre… 266

N



$$F_N = Ar$$
$$A = 4.6 \times 10^{-11}\ \mathrm{s}^{-2}$$



$$\frac{\partial F_N}{\partial r} \to \infty$$

A target for LPF?

- Accelerometers have a sensitivity of

$$\frac{\Delta a}{\Delta r} \approx 10^{-14} s^{-2}$$

- Target region for Sun/Earth saddle has size

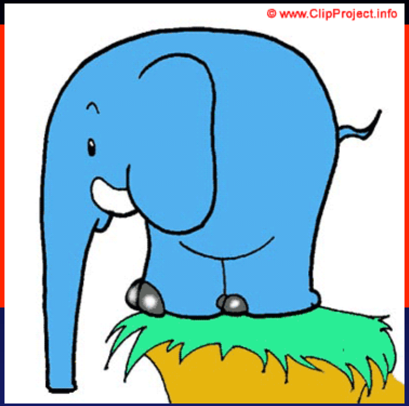
$$\Delta r \approx 400 - 600 \text{ Km}$$

The Big Picture: an ellipsoidal bubble where effects are large

$$r_0 = 383\text{Km} \qquad \text{Earth/Sun}$$

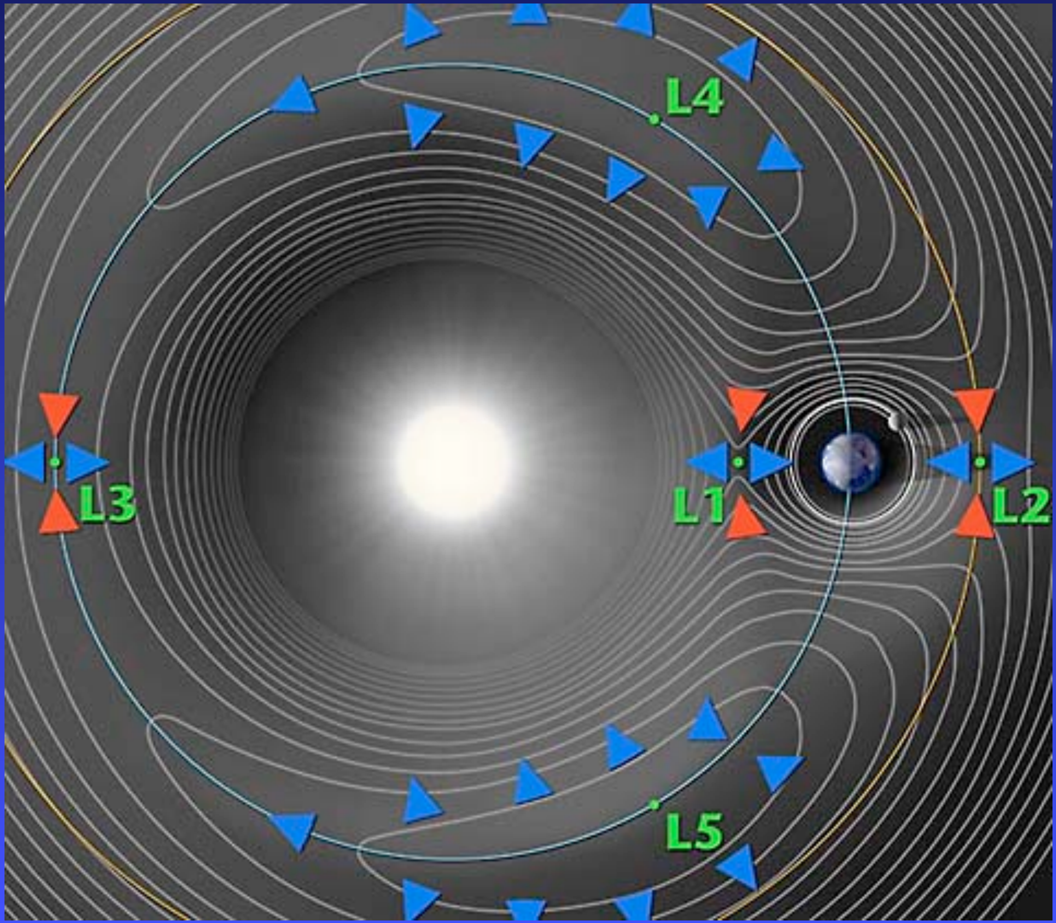
$$(r_0 = 9.6 \times 10^5\text{Km} \quad \text{Jupiter})$$

THE ELEPHANT IN THE ROOM



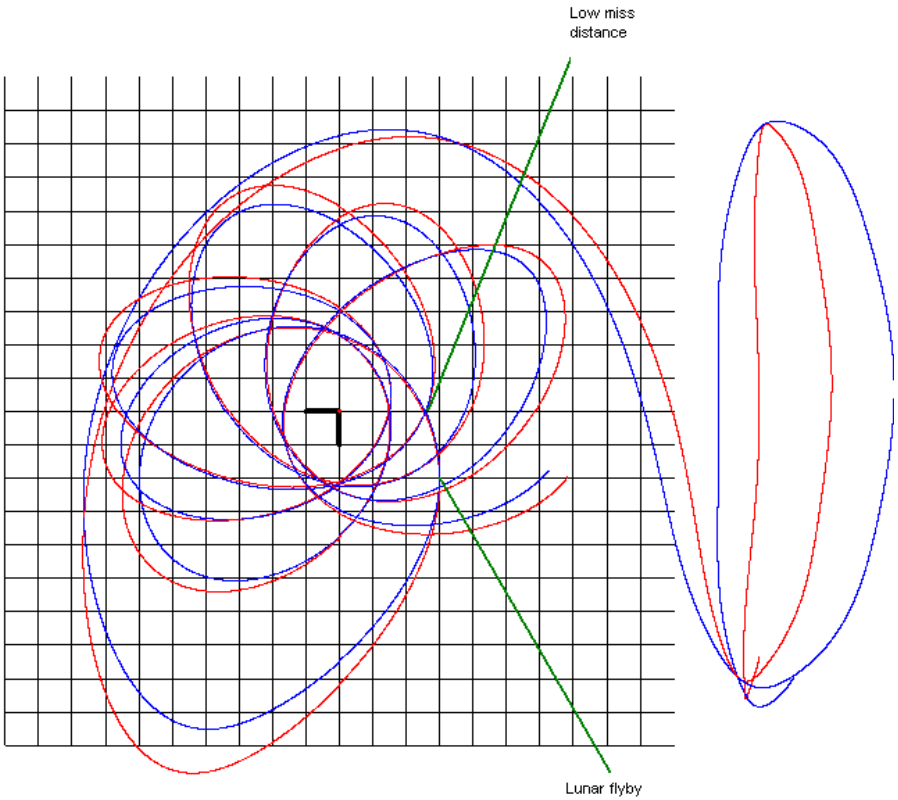
- Lagrange points

  VS.
- Saddle points.



Potential Trajectories for LISA Pathfinder

- **Trajectory with lowest miss distance found:**



- **Miss distance 600km**
- **Transfer time from nominal orbit departure 410days**
- **Lunar flyby (60000km) after 300days**

Beyond the cartoon, part I

- There are mathematical complications with this simplified calculation:
- MONDian magnetic field,
- Details of general "relativistic" MONDian theories

A taste of the complications

$$\nabla \cdot [\mu(|\nabla\phi|)\nabla\phi] = kG\rho$$

$$u = \frac{4\pi\mu}{k}\nabla\phi$$

$$\nabla \cdot u = 4\pi G\rho$$

$$\nabla \wedge \frac{u}{\mu} = 0$$

$$u = F^{(N)} + \nabla \wedge h$$

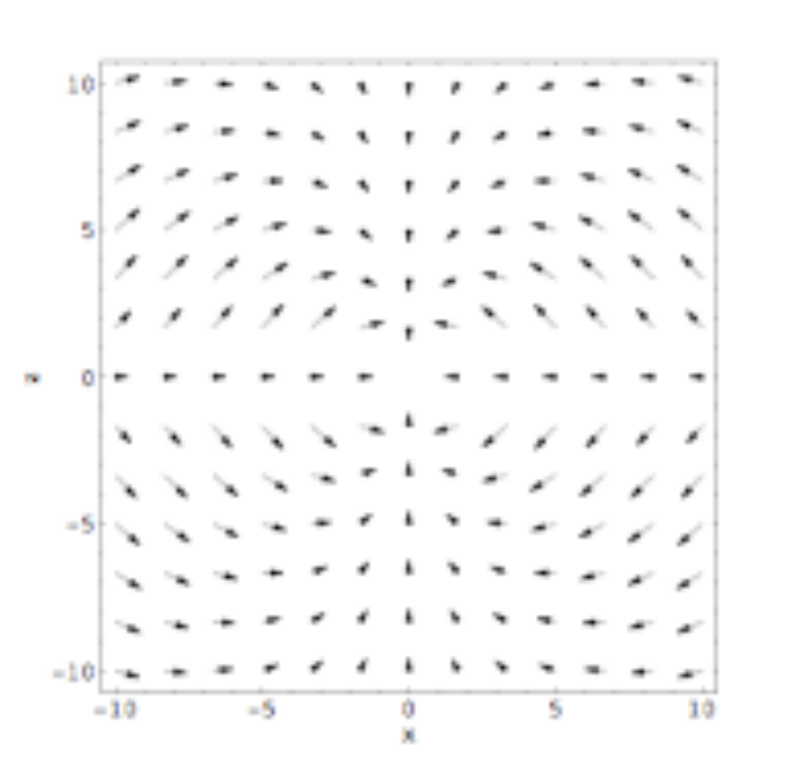
The basic picture: an ellipsoidal bubble where effects are large

$$r_0 = 383\text{Km} \qquad \text{Earth/Sun}$$
$$(r_0 = 9.6 \times 10^5\text{Km} \quad \text{Jupiter})$$

Outside (perturbative) region

- Maximal fractional effect is at the border ellipsoid
- It then falls off as

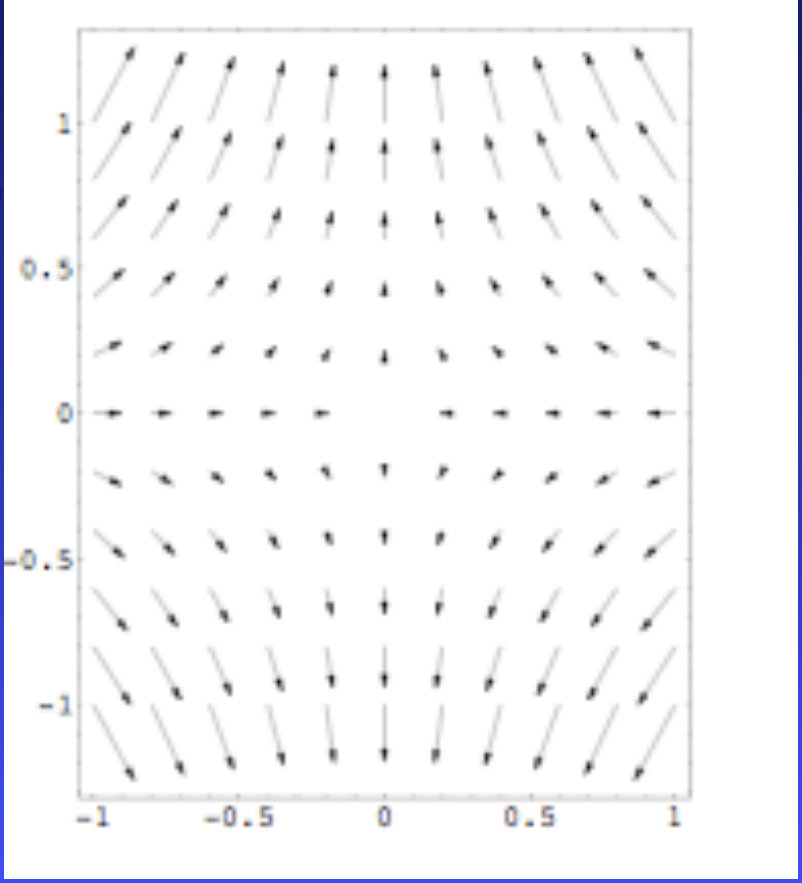
  1/r^2



The inner region profile

- The tidal stress explosion is there but it's much softer due to the curl term

$$\frac{\delta F}{F} \propto \left(\frac{r}{r_0}\right)^{-0.24}$$



In relativistic theories MOND effects are due to an extra field

- Specifically:

$$\Phi = \Phi_N + \phi$$

g_{00}

- The extra field has equation:

$$\nabla \left[\mu\left(\frac{\kappa}{4\pi a_0} |\nabla \phi| \right) \nabla \phi \right] = \kappa G \tilde{\rho}$$

The free function is heavily constrained by the following considerations:

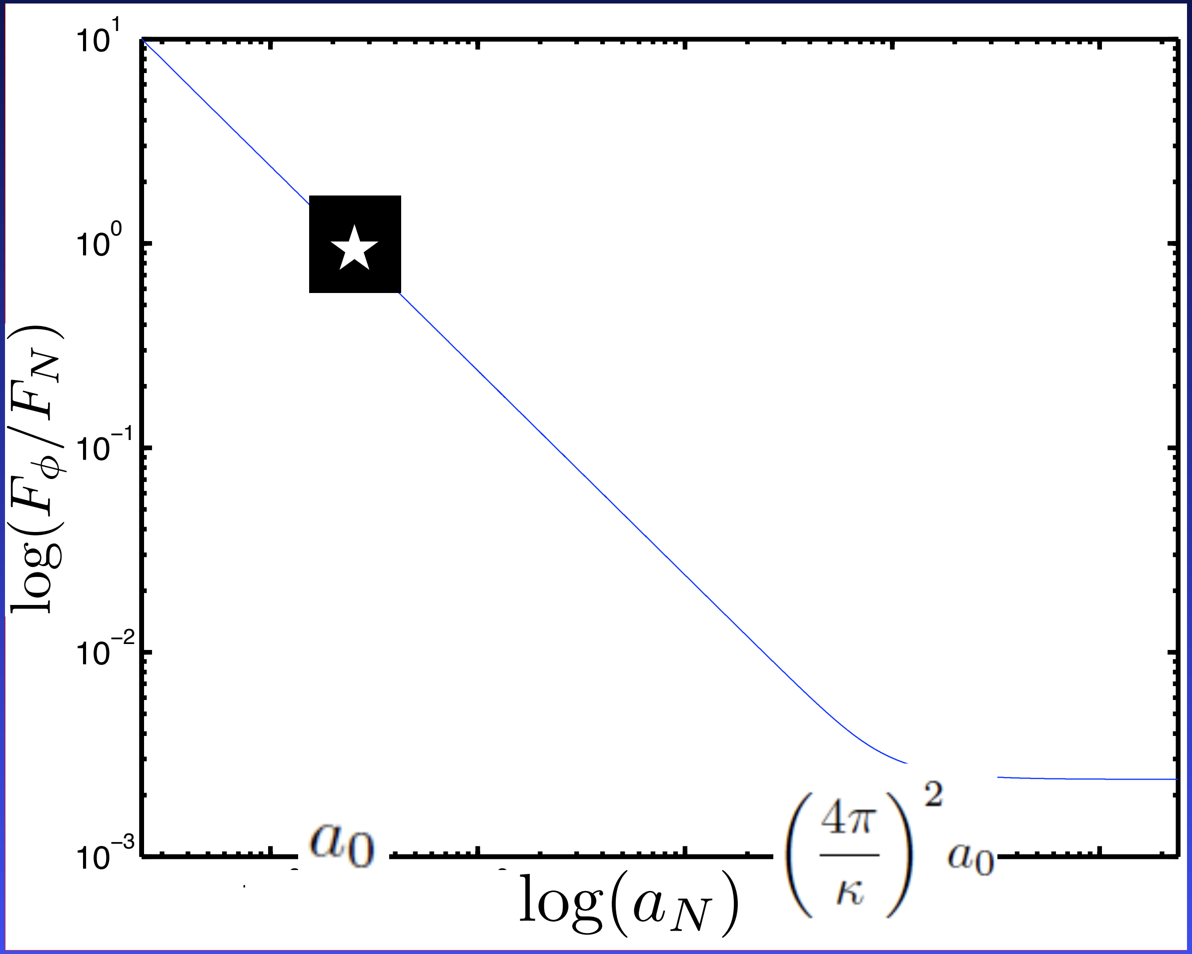
- In the Newtonian (non-relativistic, non-MONDian) limit:

$$\begin{aligned} \nabla^2\Phi_N &= 4\pi G\rho \\ \nabla(\mu\nabla\phi) &= \kappa G\rho \end{aligned}$$

$$\tilde{G} = G\left(1 + \frac{\kappa}{4\pi}\right)$$

- ϕ must then be subdominant ($\kappa \sim 0.03$)

This places constraints on the free function:



$\frac{\kappa}{4\pi}$

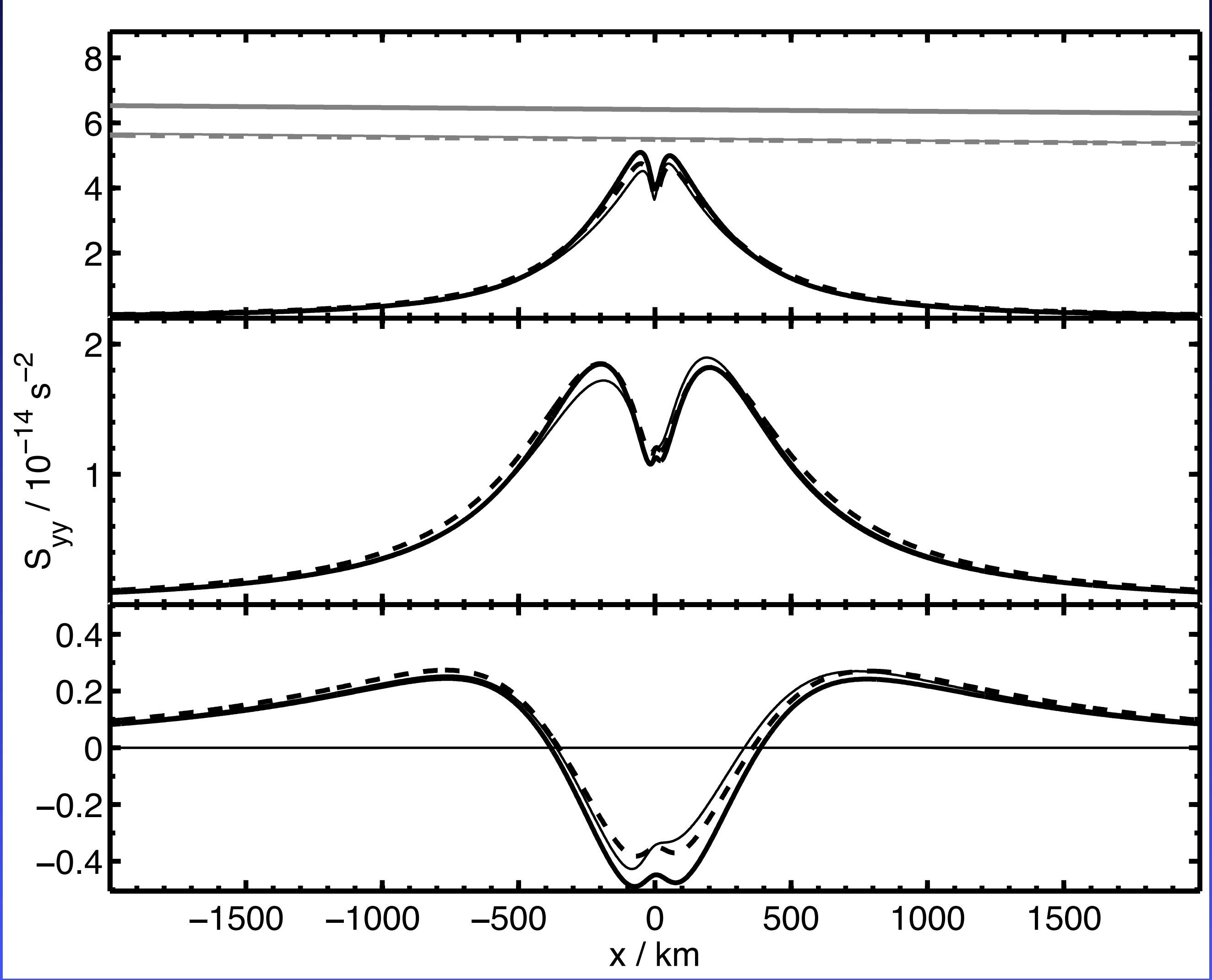
Thus MONDian regime in ϕ must start at accelerations higher than a_0

- In the Newtonian limit: $\tilde{G} = G\left(1 + \frac{\kappa}{4\pi}\right)$
- ϕ must then be subdominant ($\kappa \sim 0.03$)
- Must trigger MONDian behaviour in ϕ at much larger Newtonian accelerations:

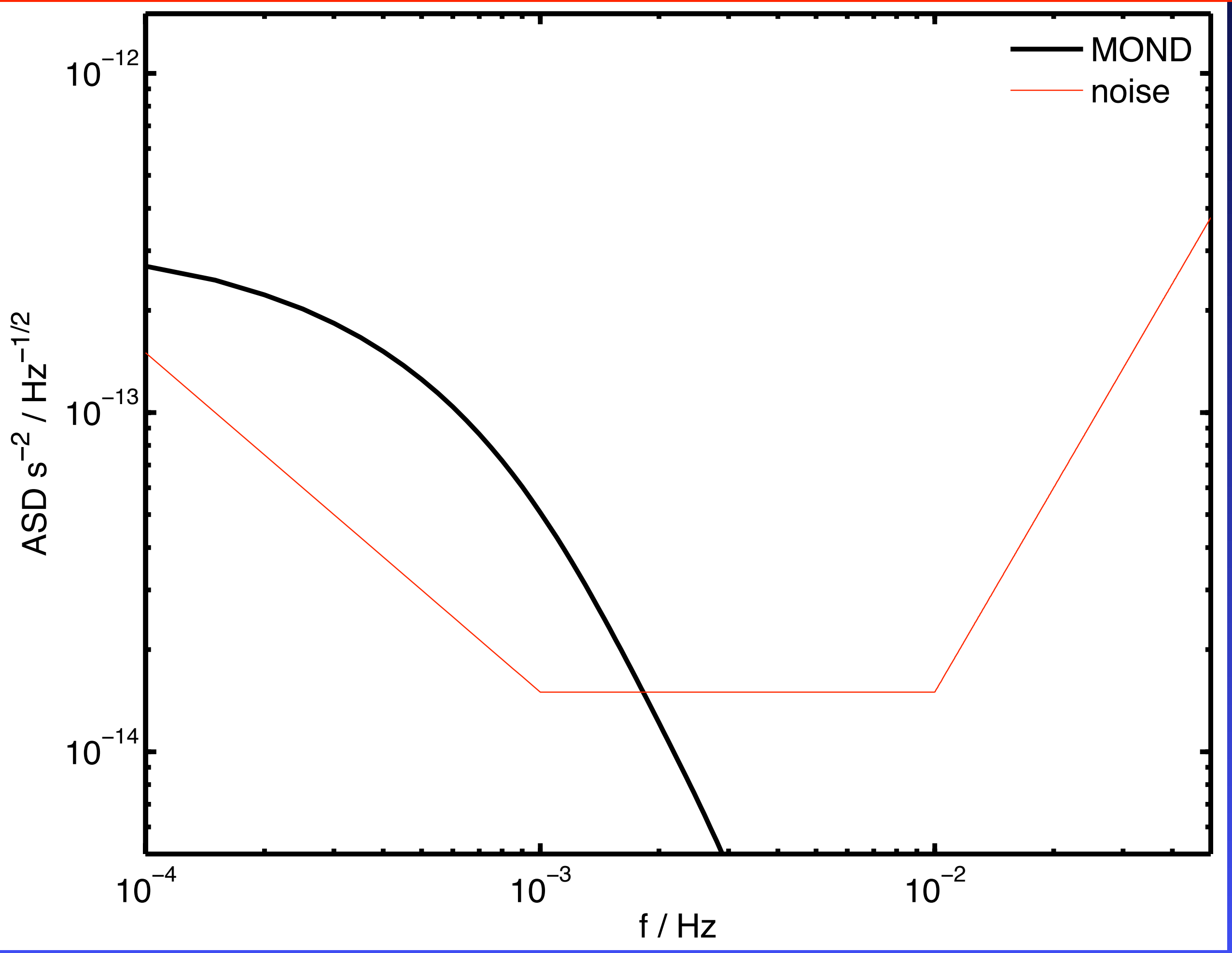
$$a_N = \left(\frac{4\pi}{\kappa}\right)^2 a_0 \sim 1.75 \times 10^5 a_0 \sim 10^{-5}\,\mathrm{ms}^{-2}$$

ANYWAY…………
What are the predictions?

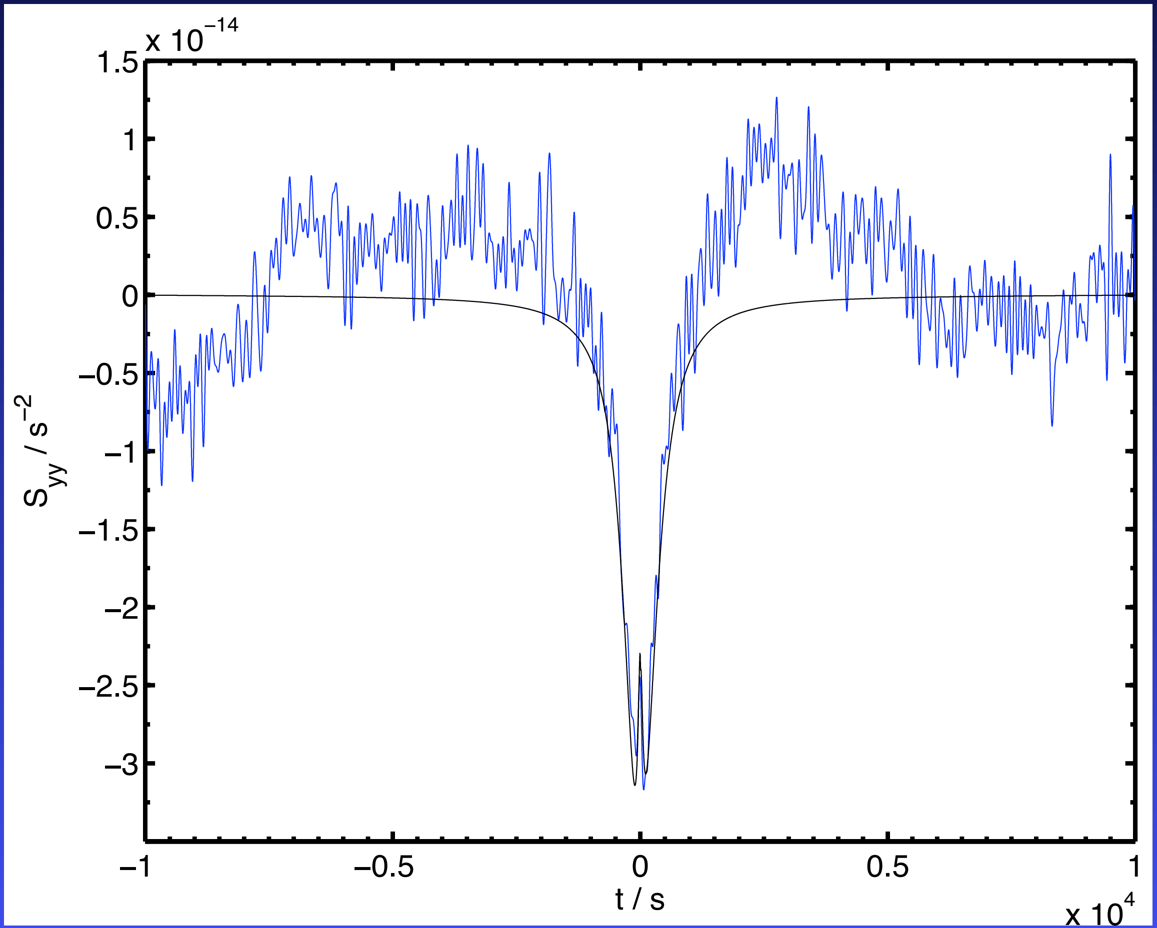
Transverse tidal stress at impact 25, 100, 400 Km of the Earth-Sun saddle



The coincidence of the century



Simulating the signal and noise



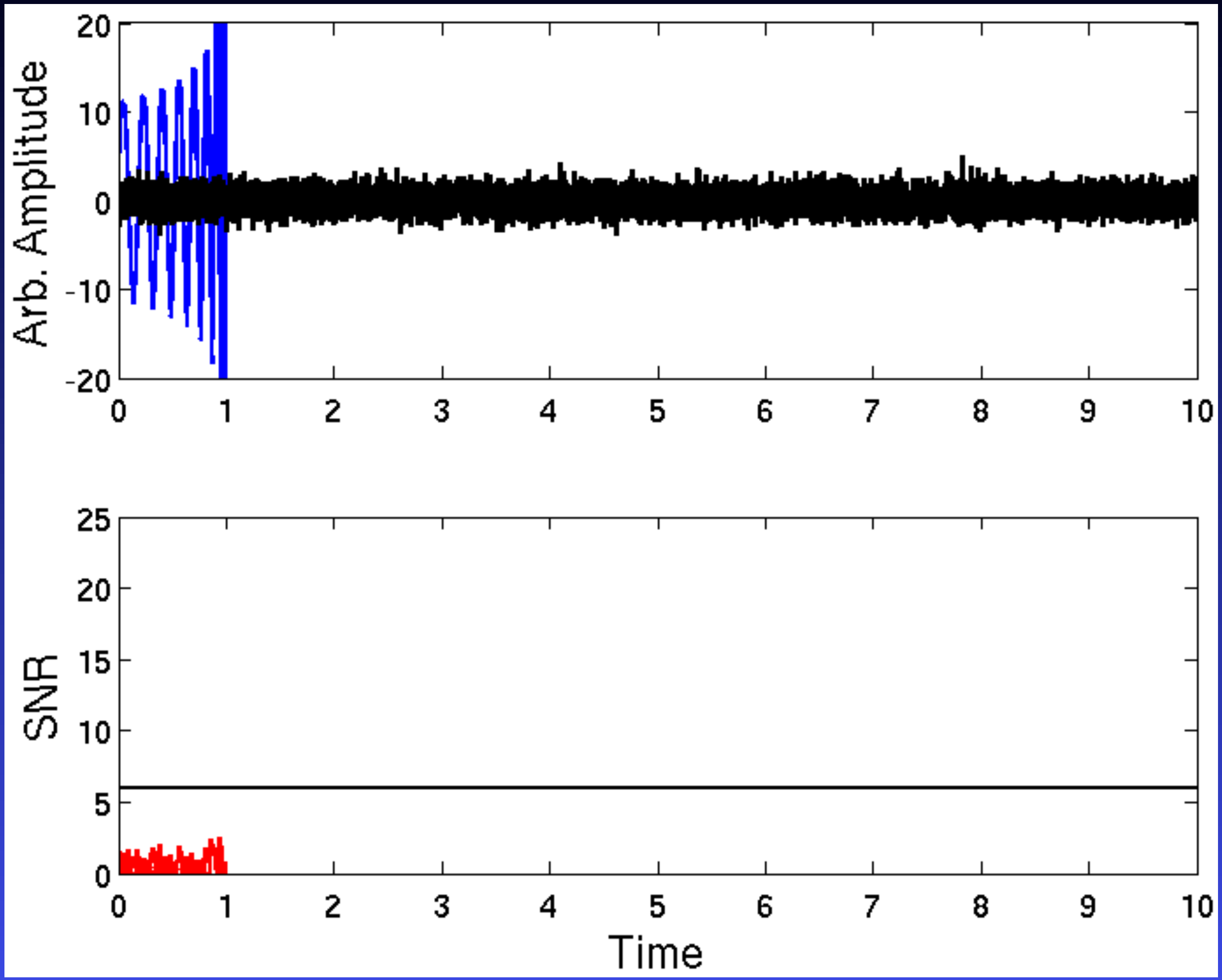
This can be quantified better:

- Borrow noise-matched filters from gravity wave detection (but with a significant simplification: we know where the template starts).
- The SNR is, as usual:

Fourier transform of Template

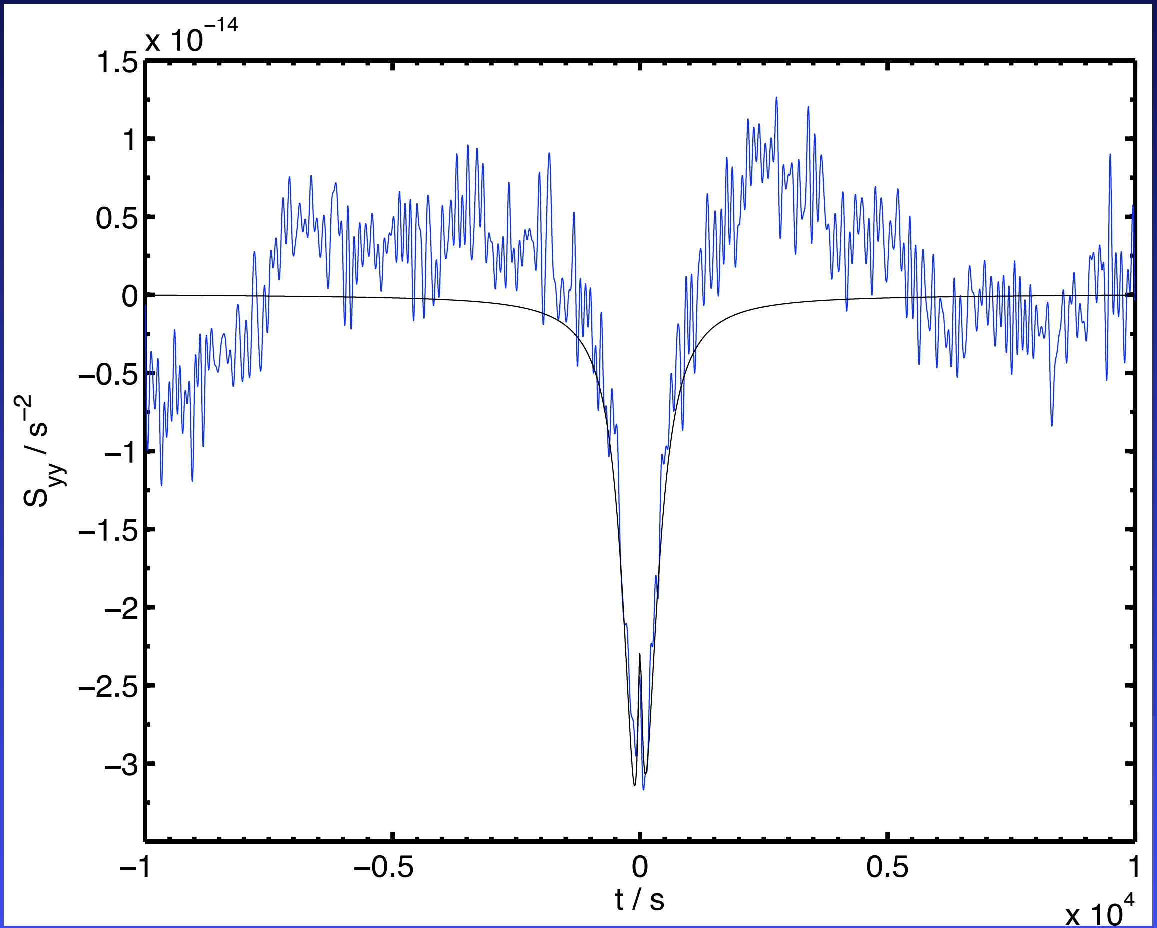
$$SNR = 2\left[\int_0^\infty df\,\frac{|\tilde{h}(f)|^2}{N(f)}\right]^{1/2}$$

Noise power spectrum

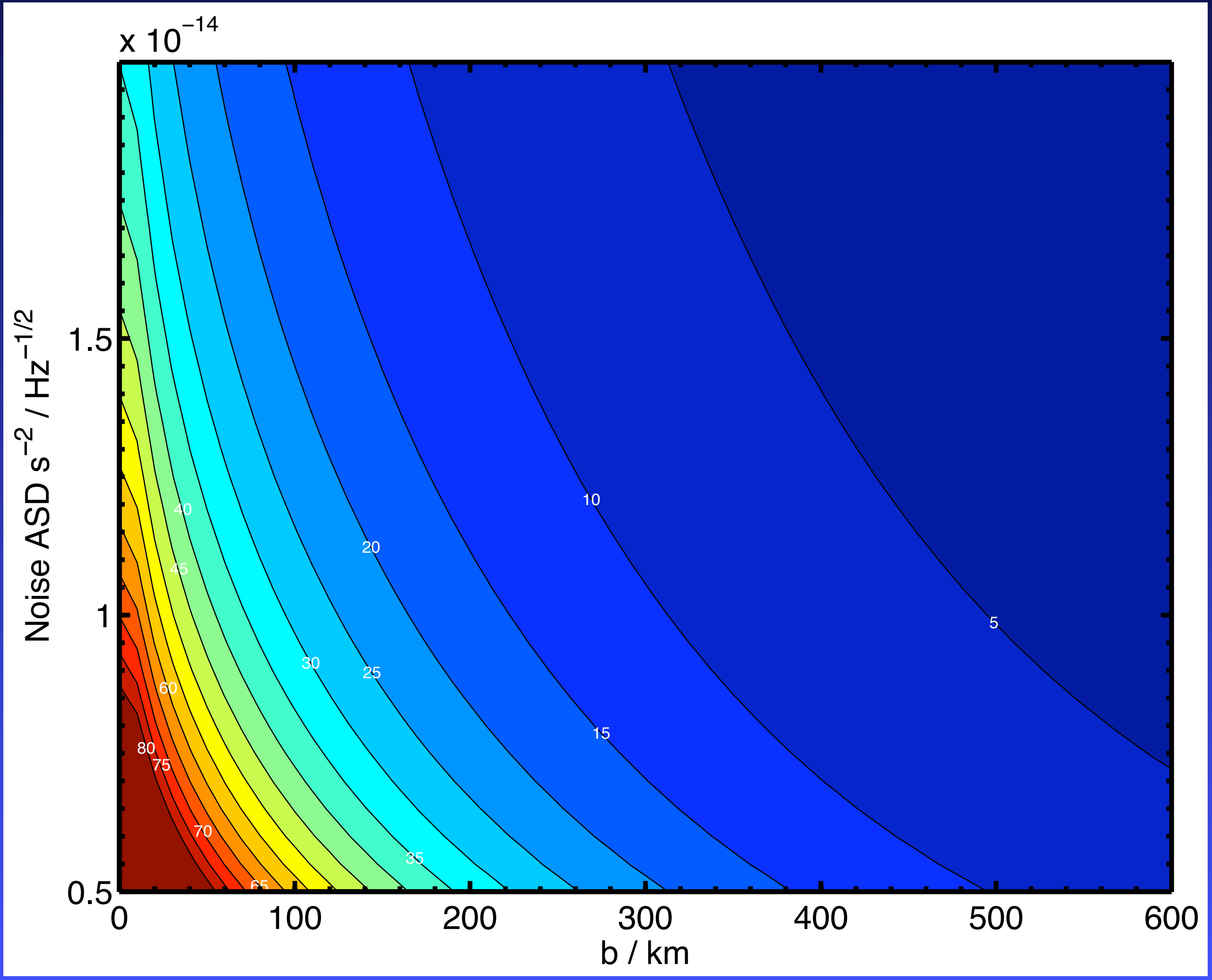


Arb. Amplitude
20
10
0
-10
-20
0 1 2 3 4 5 6 7 8 9 10
SNR
25
20
15
10
5
0
0 1 2 3 4 5 6 7 8 9 10
Time

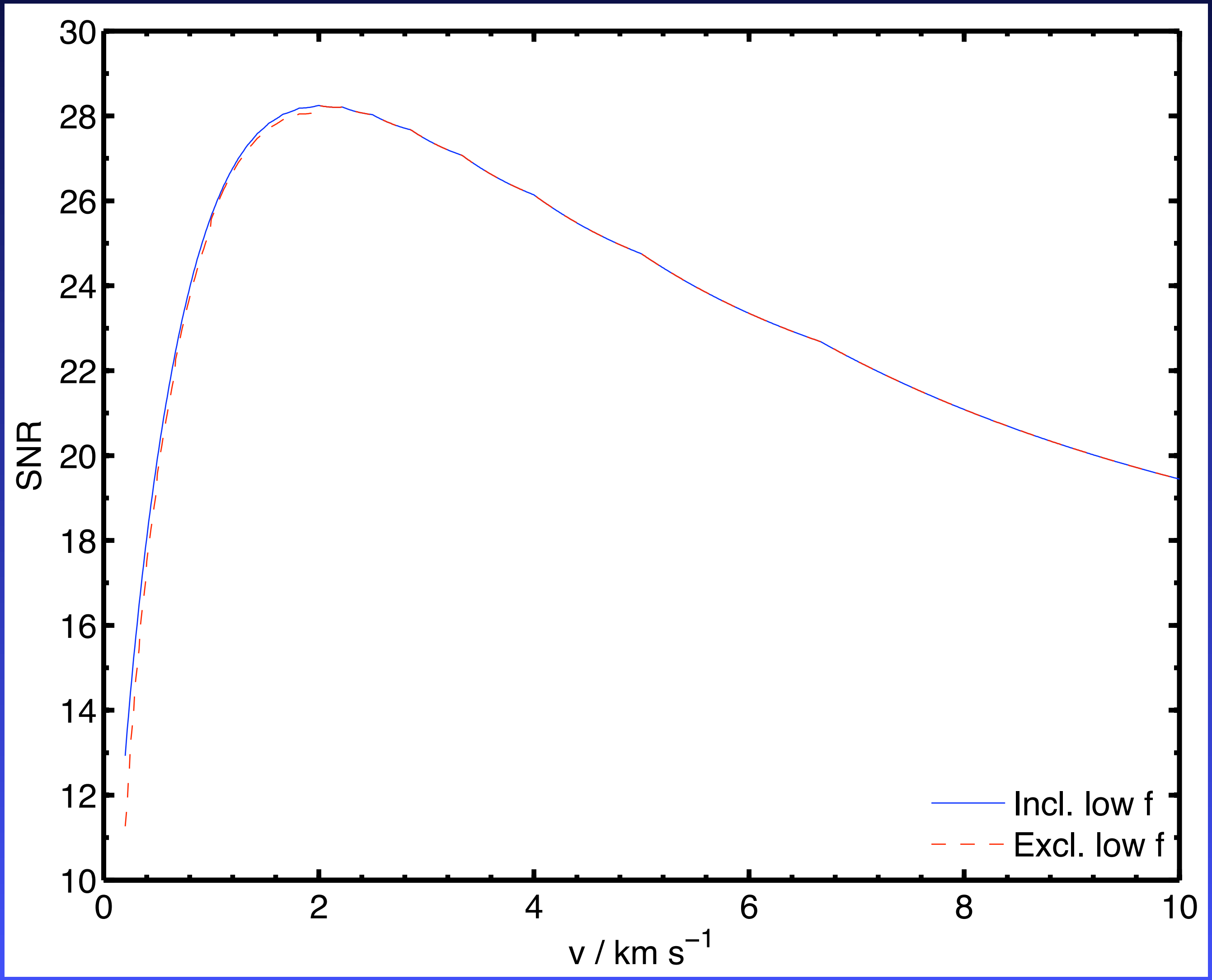
Simulating the signal and noise

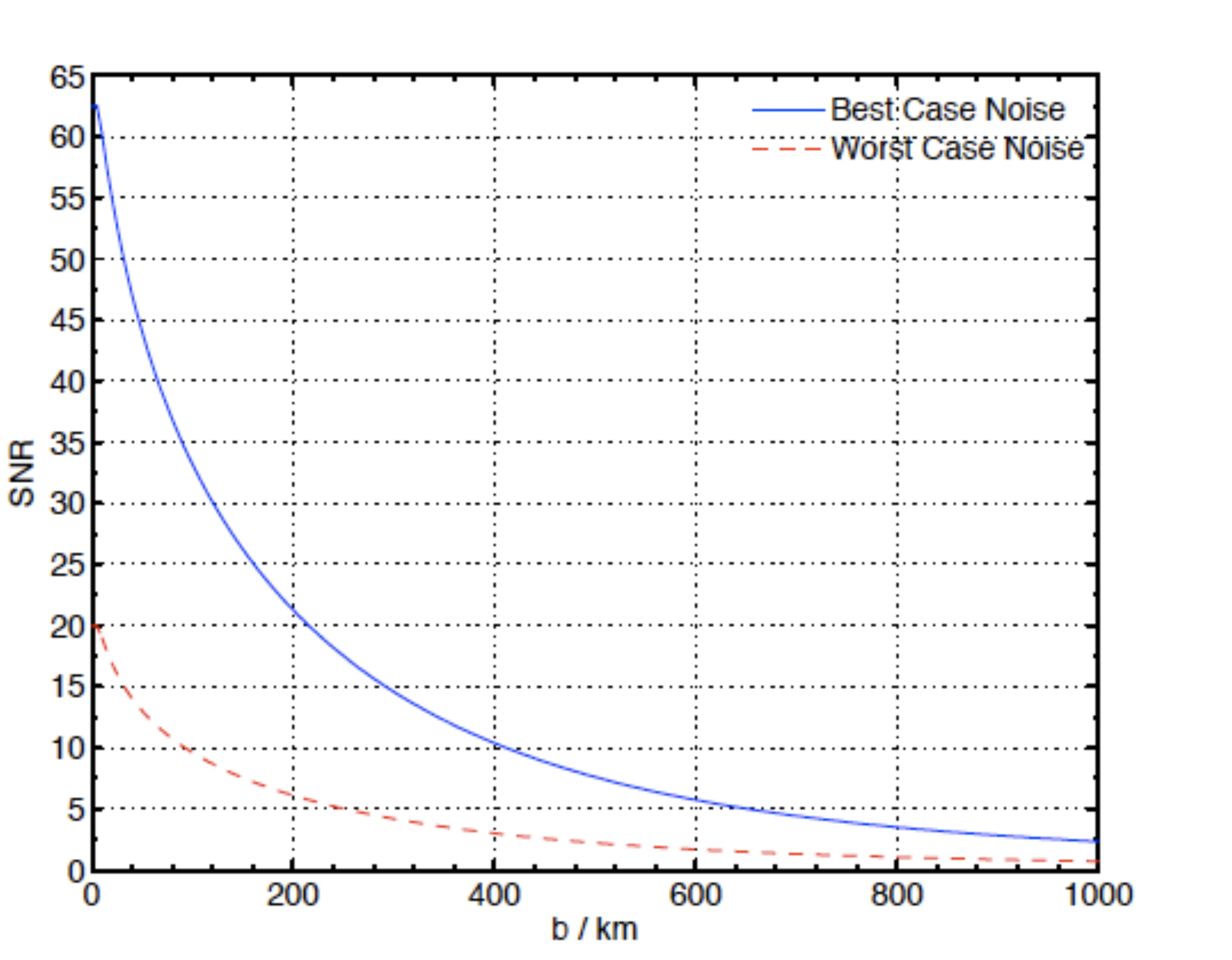


SNR for different b and noise at v=1.5Km/sec



The effect of the velocity at b=50 Km



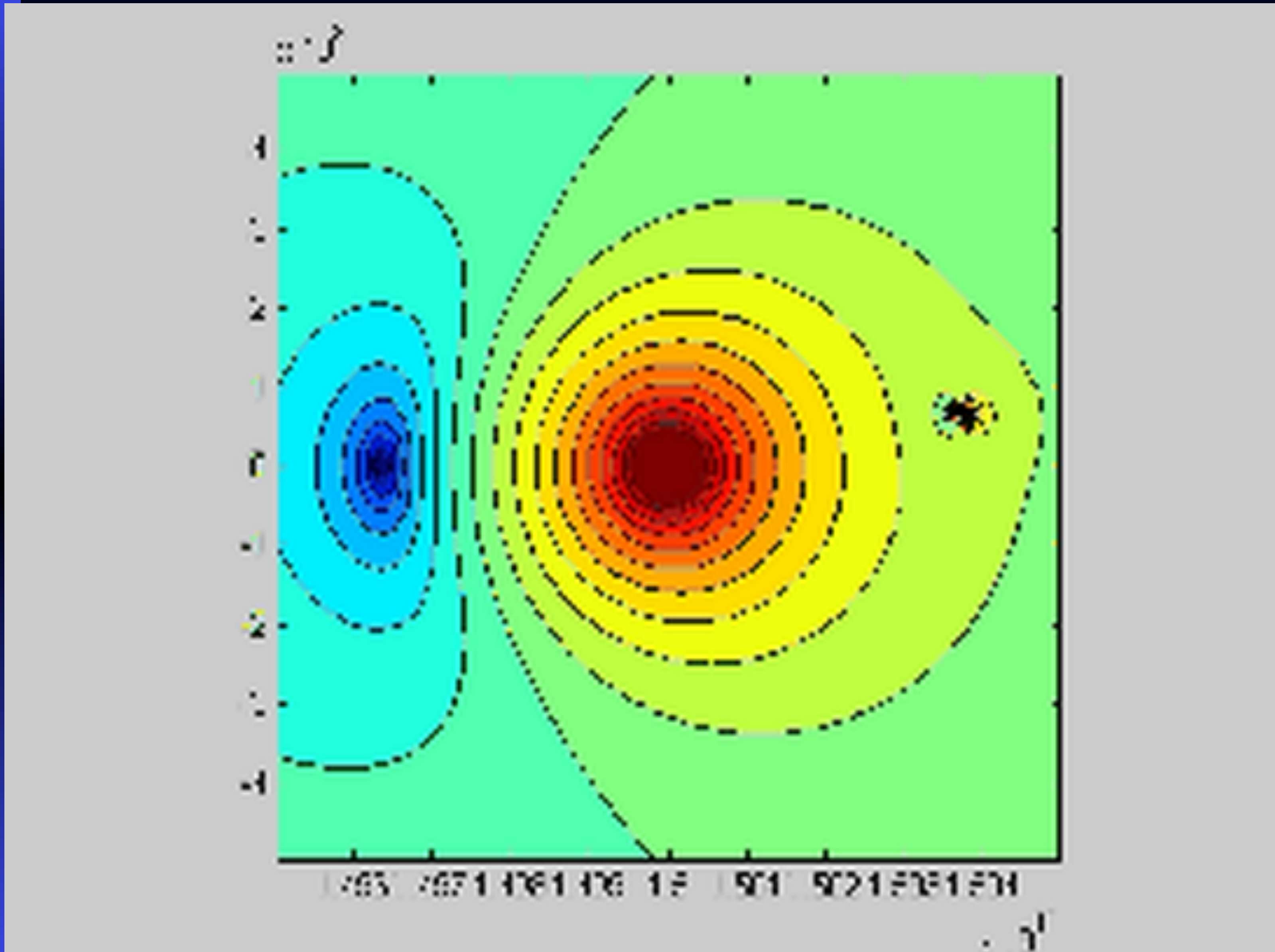


Best Case Noise
Worst Case Noise
SNR
b / km
0
5
10
15
20
25
30
35
40
45
50
55
60
65
0
200
400
600
800
1000

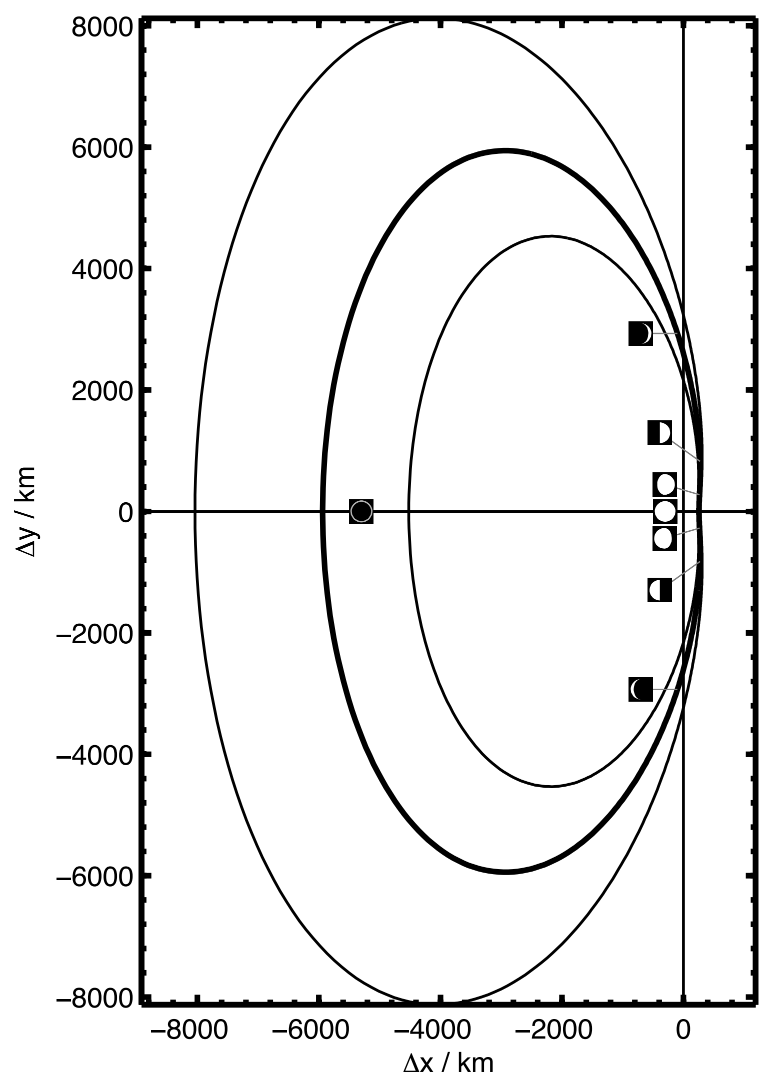
Beyond the cartoon, part II:

- A practical matter: the Moon is a clear perturbation. Is this going to be a pain?

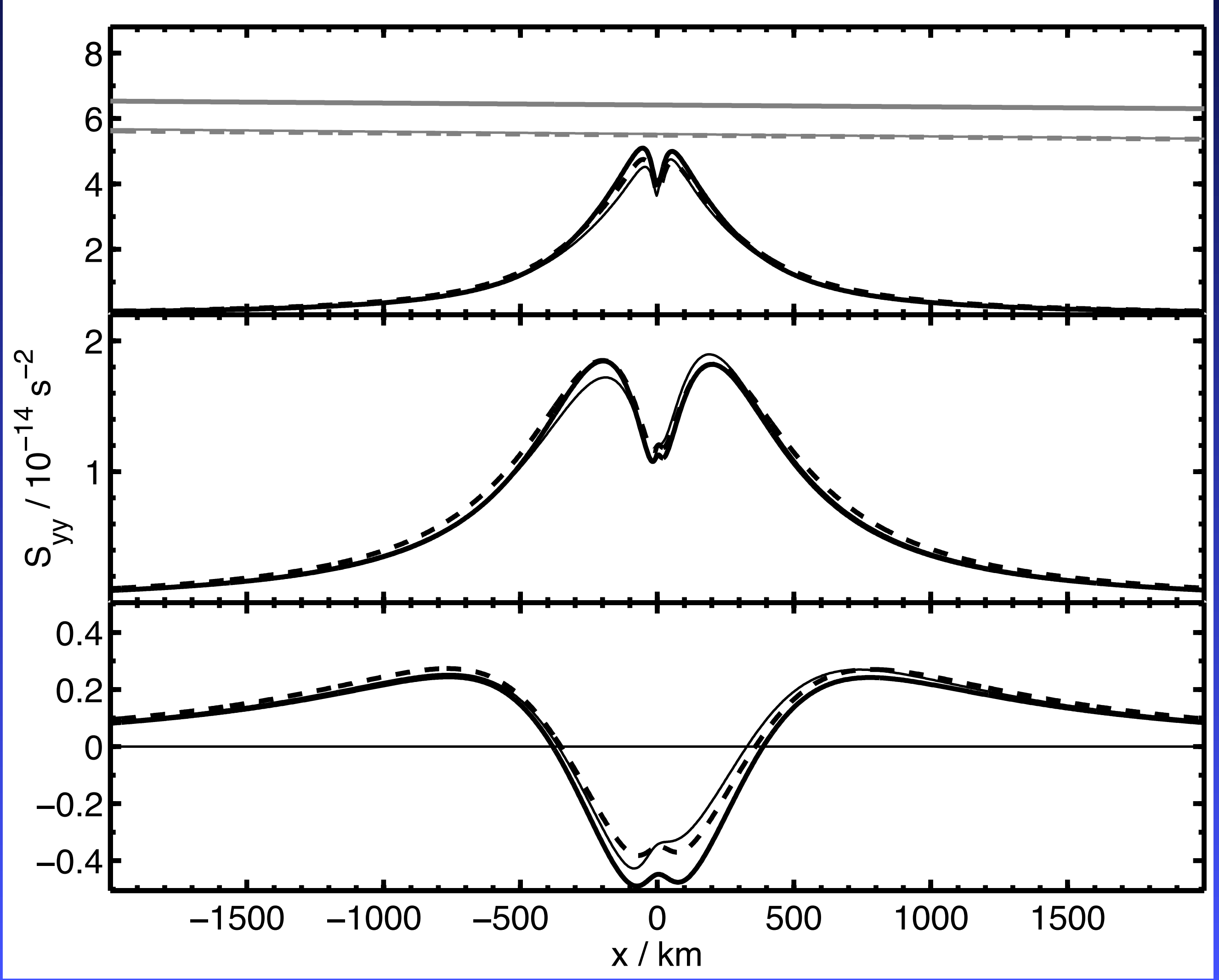
  NO!

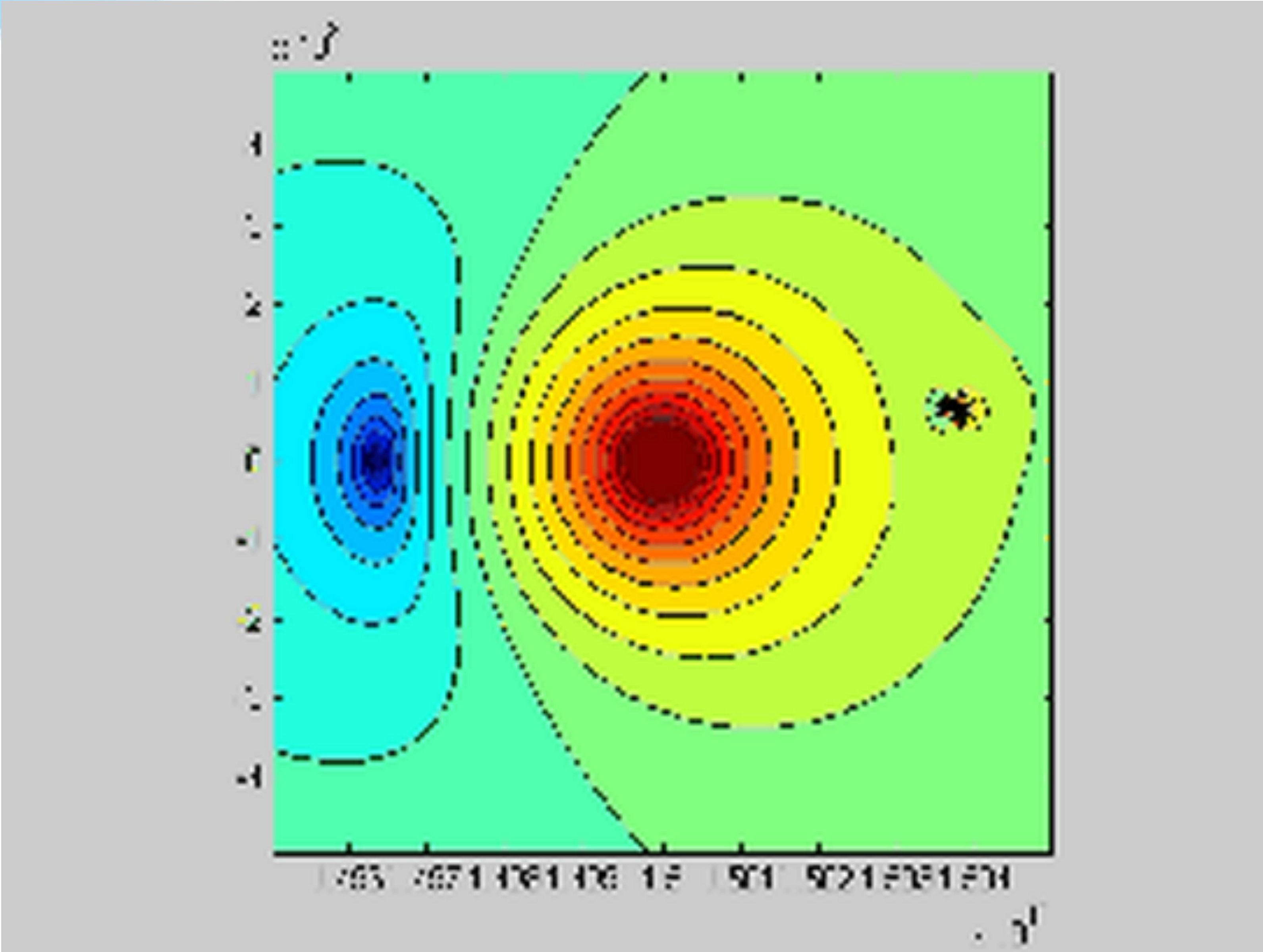


The Earth-Sun saddle is stable

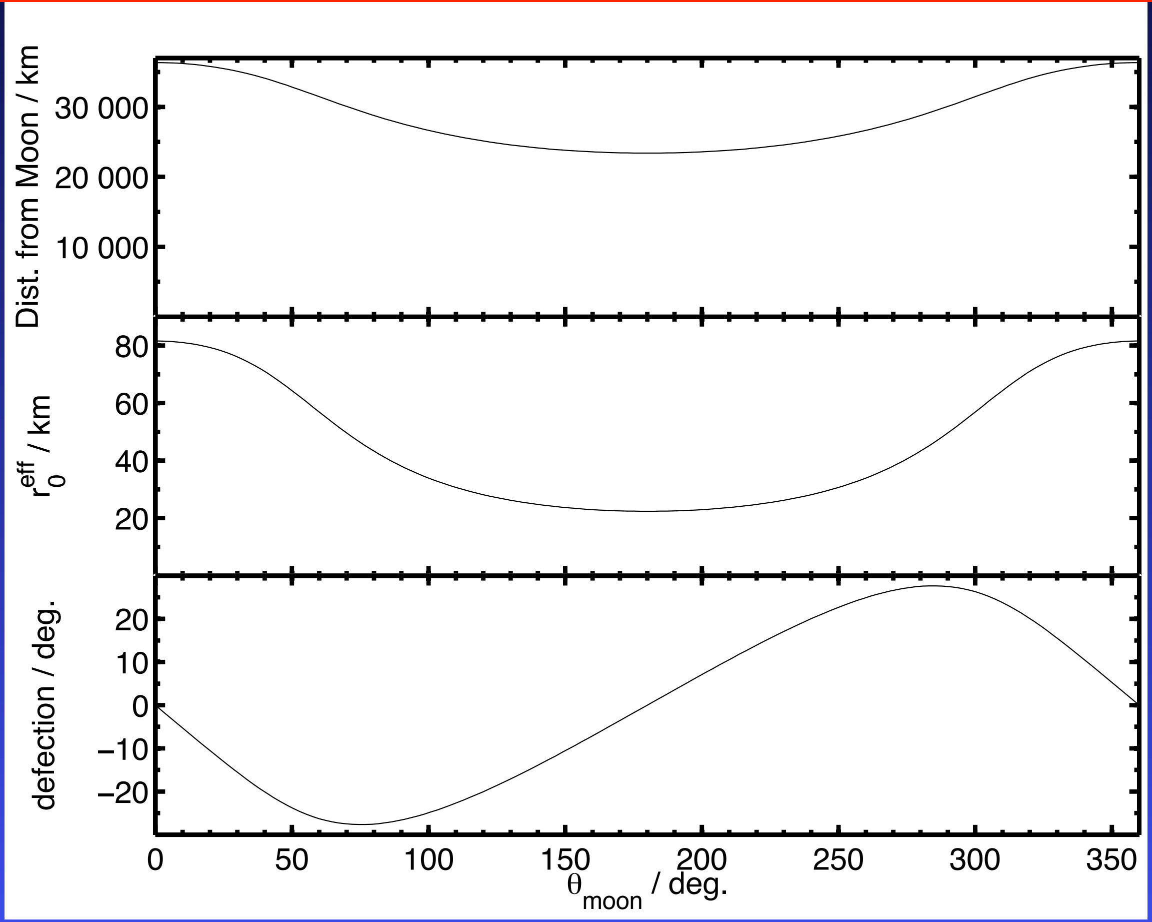


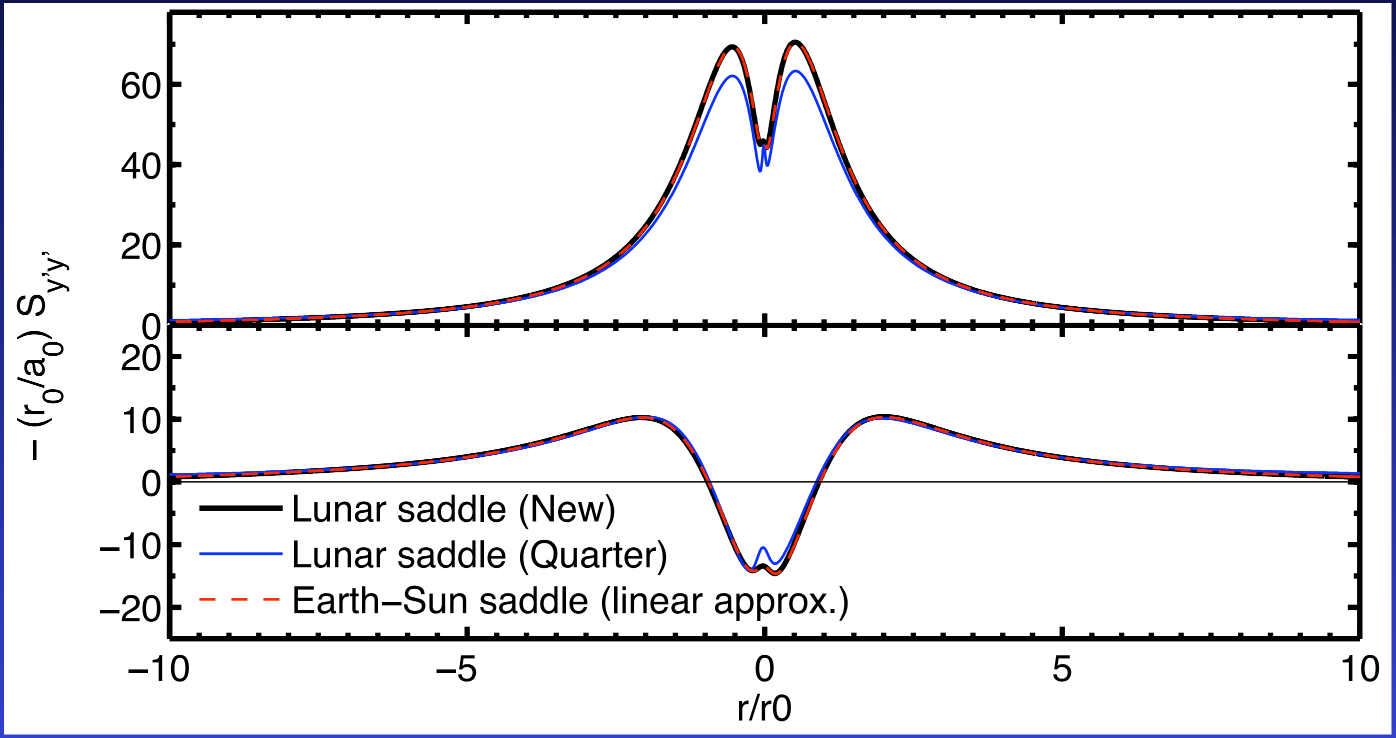
Transverse tidal stress at impact 25, 100, 400 Km of the Earth-Sun saddle





Is the lunar saddle any good?

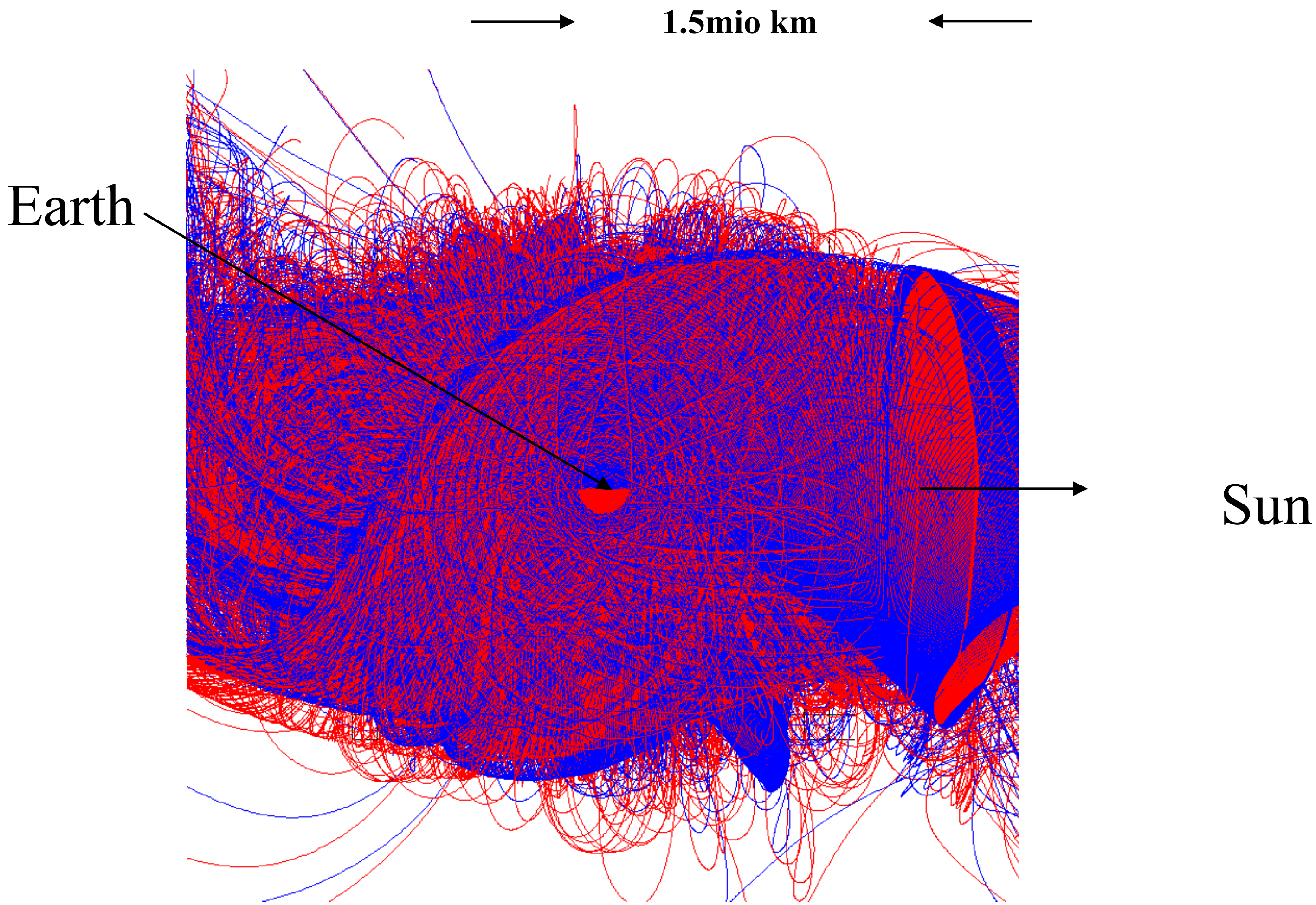




60
40
20
0
20
10
0
−10
−20
$-(r_0/a_0)\, S_{y'y'}$
Lunar saddle (New)
Lunar saddle (Quarter)
Earth−Sun saddle (linear approx.)
−10
−5
0
5
10
r/r0

Potential Trajectories for LISA Pathfinder

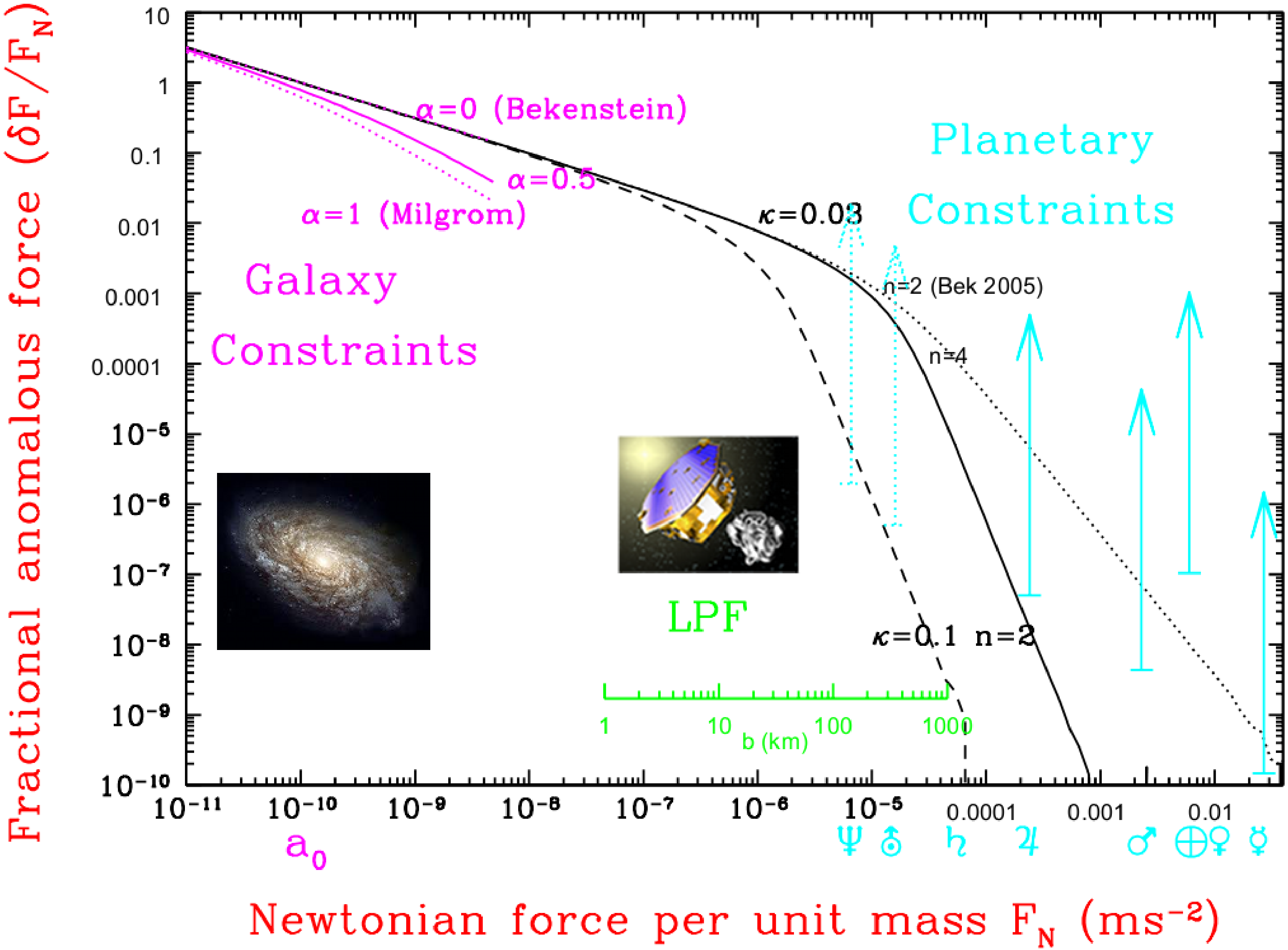
- **Illustration of the chaotic nature of the problem:**



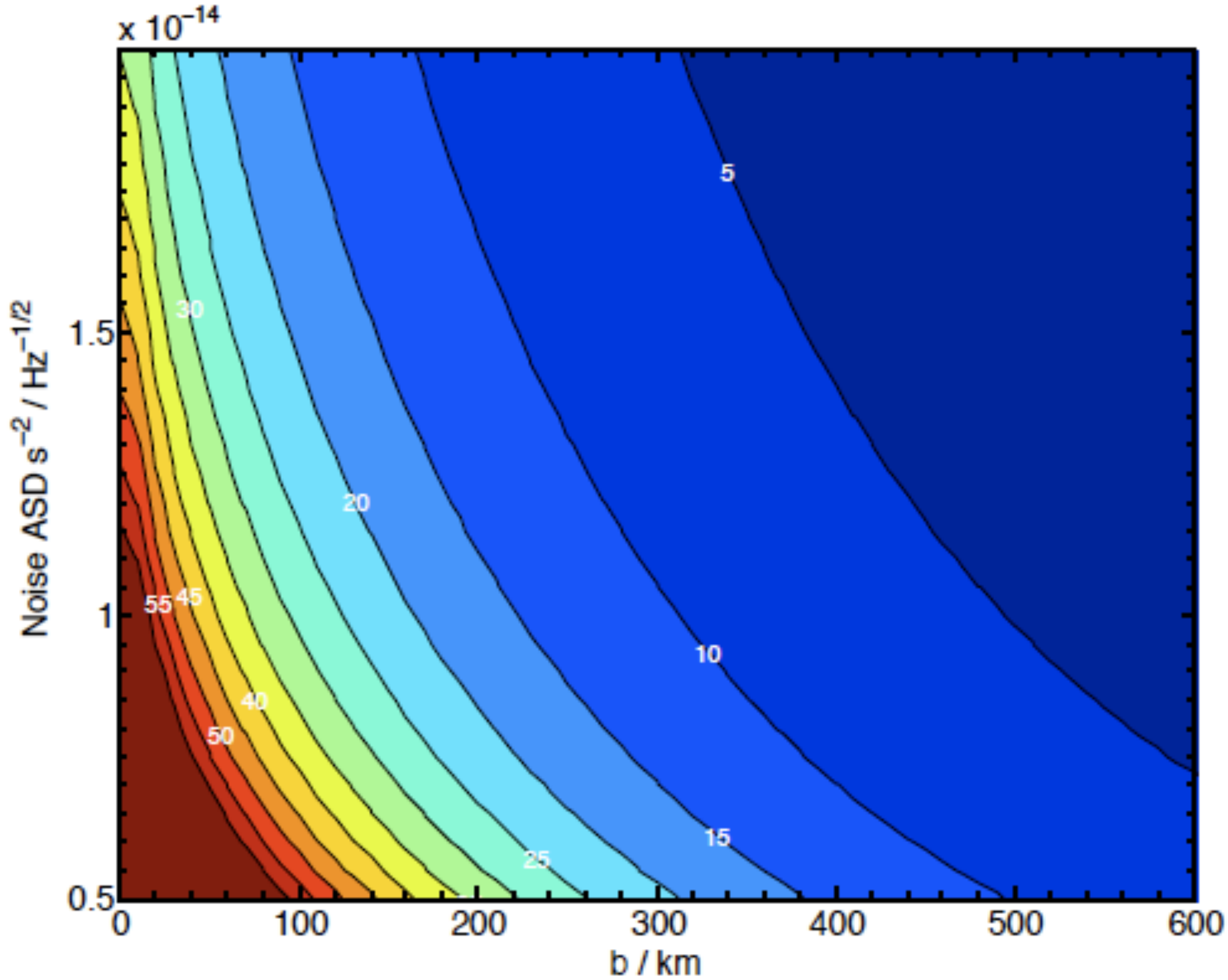
Single dV manoeuvres between 0.5m/s and 1m/s applied at 0.25 day intervals

How generic a test of MOND is this?

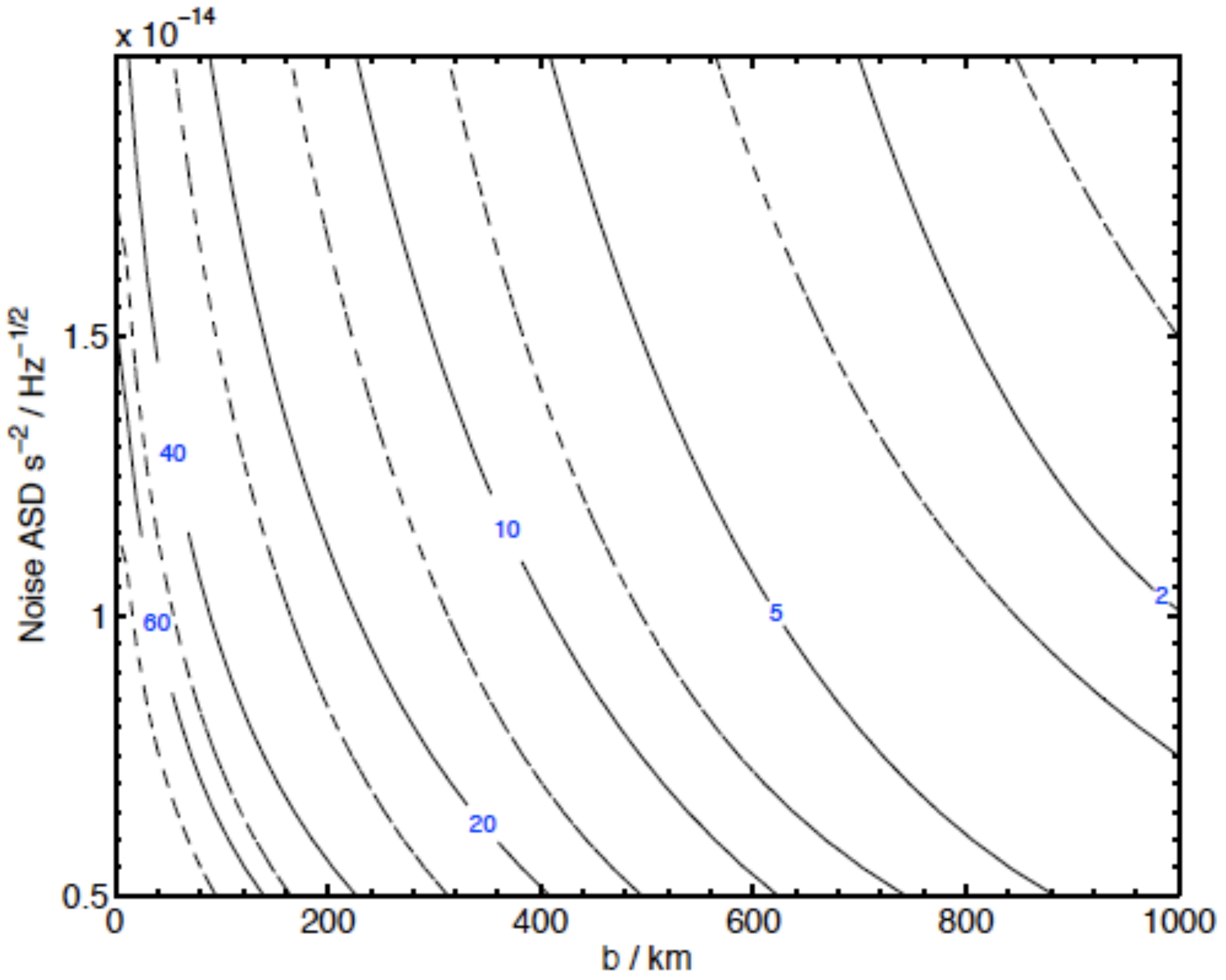
- Most of viable $\mu(y)$ in literature have essentially the same behaviour in the regime being tested
- They differ in the fall off
- They differ where ϕ takes over the Newtonian potential (not probed by LPF).



Fractional anomalous force (δF/F_N)
10
1
0.1
0.01
0.001
0.0001
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
α=0 (Bekenstein)
α=0.5
α=1 (Milgrom)
Galaxy
Constraints
Planetary
Constraints
κ=0.03
n=2 (Bek 2005)
n=4
LPF
κ=0.1 n=2
1
10
100
1000
b (km)
10⁻¹¹
10⁻¹⁰
10⁻⁹
10⁻⁸
10⁻⁷
10⁻⁶
10⁻⁵
0.0001
0.001
0.01
a₀
Ψ ♅ ♄ ♃ ♂ ⊕♀ ☿
Newtonian force per unit mass F_N (ms⁻²)

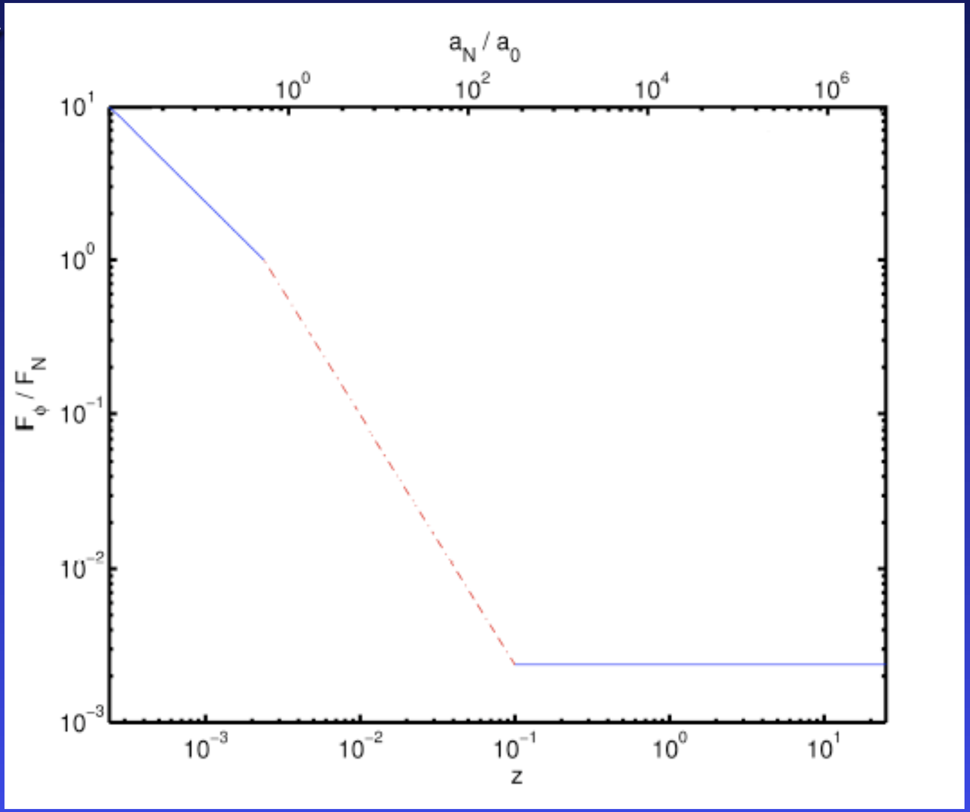

x 10^-14
Noise ASD s^-2 / Hz^-1/2
b / km
0.5
1
1.5
0
100
200
300
400
500
600
5
10
15
20
25
30
40
45
50
55

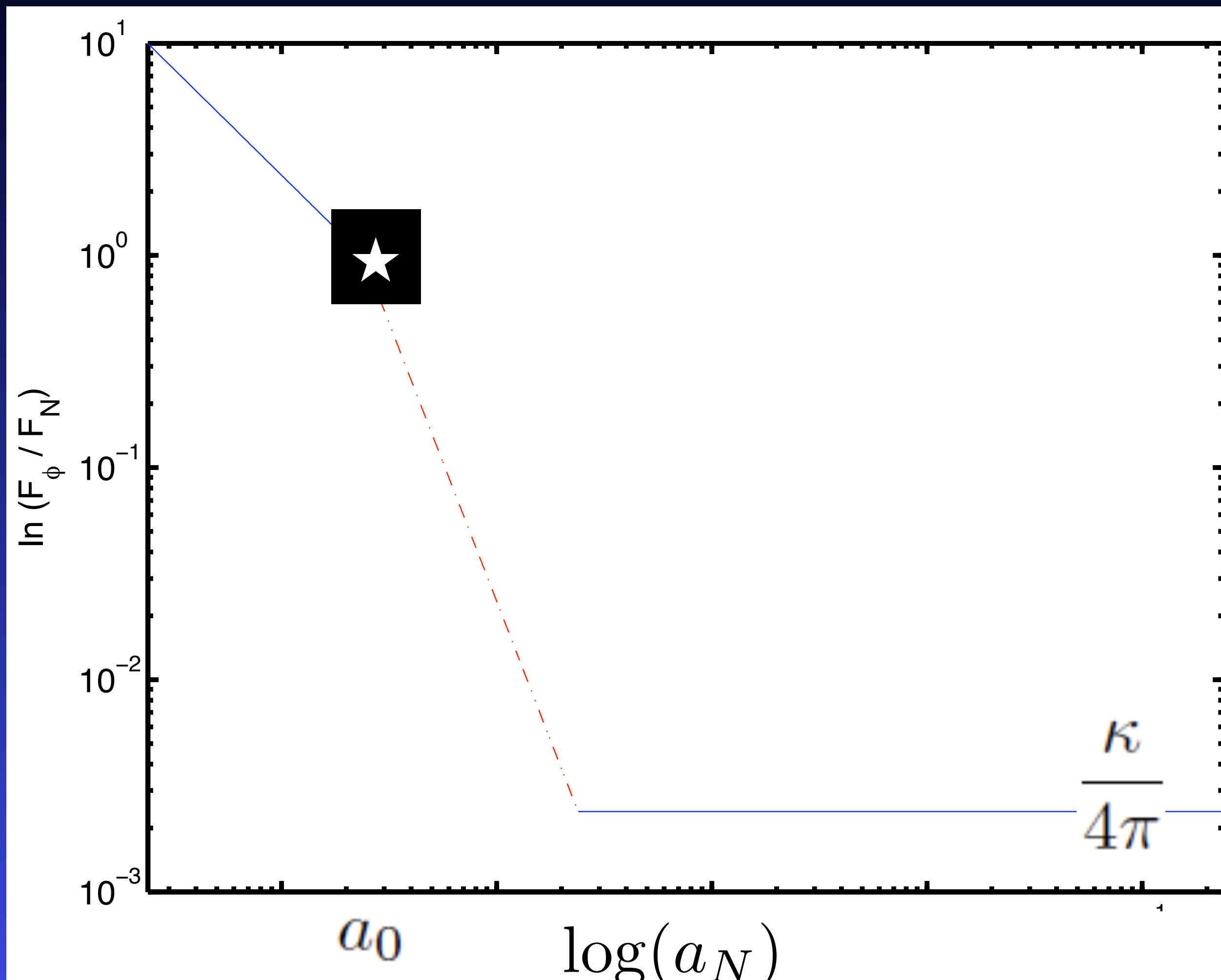
What about different types of theory?



What kind of theory could survive a negative result?

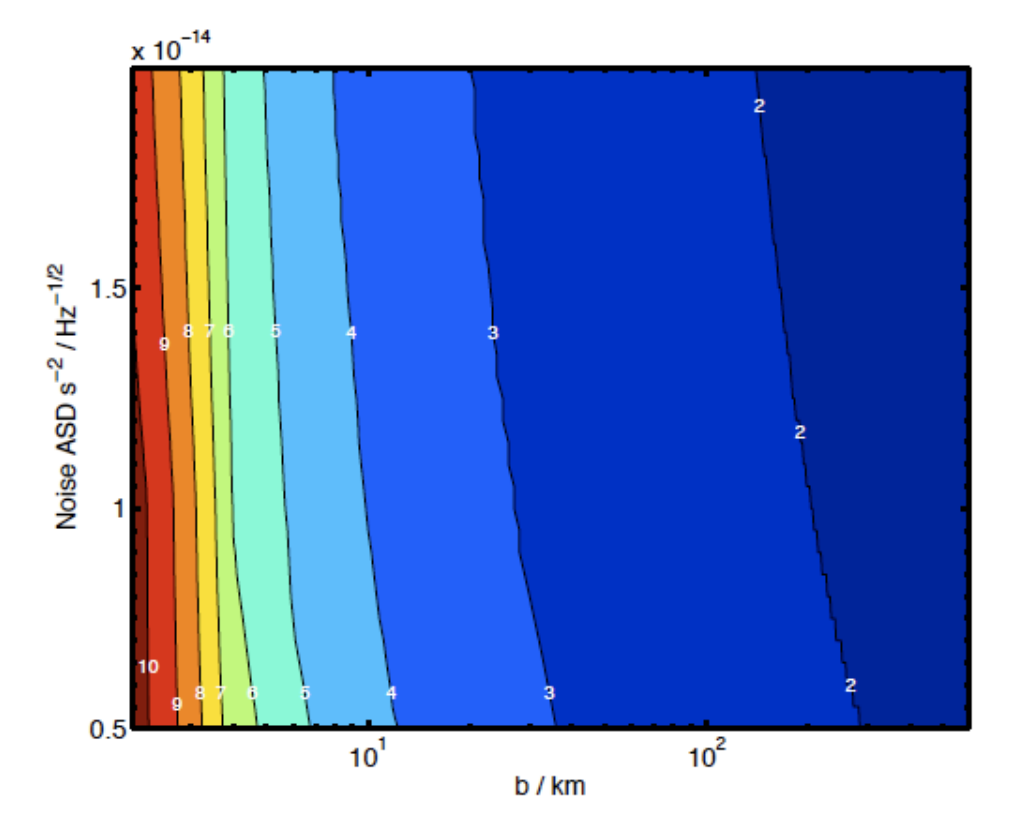
- A double power law in $\mu(y)$ would bypass a negative result





10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$\ln (F_\phi / F_N)$
$\frac{\kappa}{4\pi}$
a_0
$\log(a_N)$

A “Ridicule-O-meter”



A negative result would be pretty damning

- A double power law in $\mu(y)$ld bypass a negative result
- The intermediate power would have to be very large
- RIDICULOUS!

Conclusions:

- We have produced detailed predictions for a LPF signal for the Earth-Sun (and Moon) saddles
- SNR ratios hit double/triple figures even with modest assumptions on impact parameters and noise
- A negative result would kill MOND in most relativistic incarnations

Let’s do the experiment! It will answer many questions.

- Is dark matter a “Vulcan”?
- Is modifying gravity unwarranted lunacy?