Black-hole production from ultrarelativistic collisions

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Outline

1. The Basic Idea
2. Methodology
3. Results
4. Summary

The Basic Idea

Black-hole Formation

- One of most interesting properties in classical general relativity is “black-hole formation”.
- There are a lot of astronomical and numerical evidence that BHs exist.
- Theoretical and numerical works support that BHs are formed in a variety of environments.

  e.g.,
  - Gravitational collapse of unstable stars.
  - Merger of binary neutron stars.
- We do not have a rigorous criterion for BH formation yet.
  - For example, we cannot predict whether/when the collision of two compact objects will lead to BH formation.

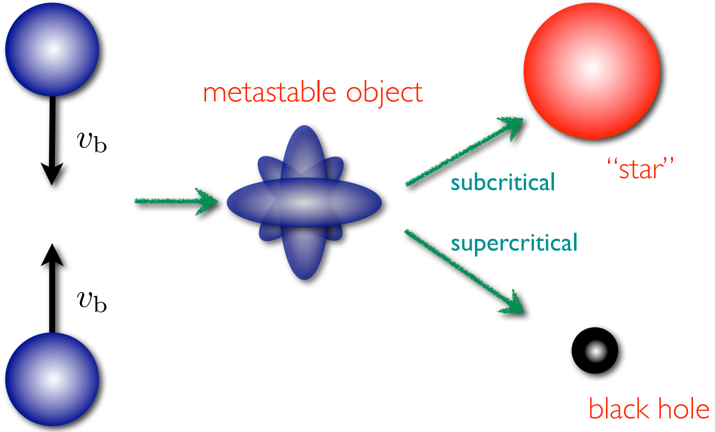
Black-hole Formation

Thorne's hoop conjecture

- Thorne provided us the intuitive *"hoop conjecture"*, which is a reasonable and the only guideline to produce a BH.
  - If mass/energy E is compressed within a hoop with radius $R_{\rm hoop}$, a BH can be formed, where $R_{\rm hoop} \leq R_{\rm s} = 2GE/c^4$.
- However the Thorne's conjecture is not mathematically rigorous.
  - In fact, we do not know whether Thorne's conjecture can be used in the collision of two compact objects.
  - Difficult to determine the contribution of "kinetic energy" in a highly nonlinear process such as a collision.

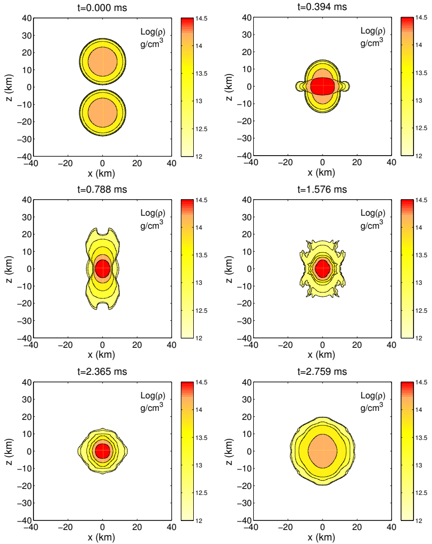
Black-hole Formation in Head-on Collision

The process in cartoon.



Black-hole Formation in Head-on Collision

Typical subcritical collision

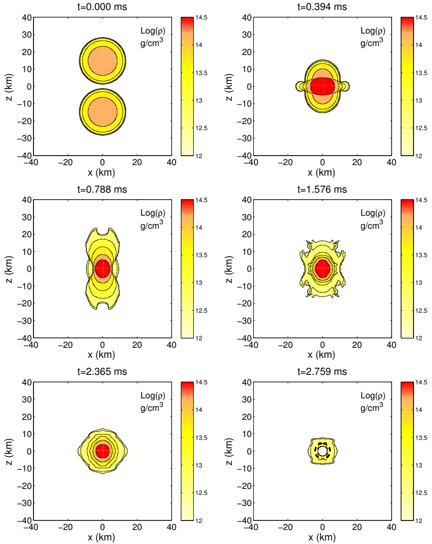


The different panels show snapshots of the rest-mass density at representative times for a subcritical binary.

Note the metastable object in panels 2-5.

Black-hole Formation in Head-on Collision

Typical supercritical collision



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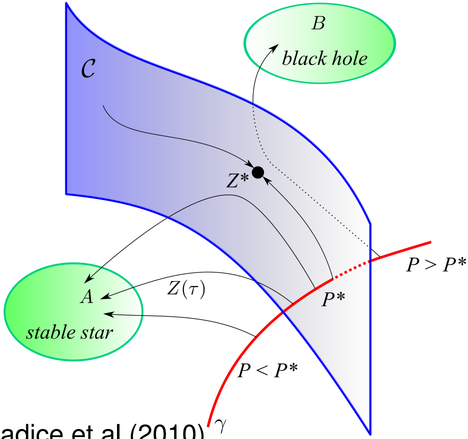
Type-I Critical Behavior in Black-hole Formation

Type-I critical behavior in a head-on collision of compact stars was first pointed out by Jin et al.(2007).



Radice et al.(2010)

Given a series of initial data parametrized by a scalar quantity P, the critical solution at $P^\star$ will separate two basins of attracting solutions.

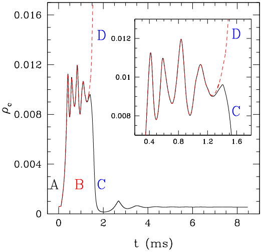
Solutions near the critical one will survive on the critical manifold for a certain time before evolving towards the corresponding basin.

- $P > P^\star \Rightarrow$ black hole
- $P < P^\star \Rightarrow$ "star"

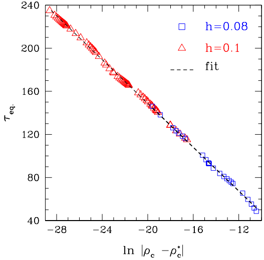
Type-I Critical Behavior in Black-hole Formation

Work by Kellerman et al.(2010).

t vs ρ_c



$|\rho_c - \rho_c^\star|$ vs τ_{eq}



Head-on collision of NSs with initial zero velocity at infinity, *i.e.* , $\gamma = 1$.

One parameter is central rest-mass density, *i.e.* , $P = \rho_c$.

- $\rho_c < \rho_c^\star \Rightarrow$ line C : "star"
- $\rho_c > \rho_c^\star \Rightarrow$ line D : black hole

Survival time of metastable object which is "state B" depends on $|\rho_c - \rho_c^\star|$:

$$\tau_{\text{eq}} \propto -\lambda \ln |\rho_c - \rho_c^\star|$$

They showed $\lambda \sim 10$.

Black-hole Formation in Ultrarelativistic Collision

Ultrarelativistic collision of black-holes

- Penrose (1971)
  - head-on collision of two Aichelburg-Sexl metrics, which is Schwarzschild metric in the limit of $\gamma_b \to \infty$.
  - upper bound of radiated energy is 29%.
- D'Eath (1978,1992)
  - head-on collision of two Aichelburg-Sexl metrics.
  - radiated energy is $\sim$ 16.4% by perturbative methods.
- Eardley et al. (2002)
  - collision of two Aichelburg-Sexl metrics for general impact parameters.
  - cross-section for black hole production is $\sigma > 32.5(GE/2c^4)^2$.
- Sperhake et al. (2008,2009), Shibata et al. (2008) and so on.

Black-hole Formation in Ultrarelativistic Collision

First important work for non-black hole collisions

which was studied by Choptuik and Pretorius (2010). They considered the collision of two classical spherical solitons, *i.e.* , boson stars, with $E = 2\gamma_{\mathrm{b}} m_0 c^2$, where $\gamma_{\mathrm{b}} \equiv 1/\sqrt{1-(v_{\mathrm{b}})^2}$.

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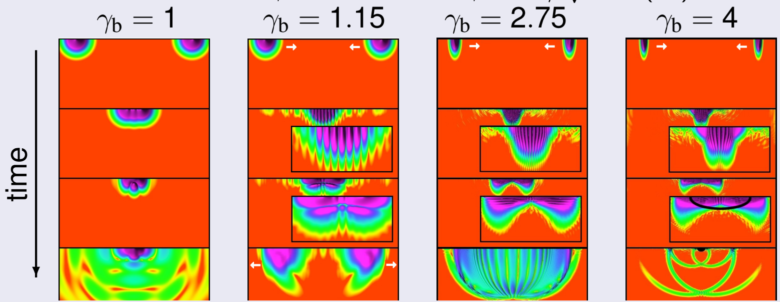

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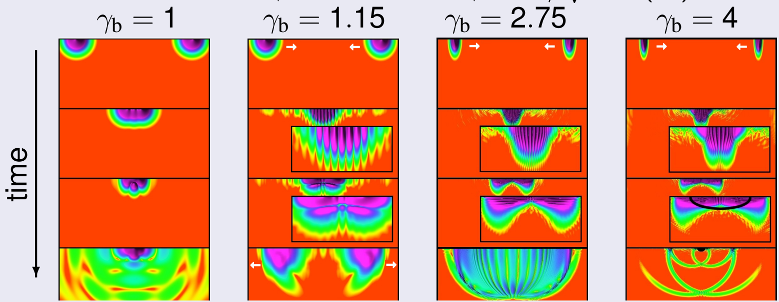


They found that a BH can be formed by sufficiently high boost with $\gamma_b \gtrsim 2.9$.

Black-hole Formation in Ultrarelativistic Collision

Work by Choptuik and Pretorius (2010)

Choptuik and Pretorius studied the collision of two **classical spherical solitons** with ultrarelativistic speeds.

More realistic description of pure fluid matter.

Our work in this presentation

We study the collision of two **selfgravitating fluid objects** with ultrarelativistic speeds.

Methodology

Numerical Method

⟹ All of our calculations have been performed in full general relativity with **axisymmetric**.

For "spacetime" part

- BSSNOK formalism (Nakamura et al. 1987, Shibata et al. 1995, Baumgarte et al. 1998)
  - "cartoon" method (Alcubierre et al. 2001)

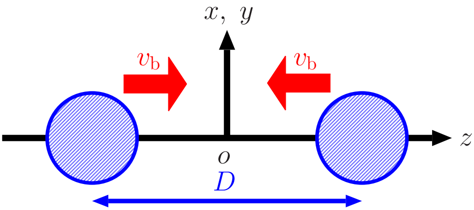
For "fluid" part

- `Whisky2D` code (Kellerman et al. 2008)
  - 2D version of 3D `Whisky` code (Baiotti et al. 2005)
  - TVD with minmod limiter
  - Harten-Lax-van Leer-Einfeldt (HLLE) solver
  - 3rd-order Runge-Kutta scheme

Numerical Method

- Equation of state
  - ideal gas: $p = (\Gamma - 1)\rho\varepsilon$ with $\Gamma = 2$
- Numerical grid structure
  - rectangular domain with $x/M_\odot \in [0, 80]$ and $z/M_\odot \in [0, 150(200)]$
  - uniform grid with $\Delta = 0.08(0.06)M_\odot$
- Boundary conditions
  - reflection boundary conditions on $z = 0$
  - radiative boundary conditions elsewhere

Initial Configuration



- Each object is TOV solution with polytropic equation of state, $p = K\rho^{\Gamma}$, $e = \rho + \frac{p}{\Gamma - 1}$, in the comoving frame, where $K = 100$ and $\Gamma = 2$.
- Each object is boosted against the center of masses frame via Lorentz transformation with v_b or $\gamma_b = \left(1 - v_b^2\right)^{-1/2}$.
  $\Rightarrow\ g_{03} \neq 0,\ K_{ij} \neq 0$

Initial Configuration

- Non-zero metrics :

$$g_{00} = \gamma_b{}^2 \left(\bar{g}_{00} + \bar{g}_{33} v_b{}^2 \right) , \quad g_{11} = \bar{g}_{11} , \quad g_{22} = \bar{g}_{22} ,$$
$$g_{33} = \gamma_b{}^2 \left(\bar{g}_{00} v_b{}^2 + \bar{g}_{33} \right) , \quad g_{03} = -\gamma_b{}^2 \left(\bar{g}_{00} + \bar{g}_{33} \right) v_b .$$

- Non-zero extrinsic curvature :

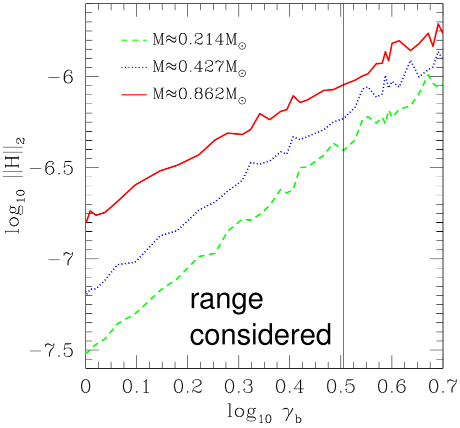
$$K_{xx} = \frac{\gamma_b}{2\alpha} \left(v_b + \beta^z \right) \frac{\partial \gamma_{xx}}{\partial \bar{z}} , \quad K_{yy} = \frac{\gamma_b}{2\alpha} \left(v_b + \beta^z \right) \frac{\partial \gamma_{yy}}{\partial \bar{z}} ,$$
$$K_{zz} = \frac{\gamma_b}{2\alpha} \left[2\gamma_{zz} \frac{\partial \beta^z}{\partial \bar{z}} + \left(v_b + \beta^z \right) \frac{\partial \gamma_{zz}}{\partial \bar{z}} \right] ,$$
$$K_{xz} = \frac{\gamma_{zz}}{2\alpha} \frac{\partial \beta^z}{\partial \bar{x}} , \quad K_{yz} = \frac{\gamma_{zz}}{2\alpha} \frac{\partial \beta^z}{\partial \bar{y}} ,$$

where "*bar*" quantities refer the metrics in the comoving frame, $\alpha = (-g^{00})^{-1/2}$ lapse function and $\beta_i = g_{0i}$ shift vector.

- Fluid velocity :

$$v^z = \frac{1}{\alpha} \left(v_b + \beta^z \right) .$$

Initial Configuration



L_2 norm of Hamiltonian constraint violation in the initial data with single boosted neutron star.

Our boosting method slightly increases the violation, but it is sufficiently small at least within our boosting and mass range.

Initial Configuration

- Standard superposition method :

$$\begin{aligned} g_{\mu\nu} &\approx g^{(A)}_{\mu\nu} + g^{(B)}_{\mu\nu} - \eta_{\mu\nu}, \\ K_{ij} &\approx K^{(A)}_{ij} + K^{(B)}_{ij} . \end{aligned}$$

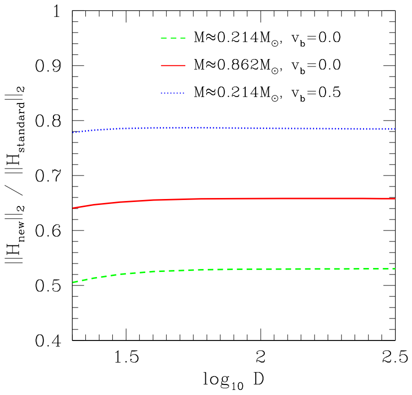
- New method based on a post-Newton expansion

$$g_{00} \approx \begin{cases} -g^{(A)}_{00} \times g^{(B)}_{00} & (\text{ for } g^{(A)}_{00} < 0 \text{ and } g^{(B)}_{00} < 0) \\ \max(g^{(A)}_{00} , g^{(B)}_{00}) & (\text{ otherwise }) \end{cases} ,$$

$$g_{ij} \approx \begin{cases} g^{(A)}_{ij} \times g^{(B)}_{ij} & (\text{ for } i = j) \\ g^{(A)}_{ij} + g^{(B)}_{ij} & (\text{ otherwise }) \end{cases} .$$

  - This approximation has high-order correction terms related with Newtonian gravitational potential ϕ unless $g^{(A,B)}_{00} < 0$.
  - It is equivalent to standard superposition method in the limit of weak gravitational field, $|\phi| \ll 1$.

Initial Configuration



Ratio of L_2 norm of Hamiltonian constraint violation in the initial data between "standard" superposition method and our "new" method.

- Violations are reduced about $20 \sim 50\%$.
- Improvement for large mass or boosting models is smaller than for another one, because of contribution of non-ϕ term.
- CTS approach provides similar improvements but much expensive (East et al. 2012).

How to measure the boost

- $\gamma_{\rm b}$ in the Lorentz transformation
  - meaningless for extended object in a general relativity.
- spatial average Lorentz factor :
  $$\langle\tilde{\gamma}\rangle \equiv \frac{\int \gamma\, dV}{\int dV}$$
- effective Lorentz factor :
  $$\langle\gamma\rangle \equiv \frac{\int T_{\mu\nu} n^{\mu} n^{\nu}\, dV}{(\int T_{\mu\nu} n^{\mu} n^{\nu}\, dV)_0}$$
  where “0” refers to quantities measured in the initial unboosted frame.
  - analogy of $E = \gamma m$ in a special relativity

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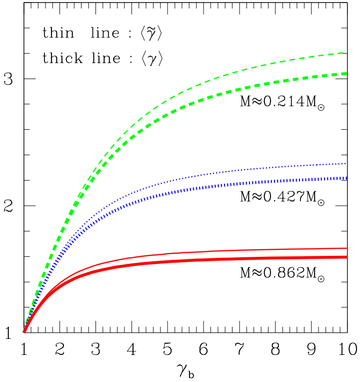
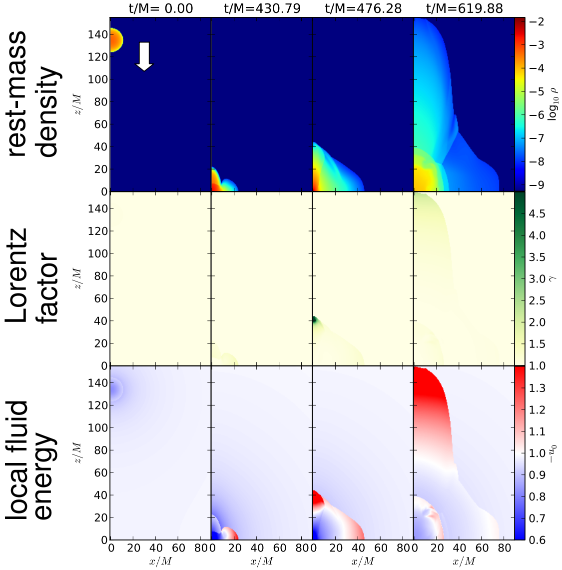


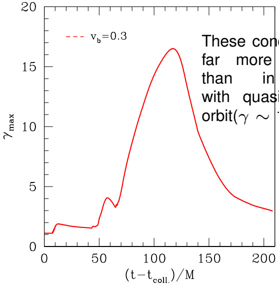
averages grow less rapidly for increasing mass: curvature is important

Results

Dynamics : mildly boosted case ($v_b = 0.3$)

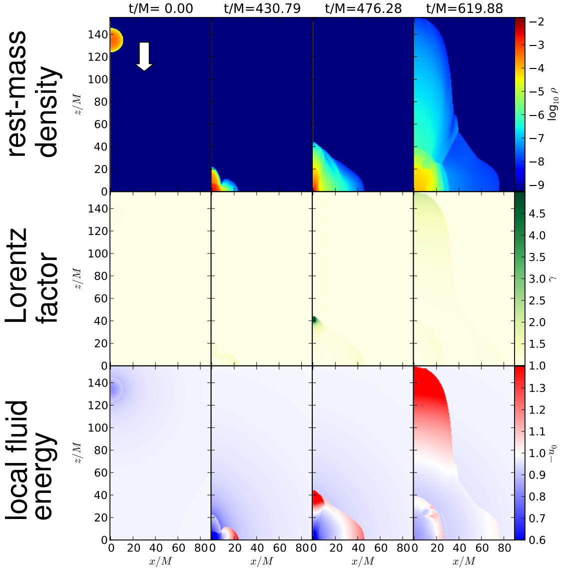


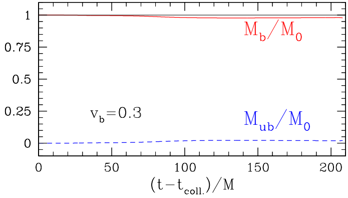
- jet-like expansion
- maximum density at the center.
- accelerating up to $\gamma \sim 16$, but settle down $\gamma \leq 3$.



These condition are far more extreme than in BNSs with quasi circular orbit($\gamma \sim 1.03$).

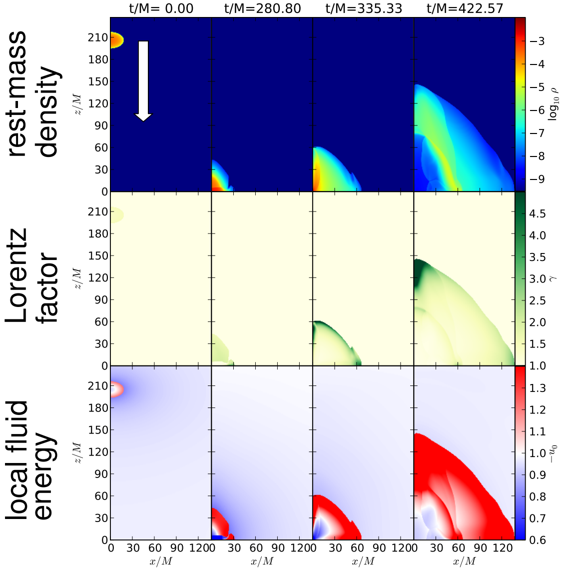
Dynamics : mildly boosted case ($v_b = 0.3$)



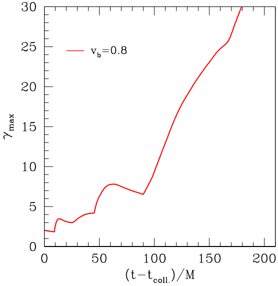


- matter with $-u_0 > 1$ is considered as unbound.
- unbound fraction is just a few percent of the total rest-mass.

Dynamics : highly boosted case ($v_b = 0.8$)



- spherical blast-wave expansion
- minimum density at the center.
- accelerating as large as $\gamma \sim 30$.



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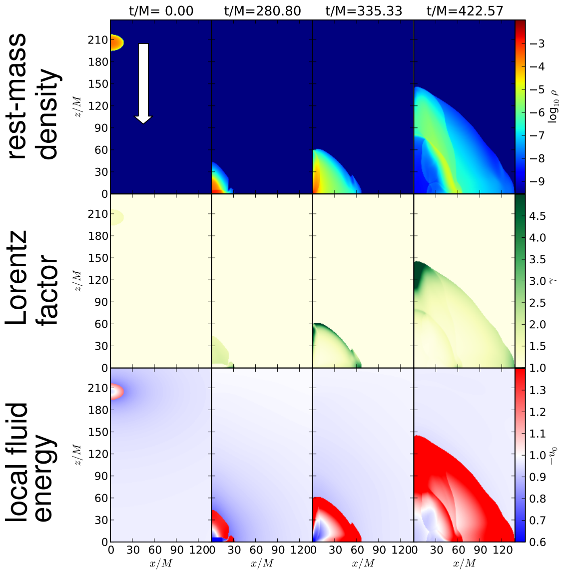
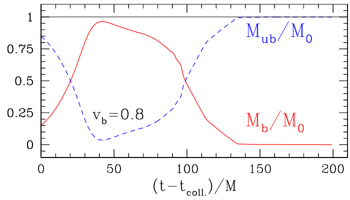




- matter with $-u_0 > 1$ is unbound.
- unbound fraction is $\sim$ 100% of the total rest-mass.
- the role played by gravitational forces is a minor one as the kinetic energy is increased.

Dynamics

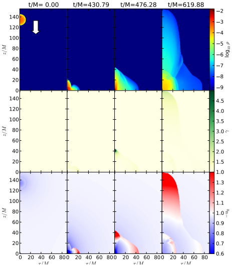
MOVIE

Dynamics

$v_b = 0.3$

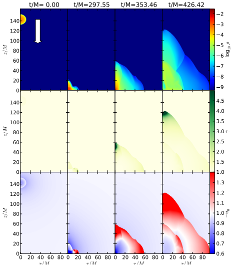
$(\gamma_b \approx 1.05)$

- Jet-like
- Maximum density is at the center.



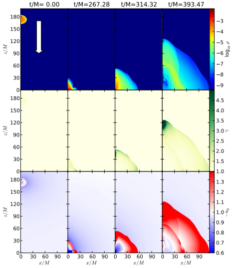
$v_b = 0.5$

$(\gamma_b \approx 1.15)$



$v_b = 0.7$

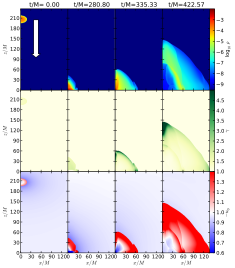
$(\gamma_b \approx 1.40)$



$v_b = 0.8$

$(\gamma_b \approx 1.67)$

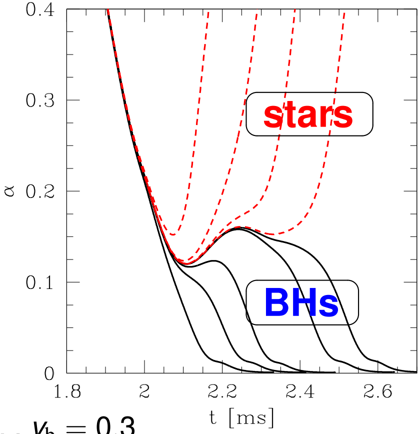
- Spherical
- The density is very low at the center.



continuous change of properties with increasing boost velocity.

Type-I Critical Behaviour

We found type-I critical behaviour also for nonzero boosts.



e.g. , $v_\mathrm{b} = 0.3$

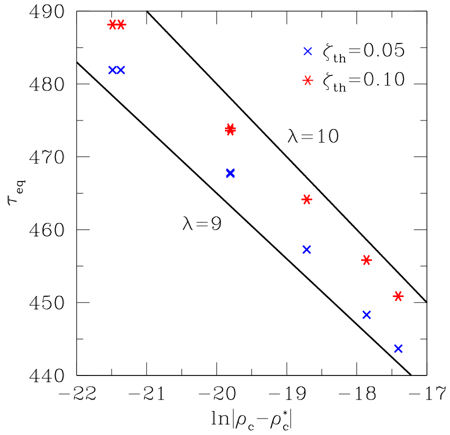
$\rho_c > \rho_c^\star \;\Rightarrow\;$ black hole,

$\rho_c < \rho_c^\star \;\Rightarrow\;$ stellar configuration,

where $\rho_\mathrm{c}^\star$ is critical value.

Survival time of metastable object :
$\tau_\mathrm{eq} \propto -\lambda \ln |\rho_c - \rho_c^\star|$

Type-I Critical Behaviour



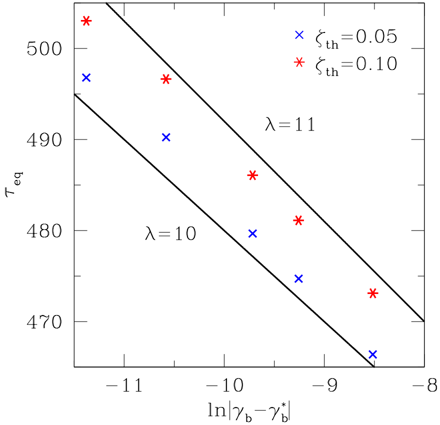
$\lambda \sim 10$
independent of the choice for the threshold value off the critical line.

$\Uparrow$

consistent with Jin et al.(2007) and Kellerman et al.(2010)

Type-I Critical Behaviour

Instead of ρ_c, we can also choose γ_b (or v_b) as one parameter of solutions for a fixed ρ_c.

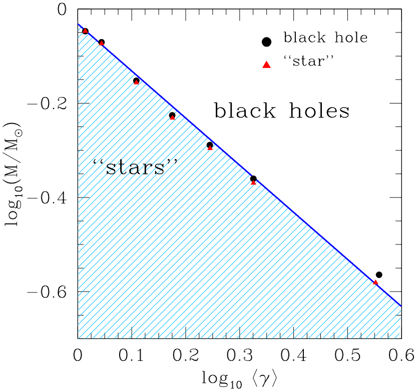


The critical exponent is still keeping $\lambda \sim 10$.

Jin et al.(2007) call this "universality".

Critical Line to Collapse to BH

For effective Lorentz factor $\langle\gamma\rangle$,



$$\frac{M_c}{M_\odot} = K\langle\gamma\rangle^{-n} \approx 0.93\langle\gamma\rangle^{-1.0}$$

Critical Line to Collapse to BH

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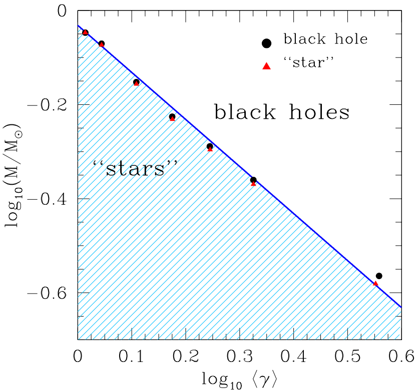

$$\frac{M_c}{M_\odot} = K\langle\gamma\rangle^{-n} \approx 0.93\langle\gamma\rangle^{-1.0}$$

⇒

$$\langle\gamma\rangle M_c \approx \text{const.}$$

Critical Line to Collapse to BH

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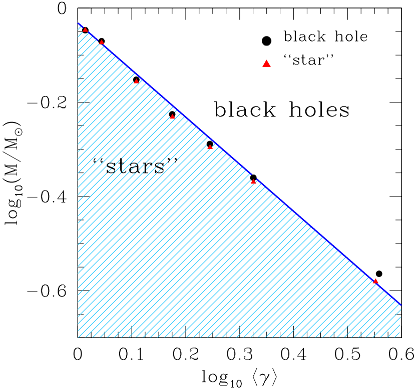

$$\frac{M_c}{M_\odot} = K\langle\gamma\rangle^{-n} \approx 0.93\langle\gamma\rangle^{-1.0}$$

Total energy to produce BH is conserved in ultrarelativistic collision.

$$\langle\gamma\rangle M_c \approx \text{const.}$$

Critical Line to Collapse to BH

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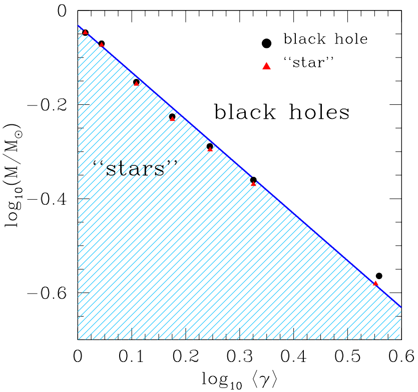


$$\frac{M_c}{M_\odot} = K\langle\gamma\rangle^{-n} \approx 0.93\langle\gamma\rangle^{-1.0}$$

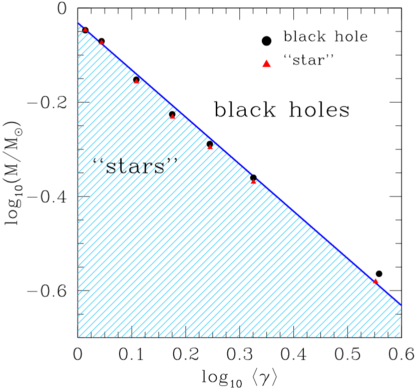
In qualitative agreement with East and Pretorius(2012), who considered fixed rest-mass models.

Total energy to produce BH is conserved in ultrarelativistic collision.

$$\langle\gamma\rangle M_c \approx \text{const.}$$

Critical Line to Collapse to BH

For effective Lorentz factor $\langle\gamma\rangle$,

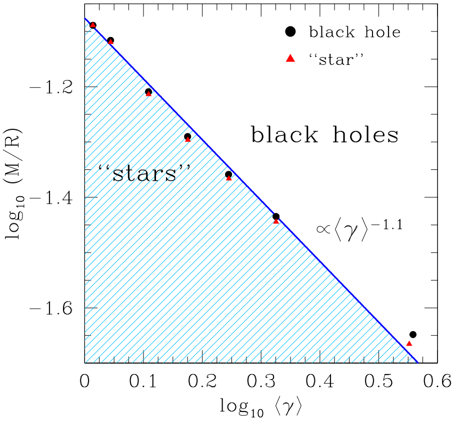


$$\frac{M_c}{M_\odot} = K\langle\gamma\rangle^{-n} \approx 0.93\langle\gamma\rangle^{-1.0}$$

- $\langle\gamma\rangle \to 1$:
  - $M_c \to 0.93\ M_\odot$.
  - $2M_c$ is only $\sim$ 12% larger than $M_{max} = 1.64 M_\odot$ in TOV.
- $\langle\gamma\rangle \to \infty$:
  - $M_c \to 0$.
  - producing zero-mass critical BH.
  - predicting the existence of type-II critical behavior ($M_{BH} = c|P - P^\star|^\lambda$).

Critical Line to Collapse to BH

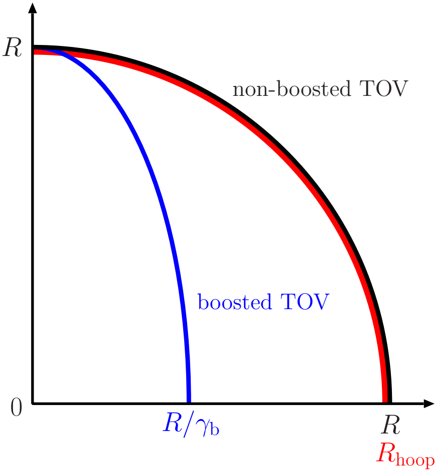
Using compactness M/R, instead of M:



power-law

$$\frac{M_c}{R_c} = K\langle\gamma\rangle^{-m} \approx 0.084\langle\gamma\rangle^{-1.1}$$

Critical Line to Collapse to BH

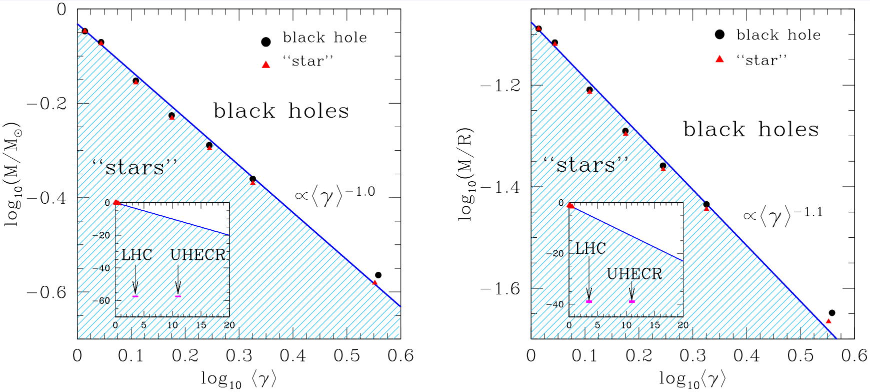


power-law

$$\frac{M_c}{R_c} = K\langle\gamma\rangle^{-m} \approx 0.084\langle\gamma\rangle^{-1.1}$$

- $R_{\rm hoop} = \dfrac{M_c\langle\gamma\rangle^m}{K}$
- If the initial object has amount of energy $M_c\langle\gamma\rangle^m/K$, a BH will be produced.

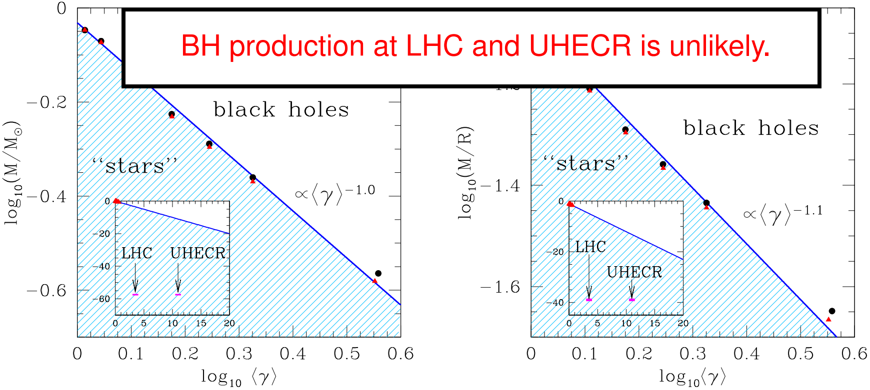
Proton-Proton Collision Case



- Our simulations do not want to represent particle collisions, but we can check where LHC regimes lay in this diagram.
- We extrapolate our critical line in very wide range.
- We neglect quantum effects and extra-dimension effects that might be important at Planck-energy scales.

Proton-Proton Collision Case

BH production at LHC and UHECR is unlikely.



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Summary

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- We studied the collision of two selfgravitating fluid objects with ultrarelativistic speeds.
- We found the dynamics strongly depends on the collisional velocities:
  - Expansion types continuously change from jet-like to spherical blast-wave with γ_{b}.
  - Fraction of unbound matter becomes $\sim$ 100% in large γ_{b}.
- We found type-I critical behavior for all γ_{b}:
  - $\tau_{\mathrm{eq}} \propto -\lambda \ln |\rho_c - \rho_c^\star|$ with $\lambda \sim 10$.
  - "universality" : $\tau_{\mathrm{eq}} \propto -\lambda \ln |\gamma_{\mathrm{b}} - \gamma_{\mathrm{b}}^\star|$ with $\lambda \sim 10$
- We found the critical line with a power law:
  - $\frac{M_{\mathrm{c}}}{M_\odot} = K \langle \gamma \rangle^{-n}$ with $K \approx 0.93$ and $n \approx 1.0$.
- Our results show proton-proton collisions in LHC and UHECR are unlikely to produce BHs within classical general relativity.