Gravitational wave emission from glitching and bursting neutron stars

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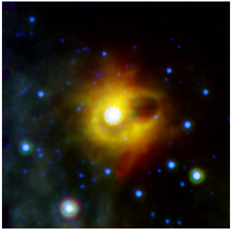
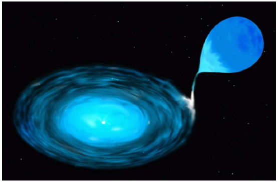
Overview: neutron stars as burst sources

- Young pulsars **glitch**.
- Magnetars **flare**.
- Low-mass X-ray binaries **burst**.

Crab pulsar



SGR 1900+14

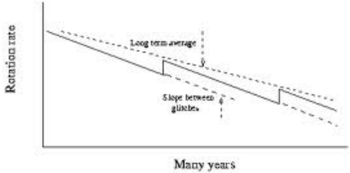


LMXB

- Which **normal modes** are excited, to what amplitudes, and what are the GW/EM implications?

What is a glitch?

- Most of the time pulsar spin frequencies gradually decrease.
- Occasionally some younger pulsars undergo sudden spin-ups.



- Fractional variation in spin frequency small, e.g.

$$\frac{\Delta\Omega}{\Omega} \sim 10^{-6}$$

for Vela; this is considered a violent glitchier!

- Data base of Espinoza et al (2011) has 315 glitches in 102 pulsars.

Glitch energies: a naive estimate

- In absence of detailed model, can make a 'naive' estimate:

$$E_{\rm glitch} = I\Omega\Delta\Omega = I\Omega^2 \frac{\Delta\Omega}{\Omega}.$$

- Parameterising with Vela in mind:

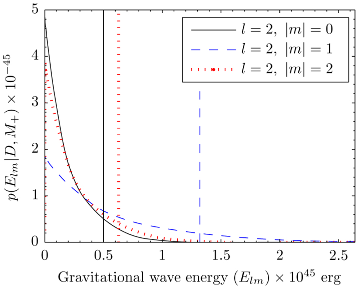
$$E_{\rm glitch} \sim 4.95 \times 10^{42}\,{\rm erg} \left(\frac{f_{\rm spin}}{11.2\,{\rm Hz}}\right)^2 \left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right),$$

$$h_{\rm rss} \sim 10^{-22}\,{\rm Hz}^{1/2} \left(\frac{287\,{\rm pc}}{r}\right) \left(\frac{f_{\rm spin}}{11.2{\rm Hz}}\right) \left(\frac{1\,{\rm kHz}}{f_{\rm GW}}\right) \left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right)^{1/2}.$$

- But can all this energy be put into modes, and if so which?

The Vela glitch paper: f-modes

- Search for 'fundamental' f-mode excitation following a Vela glitch was carried out (Abadie+ 2011).
- Looked for damped sinusoids with $f \sim$ 1–3 kHz, $\tau \lesssim 0.5$ s.
- Found energy release $\Delta E_{\rm GW} \lesssim 10^{45}$ erg.



- How is actual excitation energy spread over different m-values, and how to search for such a signal? (Work with James Clark).
- And who says f-mode is the one most excited . . . ...

Transient sources: r-modes

- For r-modes, mode frequency $\sim$ rotation frequency:

$$|f_{\rm mode}| \approx \frac{4}{3} f_{\rm star}.$$

- Decay time determined by dissipation timescale, not gravitational radiation reaction. For Levin & Ushomirsky (2001) model of the viscous boundary layer:

$$\tau_{\rm viscosity} = \tau_{\rm LU} \sim 700\,{\rm s}\,\frac{T_8}{F^{1/2}}\left(\frac{11.2\,{\rm Hz}}{f_{\rm spin}}\right)^{1/2}\left(\frac{\delta v}{v}\right)^{-2}.$$

- In the regime of a *transient* GW search.

Transient sources: r-modes cont . . .

- R-mode transient search investigated by Santiago-Prieto, Heng, DIJ & Clarke (2012).
- Not all of the glitch energy goes into GWs:

$$\Delta E_{\rm GW} = \frac{\tau}{|\tau_{\rm GRR}|} E_{\rm mode}.$$

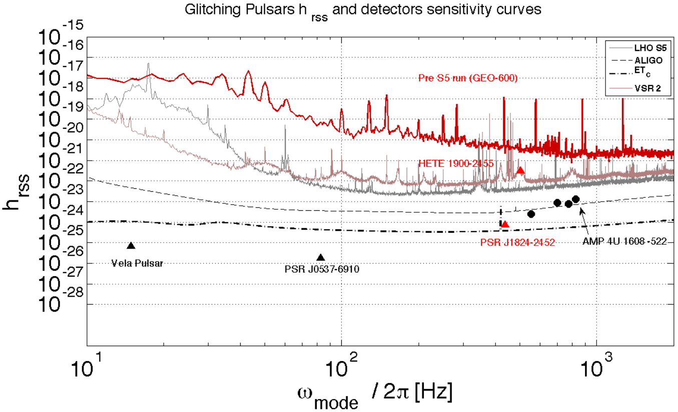
- In fact, this is very inefficient:

$$\frac{\tau_{\rm LU}}{\tau_{\rm GRR}} \approx \frac{\Delta E_{\rm GW}}{E_{\rm glitch}} = 6.76 \times 10^{-7} \frac{T_8}{F^{1/2}} \left(\frac{f_{\rm spin}}{62\,{\rm Hz}}\right)^{11/2} \left(\frac{\delta u}{u}\right)^{-2}.$$

- Not encouraging, but must remember damping rather uncertain.

Transient sources: r-modes cont . . .

Making use of real data (plot from Santiago-Prieto):



The starquake model

- Outer part of neutron star is a solid **elastic** crust.
- As star spins down, **strains** build up in crust.
- Once critical breaking strain reached, crust **fractures**, and settles down to new less oblate state.
- Conservation of angular momentum demands a corresponding **increase** in the spin frequency.

The starquake model: simple estimates

- For star of ellipticity ϵ, angular momentum $J = I(1+\epsilon)\Omega$, can write energy as:

$$E = E_{\text{sphere}} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2 + \frac{J^2}{2I(1+\epsilon)}.$$

  where $A \sim$ gravitational binding energy, $B \sim$ electrostatic binding energy of solid crust, and crust relaxed when $\epsilon = \epsilon_0$.

- Minimising at fixed angular momentum gives 2-parameter family:

$$\epsilon = \epsilon(\Omega, \epsilon_0) = \frac{I\Omega^2}{4(A+B)} + \frac{B}{A+B}\epsilon_0 \equiv \epsilon_\Omega + \epsilon_{\text{def}}.$$

The starquake model: simple estimates cont . . .

- Can use model to estimate glitch energy:

$$\Delta E \sim \Delta E^{\rm naive} \frac{2Bu}{I\Omega^2}$$

  where $u \sim$ strain in crust.

- Parameterising:

$$\Delta E \sim 4 \times 10^{-2} \Delta E^{\rm naive} \left(\frac{B}{10^{48}\,\rm erg} \right) \left(\frac{u}{0.1} \right) \left(\frac{11.2\,\rm Hz}{f_{\rm spin}} \right)^2 .$$

The problem with the starquake model

- Strain relieved at glitch is of order

$$\Delta u_{\rm G} \sim \frac{\Delta\Omega_{\rm G}}{\Omega}.$$

- Can use energy model to show that strain build up in interval $\Delta_{\rm IG} t$ between glitches is of order

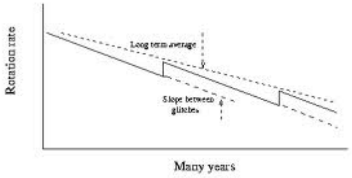
$$\Delta u_{\rm IG} \sim \epsilon_\Omega \frac{\Delta t_{\rm IG}}{\tau}$$

  where $\tau \sim \Omega/\dot{\Omega}$, the characteristic age.

- For Crab, $\Delta u_{\rm IG} \sim \Delta u_{\rm G}$.
- For Vela, $\Delta u_{\rm IG} \ll \Delta u_{\rm G}$.
- This is clearly a problem–Vela can't replenish strain between its glitches!

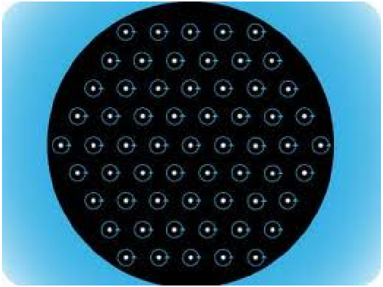
What is a glitch? A closer look

- Some pulsars have undergone *multiple* glitches.
- $\sim 1\%$ of spin-down reversed in glitches.
- Taken as evidence that about 1% of moment of inertia decoupled from smooth spin-down.



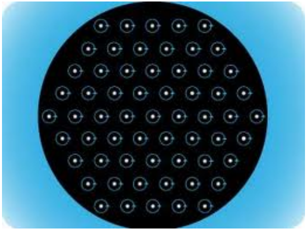
Basics: Superfluid neutron stars

- Can model star as a mixture of:
  1. Superfluid neutrons;
  2. Charged particles (protons & electrons).
- The superfluid neutrons rotate by forming an array of vortices:



Radio pulsars: glitches

- Area density of vortices determine rotation rate.
- For smooth spin-down, vortices migrate outwards at a rate $\propto \dot{\Omega}$, to allow for smoothly decreasing area density.
- *Pinning model*: some of the superfluid vortices are rigidly attached to the solid phase, preventing them from undergoing smooth spindown.
- When a sufficiently large angular velocity lag has built up catastrophic unpinning occurs, corotation is established, spinning up the charged part of the star.



The pinned superfluid model: simple estimates

- Conserve angular momentum between two rigidly rotating components, the ‘crust’ and the ‘superfluid’; find

$$\Delta E \sim 10^{-4} \Delta E^{\rm naive} \left(\frac{10^{-2}}{I_{\rm s}/I_{\rm c}} \right) \left(\frac{\Delta\Omega/\Omega}{10^{-6}} \right).$$

More accurate treatments

- van Eysden and Melatos (2008) looked at two-component of infinitely long cylinder.
- Sidery, Passamonti, Andersson (2009) looked at two-component spherical spin-up using time evolutions.
- Want to go back to basics and build up mode excitation model to:
  1. Assess validity of simple estimates;
  2. Clearly identify modes excited;
  3. Assess GW detectability;
  4. Assess EM signature (e.g. in radio pulsar data).

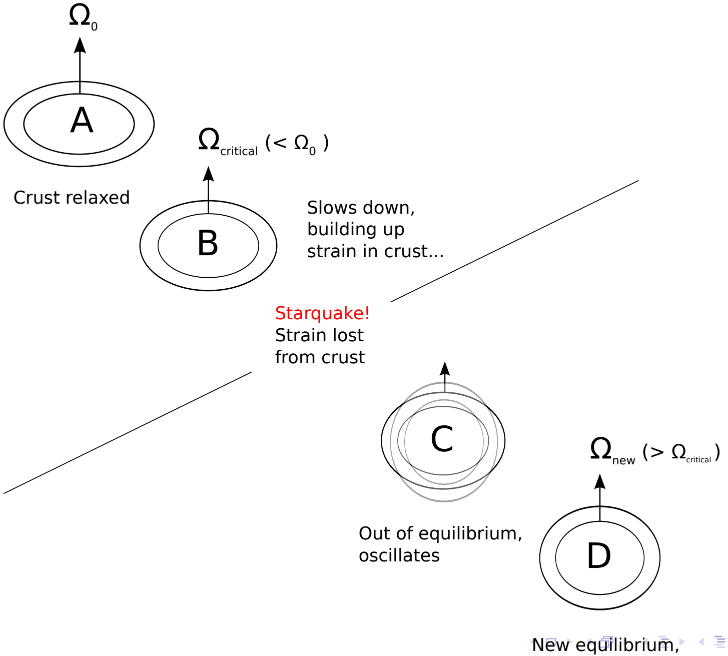
Towards accurate treatment of starquakes

- Subject of Lucy Keer's PhD.
- Treat neutron star as a self-gravitating incompressible elastic sphere, satisfying:

$$\begin{aligned} \rho \frac{dv_a}{dt} &= -\nabla_a P - \rho \nabla_a \Phi + \mu \nabla^2 \xi_a, & (1) \\ \nabla^2 \Phi &= 4\pi G \rho, & (2) \\ \nabla_a \xi^a &= \nabla_a v^a = 0, & (3) \\ P &= P(\rho). & (4) \end{aligned}$$

- Divide solution into two parts:
  1. An axisymmetric stationary rotating background;
  2. Perturbations with frequency ω, i.e. modes.

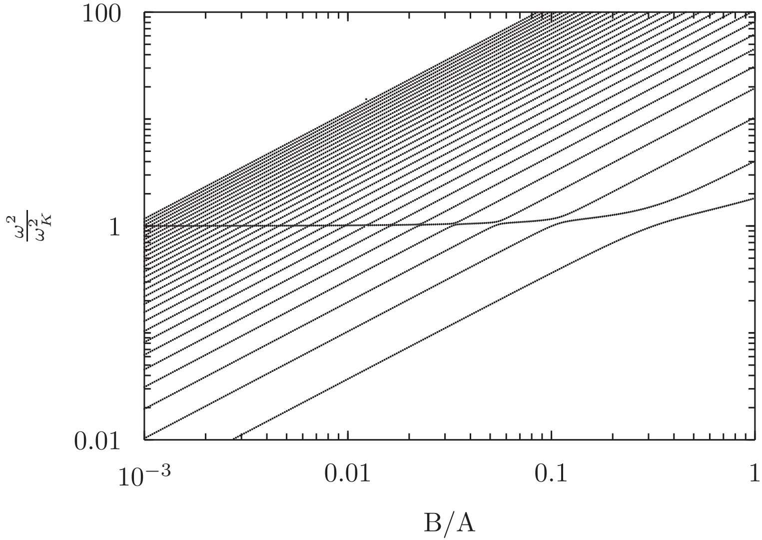
The main idea in pictures



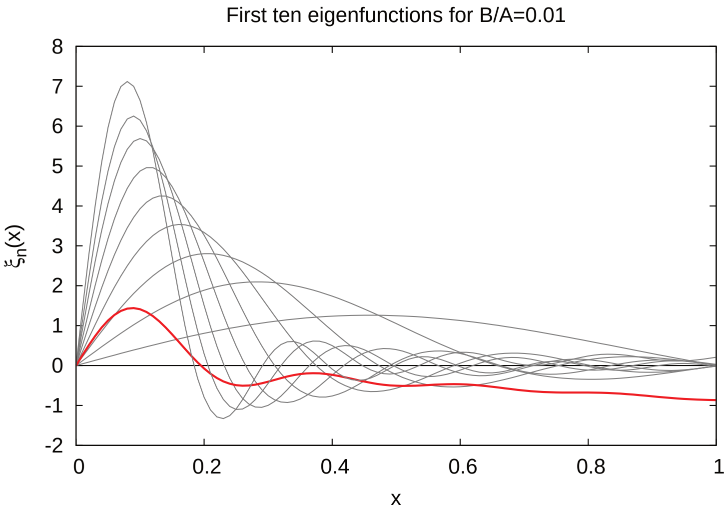
The main idea in words

- Can can compute structure of stars A, B and D as equilibrium solutions.
- C is out of equilibrium; need to supply physical model of glitch to relate C to B.
  - We currently model C as same as B, but with all shear strain removed.
  - Not very realistic (or optimistic), but simple.
  - Have completed $\Omega_B = \Omega_C = \Omega_D = 0$ case, now working on more realistic rotating case.
- Write ‘initial data’ (C-D) in terms of normal modes of D.

The modes: frequencies

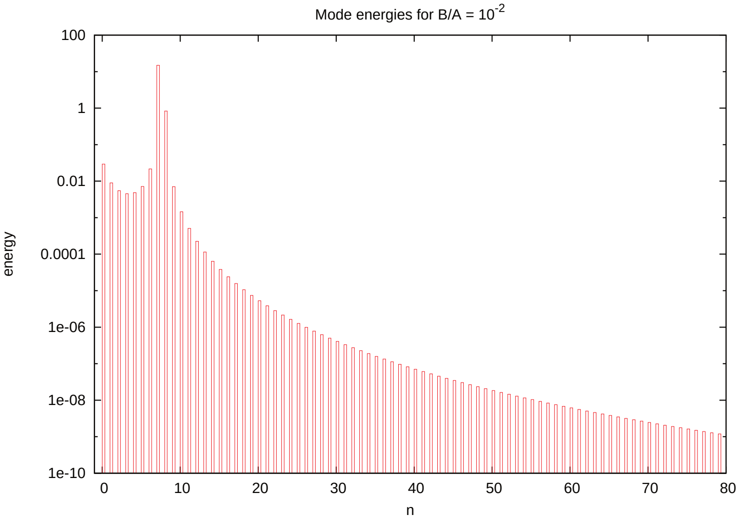


The modes: eigenfunctions



Projecting the initial data onto the modes

energytest2.pdf



Projecting the initial data onto the modes

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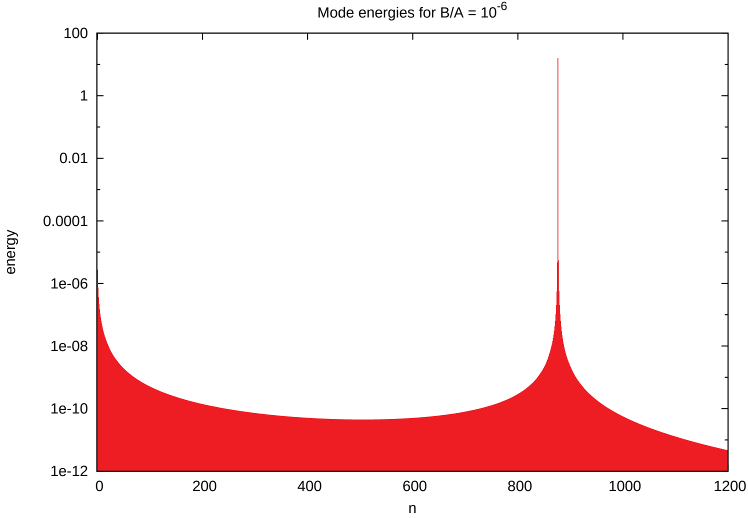

Towards more realistic models

- Have calculated mode excitation for a highly idealised glitch model of a simple star.
- Results consistent with simple $E = E(\Omega, \epsilon_0)$ calculations.
- Want to increase realism by:
  - Allowing for rotation in final state (in progress);
  - Having fluid core/elastic crust;
  - Allowing for incompressibility;
  - **Having more realistic model of starquake itself.**
- No surprises yet, but for each new ingredient in stellar model, you get new set of modes.
- Will hopefully soon go beyond regime where simple estimates are a good guide.