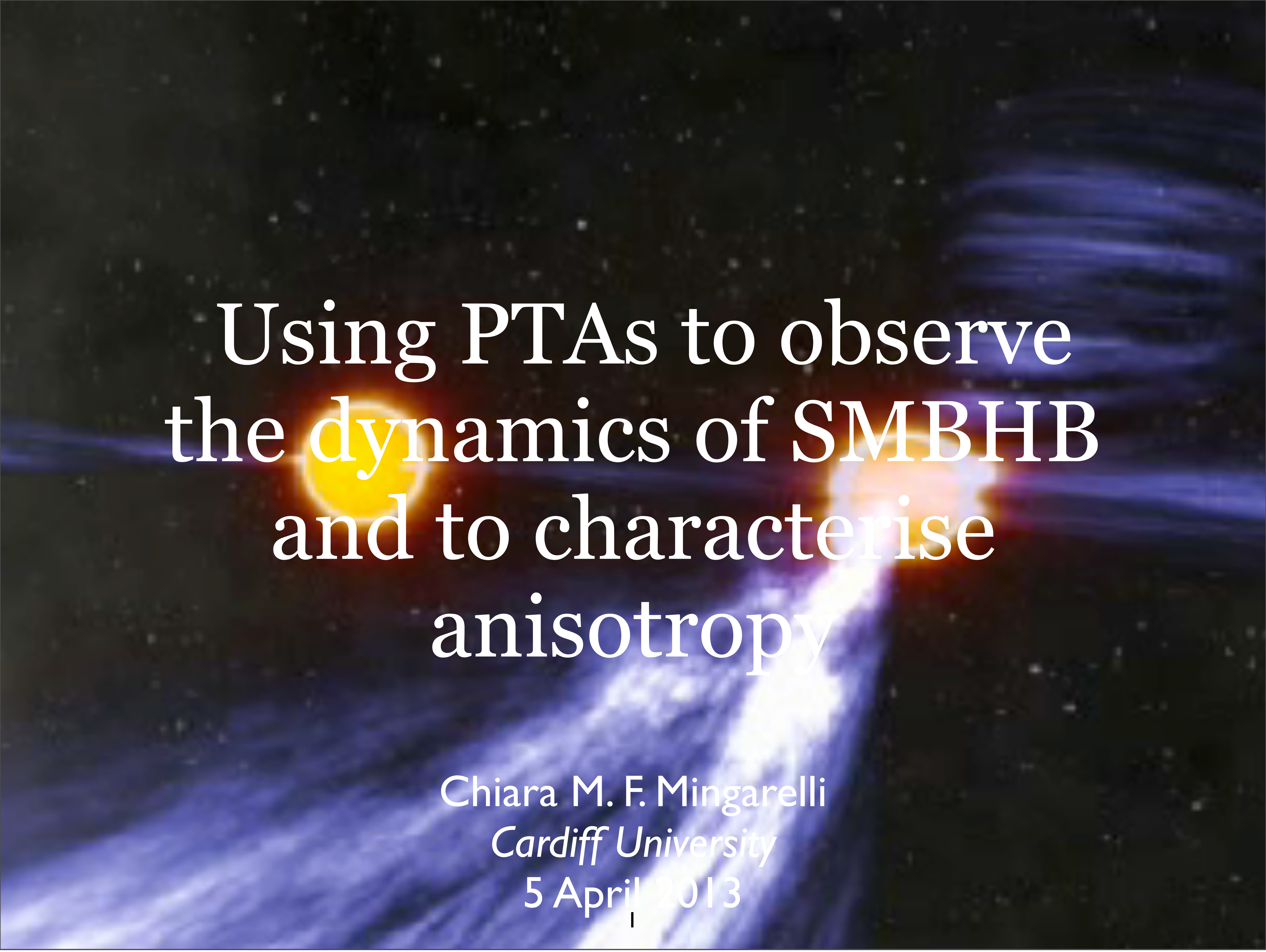

Using PTAs to observe the dynamics of SMBHB and to characterise anisotropy

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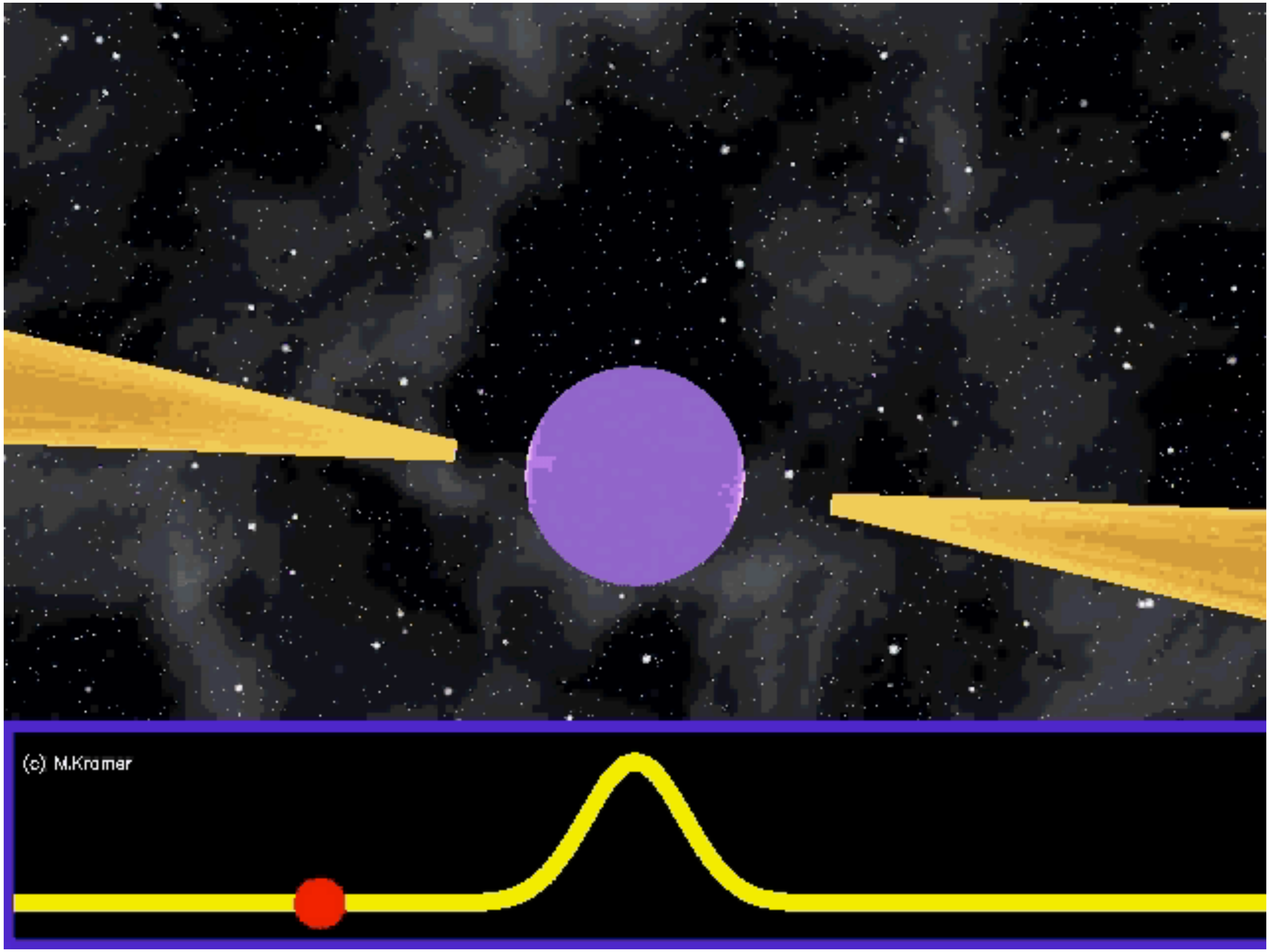
5 April 2013

Outline

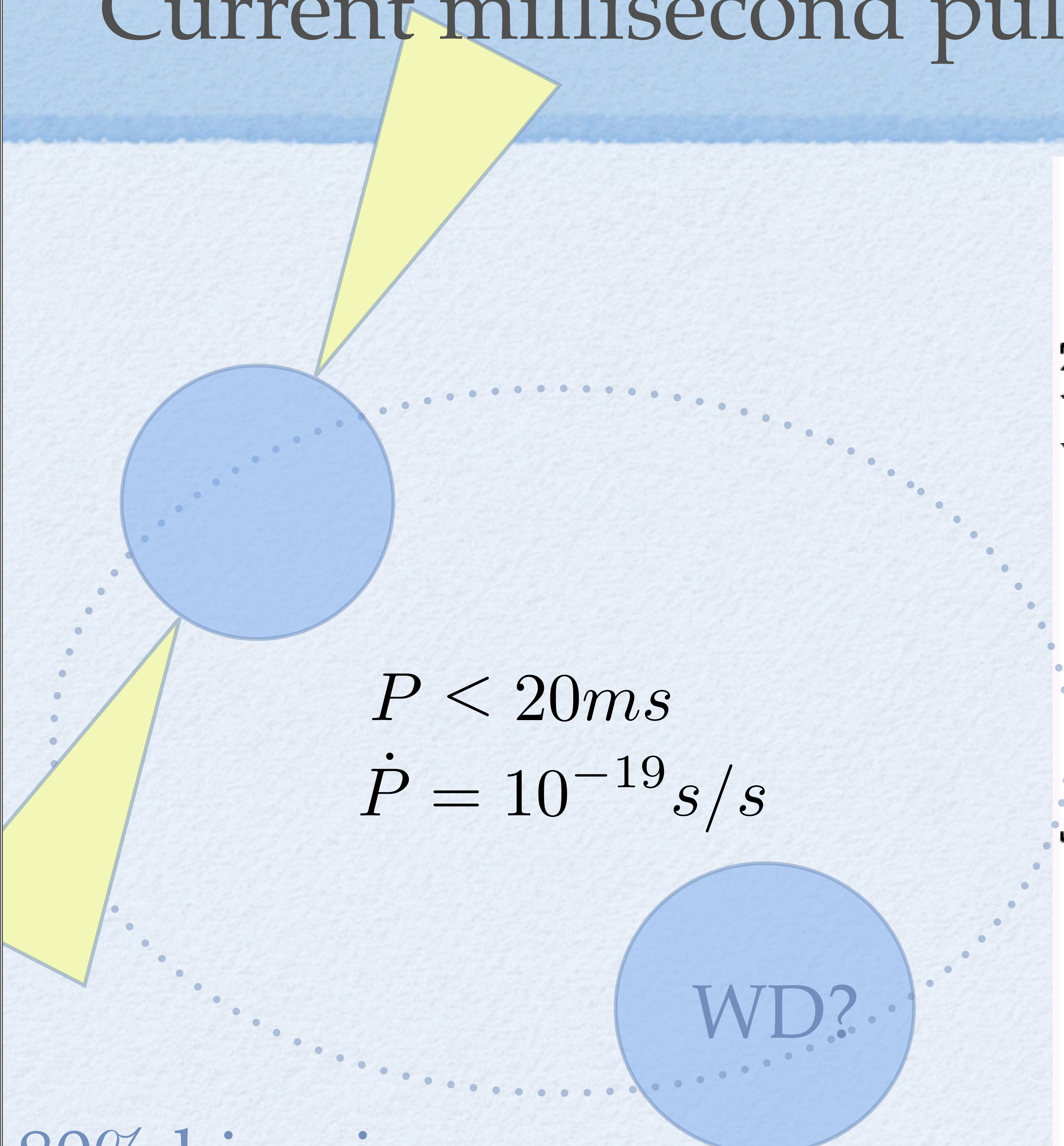
- What is a Pulsar Timing Array?
- PTA Geometry and Signals
- The PTA overlap reduction function
- Using PTAs to characterise anisotropy in the GW background (generalised ORF)
- How we can use PTAs as time machines
- Testing the pN expansion with PTAs

Pulsars

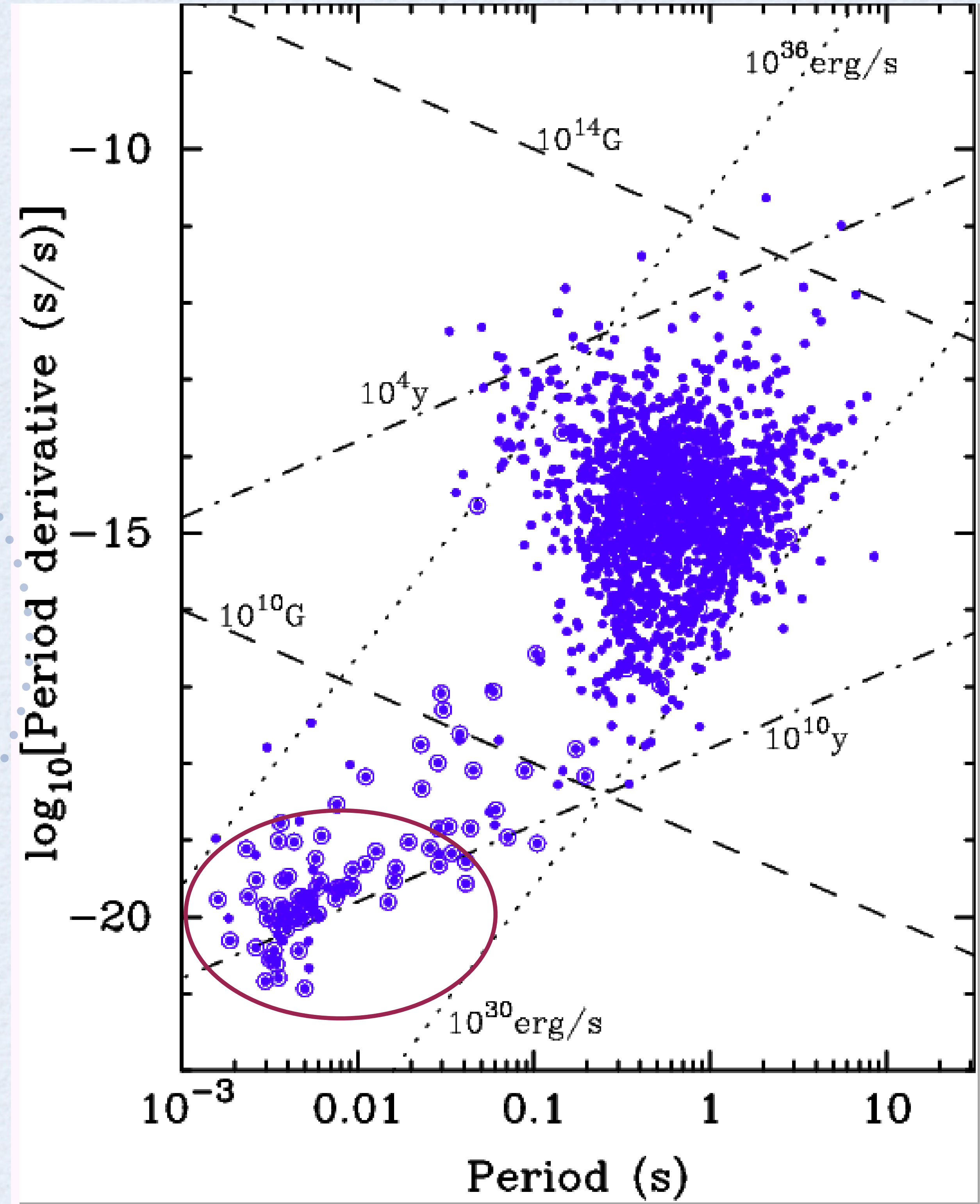


credit: Michael Kramer, MPI Bonn

Current millisecond pulsar population



$$P < 20ms$$
$$\dot{P} = 10^{-19} s/s$$



80% binaries
about 1 kpc away

C.M.F Mingarelli

GW detection

Animation from John Rowe Animation/Australia Telescope National Facility, CSIRO

Pulsar Timing Array

- Sources include supermassive black hole binaries, cosmic strings and relic GWs from inflation.
- Frequency band defined by total observation time and cadence of observation.
- GW signal correlated between pulsars, pulsar noise is not.

The Signal

- Consider a GW metric perturbation in the transverse and traceless gauge:

$$h_{ab}(t,\hat{\Omega}) = e^{+}_{ab}(\hat{\Omega})h_{+}(t,\hat{\Omega}) + e^{\times}_{ab}(\hat{\Omega})\,h_{\times}(t,\hat{\Omega}),$$

- the $h_A(t,\hat{\Omega})$ are the polarisation amplitudes, and the $e^A_{ab}(t,\hat{\Omega})$ are the polarisation tensors

$$e^{+}_{ab}(\hat{\Omega}) = \hat{m}_a\hat{m}_b - \hat{n}_a\hat{n}_b\,,$$
$$e^{\times}_{ab}(\hat{\Omega}) = \hat{m}_a\hat{n}_b + \hat{n}_a\hat{m}_b\,.$$

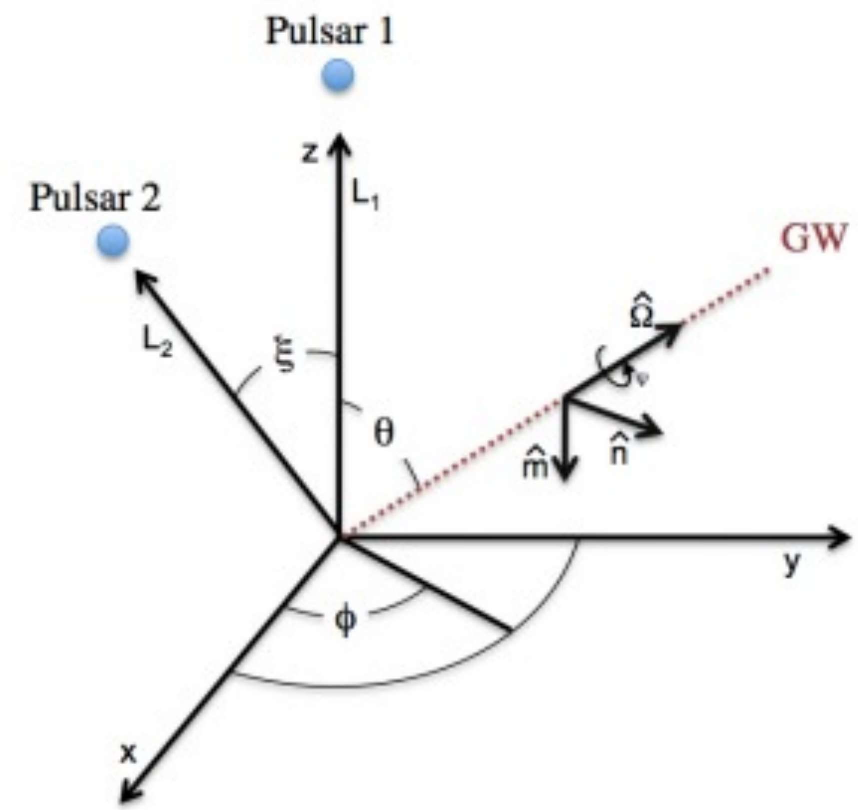


image: S. Chamberlin, X. Siemens (2011)

Plane Wave

- Can now write the metric perturbation at coordinates t and **x** in terms of a plane wave expansion

$$h_{ab}(t,\vec{x}) = \sum_A \int_{-\infty}^{\infty} df \int_{S^2} d\hat{\Omega}\, h_A(f,\hat{\Omega})\, e^{i2\pi f(t-\hat{\Omega}\cdot\vec{x})}\, e^A_{ab}(\hat{\Omega})\,,$$

- For a stationary, Gaussian and unpolarised background, the polarisation amplitude satisfy

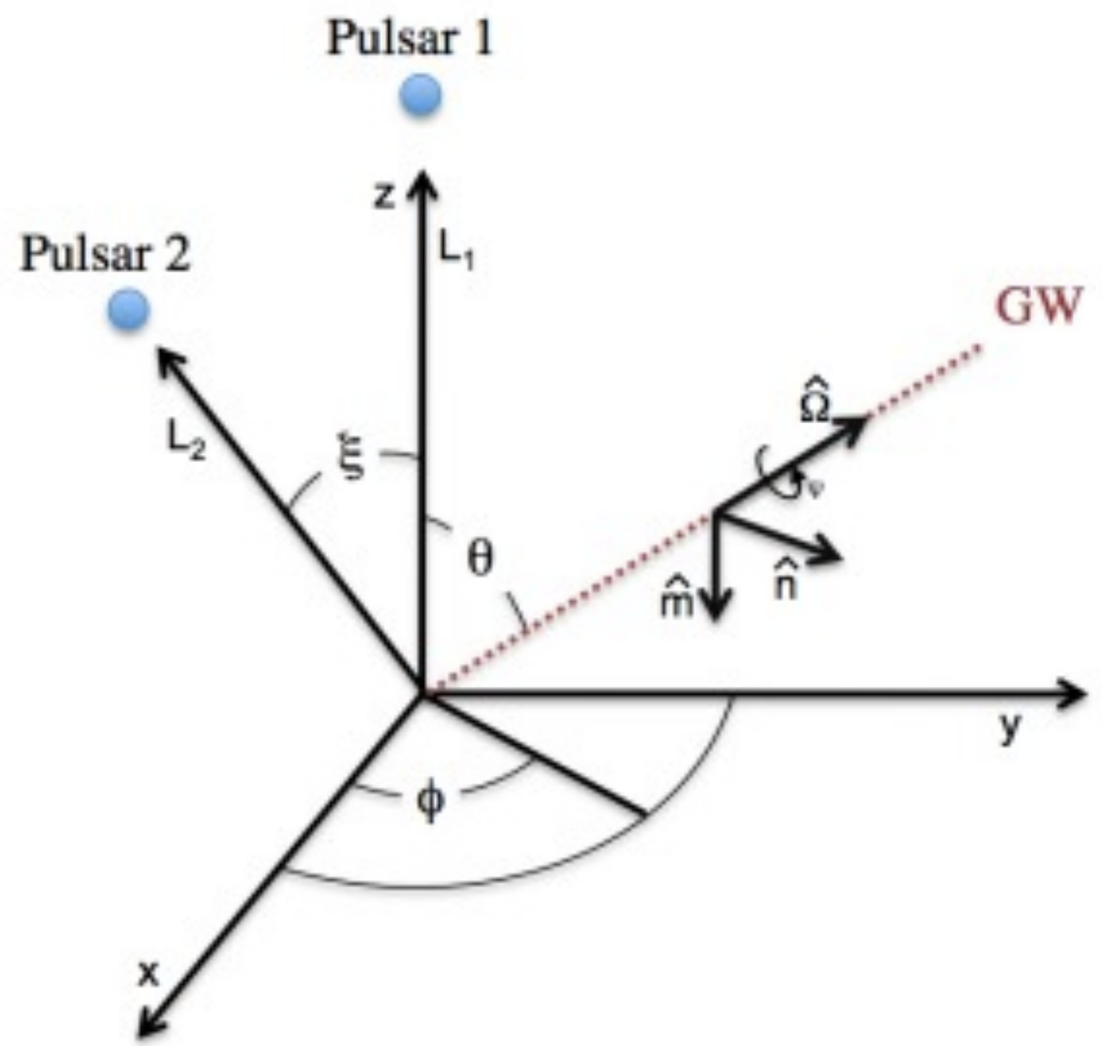
$$\langle h^*_A(f,\hat{\Omega}) h_{A'}(f',\hat{\Omega}')\rangle = \delta^2(\hat{\Omega},\hat{\Omega}')\delta_{AA'}\delta(f-f')H(f)P(\hat{\mathbf{\Omega}})\,,$$

H(f) is the spectral content and the power distribution is $P(\hat{\Omega})$ and

$$\Omega_{\rm gw}(f) = \frac{32\pi^3}{3H_0^2} f^3 H(f)$$

is the density parameter.

Timing Residuals r(t)



$$r(t) = \int_0^t dt' z(t', \hat{\Omega})$$

$$z(t, \hat{\Omega}) \equiv \frac{\nu(t) - \nu_0}{\nu_0}$$
$$= \frac{1}{2} \frac{\hat{p}^a \hat{p}^b}{1 + \hat{p}^a \hat{\Omega}_a} \Delta h_{ab}(t; \hat{\Omega}) \, .$$

$$\Delta h_{ab}(t) \equiv h_{ab}(t_{\rm p}, \hat{\Omega}) - h_{ab}(t, \hat{\Omega})$$

S. Detweiler, ApJ. 234, 1100 (1979).

Overlap Reduction Function

- The metric perturbation can now be written as

$$\Delta h_{ab}(t,\hat{\Omega}) = \sum_A \int_{-\infty}^{\infty} df \int_{S^2} d\hat{\Omega}\, e_{ab}^A(\hat{\Omega})\; h_A(f,\hat{\Omega})\, e^{i2\pi f t} \left[e^{i2\pi f L(1+\hat{\Omega}\cdot\hat{p})} - 1\right]$$

- and the redshift z(t) can now be defined as

$$z(t) = \int_{-\infty}^{\infty} df \int_{S^2} d\hat{\Omega}\; F^A(\hat{\Omega}) h_A(f,\hat{\Omega}) e^{i2\pi f t} \left[e^{i2\pi f L(1+\hat{\Omega}\cdot\hat{p})} - 1\right]$$

where $F^A(\hat{\Omega})$ is the Antenna Beam Pattern,

$$F^A(\hat{\Omega}) = \left[\frac{1}{2}\frac{\hat{p}^a\hat{p}^b}{1+\hat{p}^a\hat{\Omega}_a}\; e_{ab}^A(\hat{\Omega})\right].$$

Overlap Reduction Function

- The search for a stochastic background contribution in pulsar timing data relies on correlations in the residual from different pulsars induced by GWs.

- These correlations in turn depend on the on the spectral properties of the radiation and the antenna beam pattern convolved with the angular distribution of the signal in the sky.

Overlap Reduction Function

$$\langle r_a^*(t_j) r_b(t_k)\rangle = \left\langle \int^{t_j} dt' \int^{t_k} dt'' z_a^*(t', \hat{\Omega}) z_b(t'', \hat{\Omega}) \right\rangle$$

$$= \left\langle \int^{t_j} dt' \int^{t_k} dt'' \int_{-\infty}^{+\infty} df' \int_{-\infty}^{+\infty} df'' \tilde{z}_a^*(f') \tilde{z}_b(f'')\, e^{-i2\pi(f't' - f''t'')} \right\rangle$$

$$= \int^{t_j} dt' \int^{t_k} dt'' \int_{-\infty}^{+\infty} df e^{-i2\pi f(t'-t'')} H(f)\, {}^{(ab)}\Gamma(f)$$

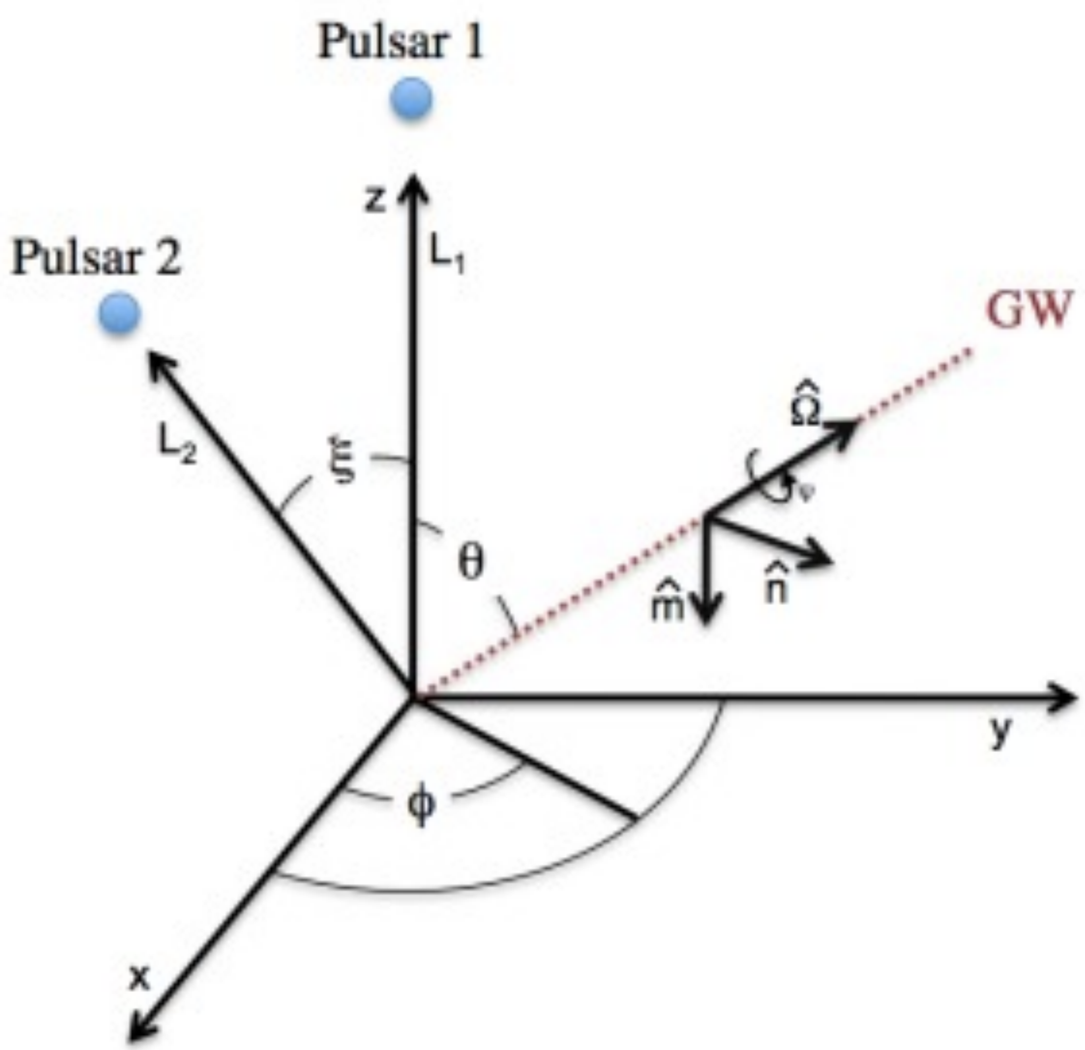
where

$${}^{(ab)}\Gamma(f) \equiv \int d\hat{\Omega} P(\Omega) \kappa_{ab}(f, \hat{\Omega}) \left[\sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) \right],$$

and

$$\kappa_{ab}(f, \hat{\Omega}) \equiv \left[1 - e^{i2\pi f L_a(1+\hat{\Omega}\cdot\hat{p}_a)}\right] \left[1 - e^{-i2\pi f L_b(1+\hat{\Omega}\cdot\hat{p}_b)}\right].$$

PTA Geometry



$$\hat{\Omega} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$
$$\hat{m} = (\sin\phi, -\cos\phi, 0)$$
$$\hat{n} = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$$

image: S. Chamberlin, X. Siemens (2011)

This choice of coordinates zeroes one of the antenna beam patterns $F_1^{\times} = 0$

The GW background

Defined in terms of characteristic strain:

$$h_c(f) = A \left(\frac{f}{yr^{-1}} \right)^{\alpha}$$

where

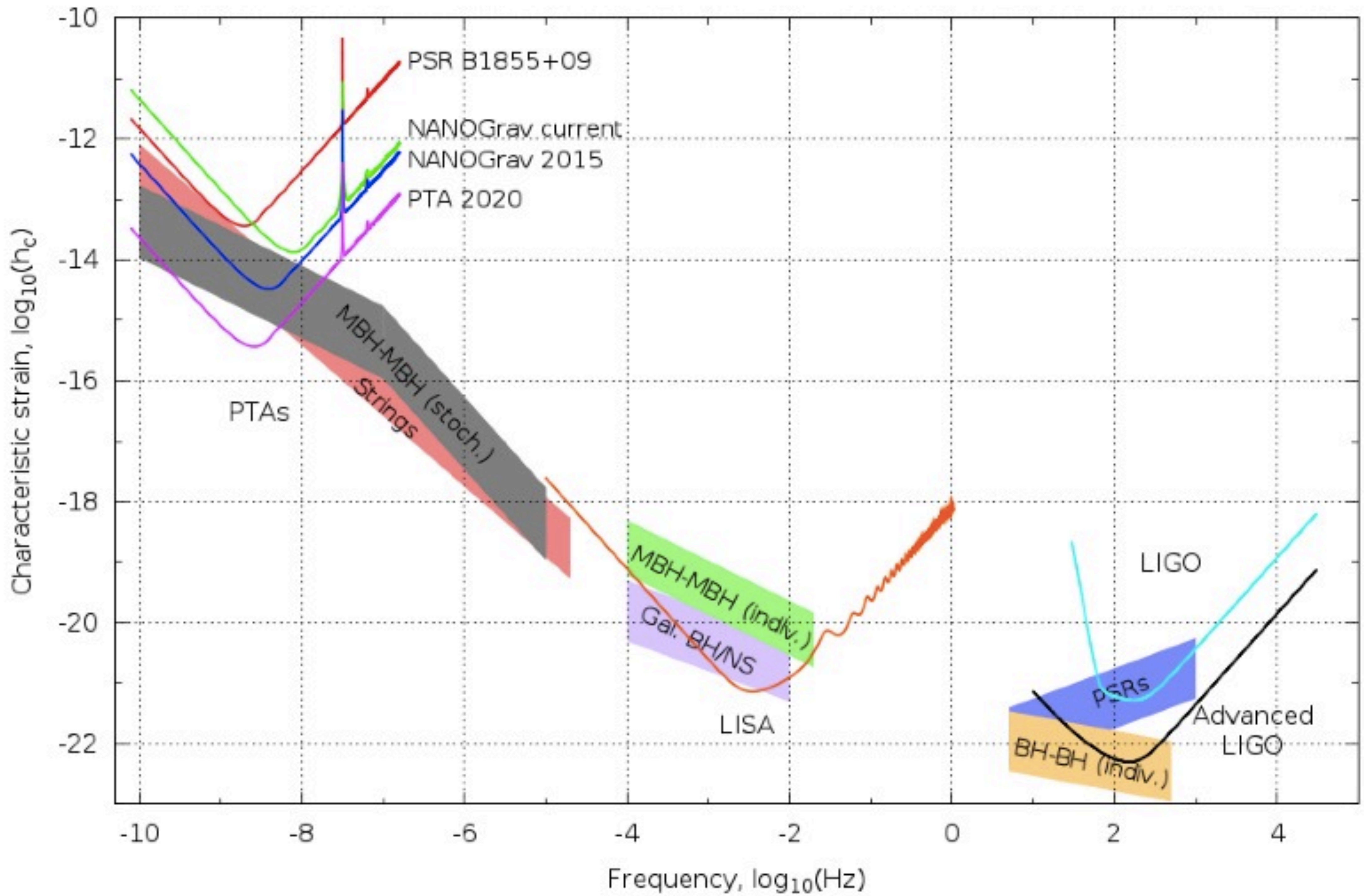
$$P(f) = \frac{1}{12\pi^2} \frac{1}{f^3} h_c(f)^2 \quad ; \quad \int_0^{\infty} P(f) df = R(t)^2$$

$P(f)$ is the one-sided power spectrum of the induced residuals, R(t), and the energy density of the background is

$$\Omega_{gw}(f) = \frac{2}{3} \frac{\pi^2}{H_0^2} f^2 h_c(f)^2$$

Jenet *et al*, ApJ 653:1571-1576 (2006)

GW Spectrum



P. Demorest *et al*, 2009

Isotropic Background (?)

- Background always considered to be isotropic.
- If background is detected, need a way to analyse it, hence, ensuring it is origin is cosmological as predicted.
- Can use spherical harmonics to look for an angular power distribution (i.e. anisotropy)

Anisotropy

- We decompose the angular distribution function on the basis of the spherical harmonic functions

$$P(\hat{\Omega}) \equiv \sum_{lm} c_l^m Y_l^m(\hat{\Omega})$$

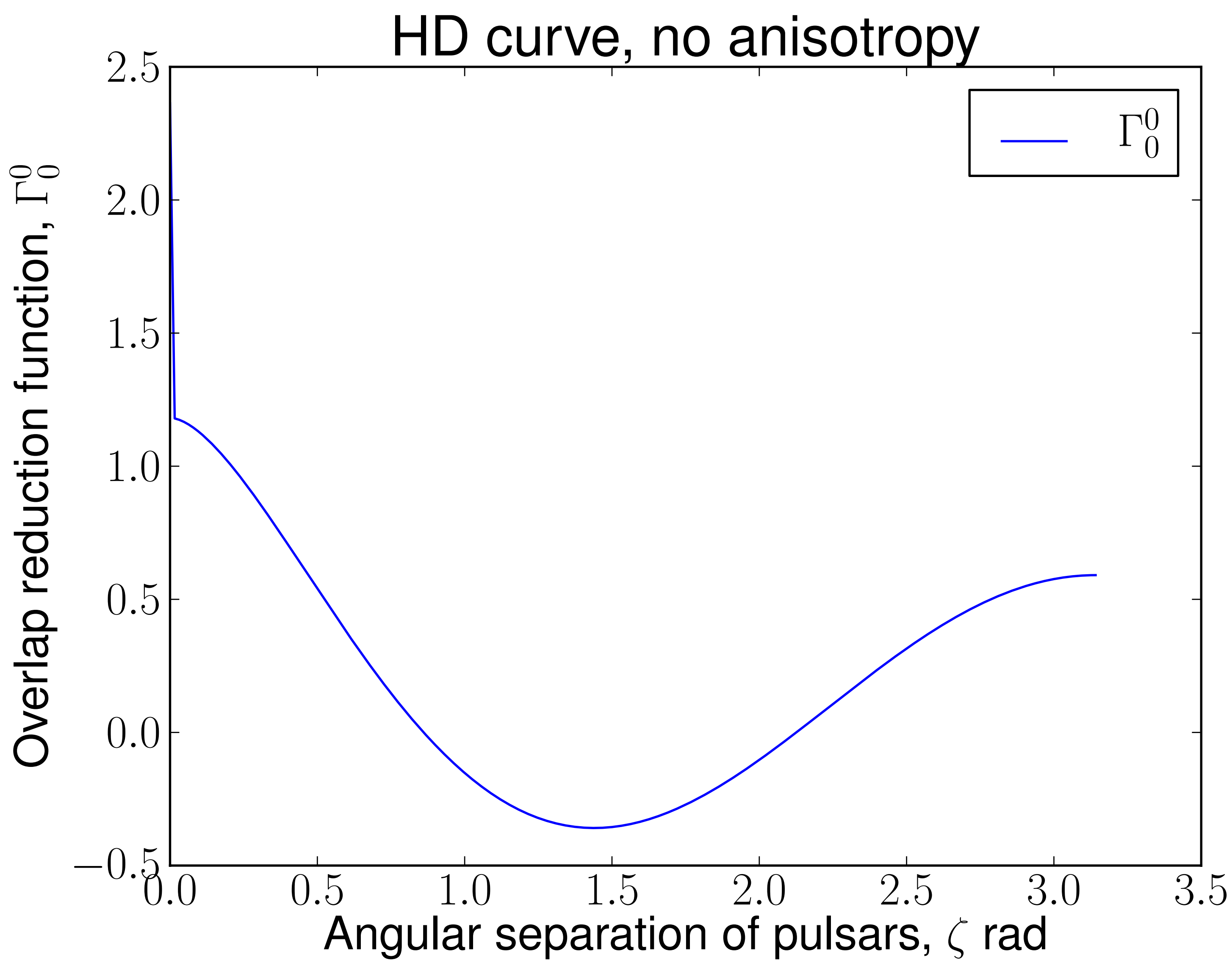
- Now write generalised correlation fns as:

$$\Gamma_l^m(f, \hat{\Omega}) \equiv \int d\hat{\Omega} P(\Omega) \kappa_{ab}(f, \hat{\Omega}) \left[\sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) \right]$$

New Overlap Reduction Functions

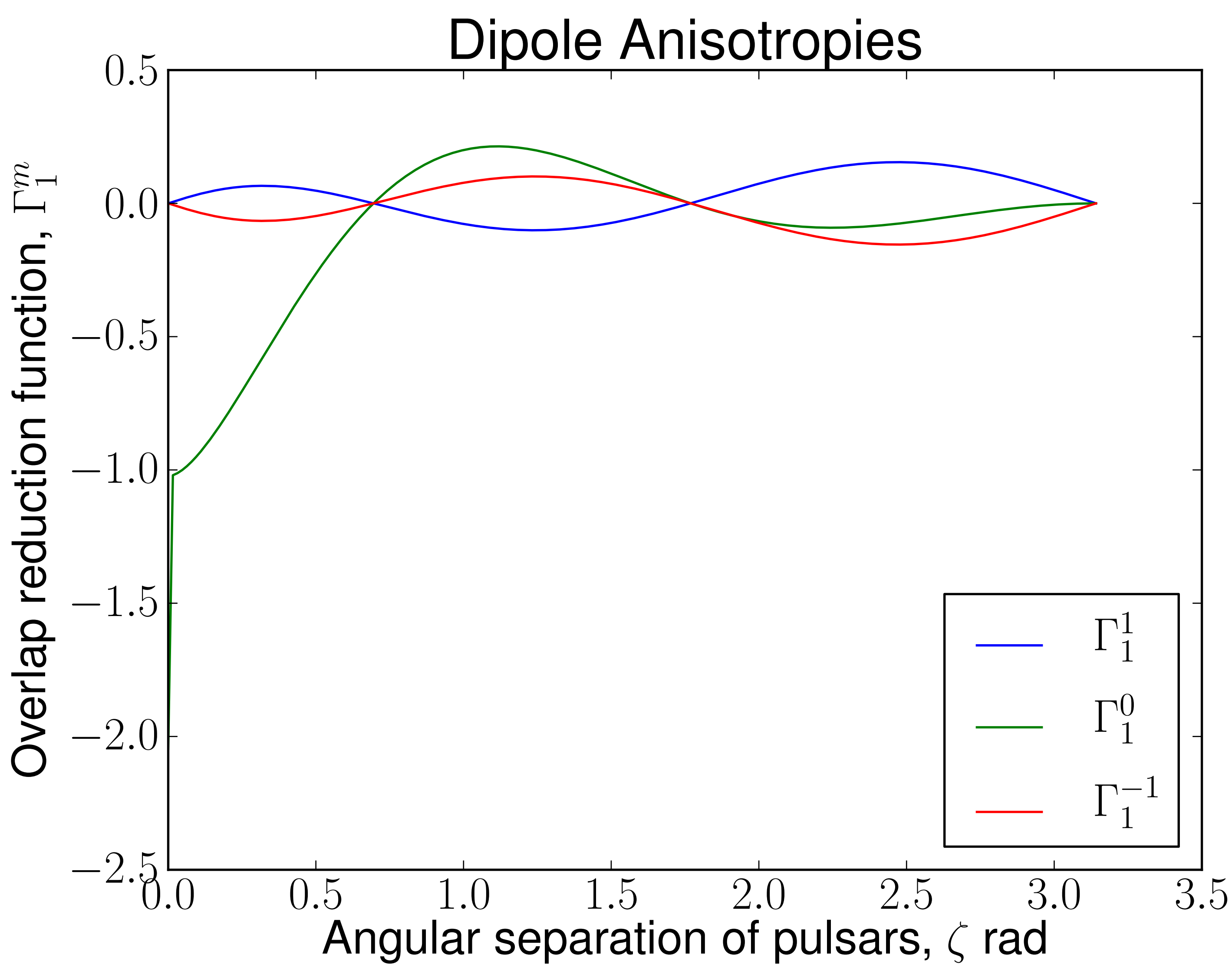
- We then analytically compute these $\Gamma_l^m(\xi)$ in a "computational frame" and then rotate them back into the cosmic "rest frame" (see Allen and Ottewill 1997)
- Compute using "Earth term" only (large fL)
- New functions computed up to quadrupole
- New search parameters are c_{lm}
- Functions not pre-normalised

Isotropy or HD Curve



$$\Gamma_0^0 = \frac{\sqrt{\pi}}{2}\left[1 + \frac{\cos\zeta}{3} + 4(1-\cos\zeta)\ln\left(\sin\frac{\zeta}{2}\right)\right]$$

Dipole



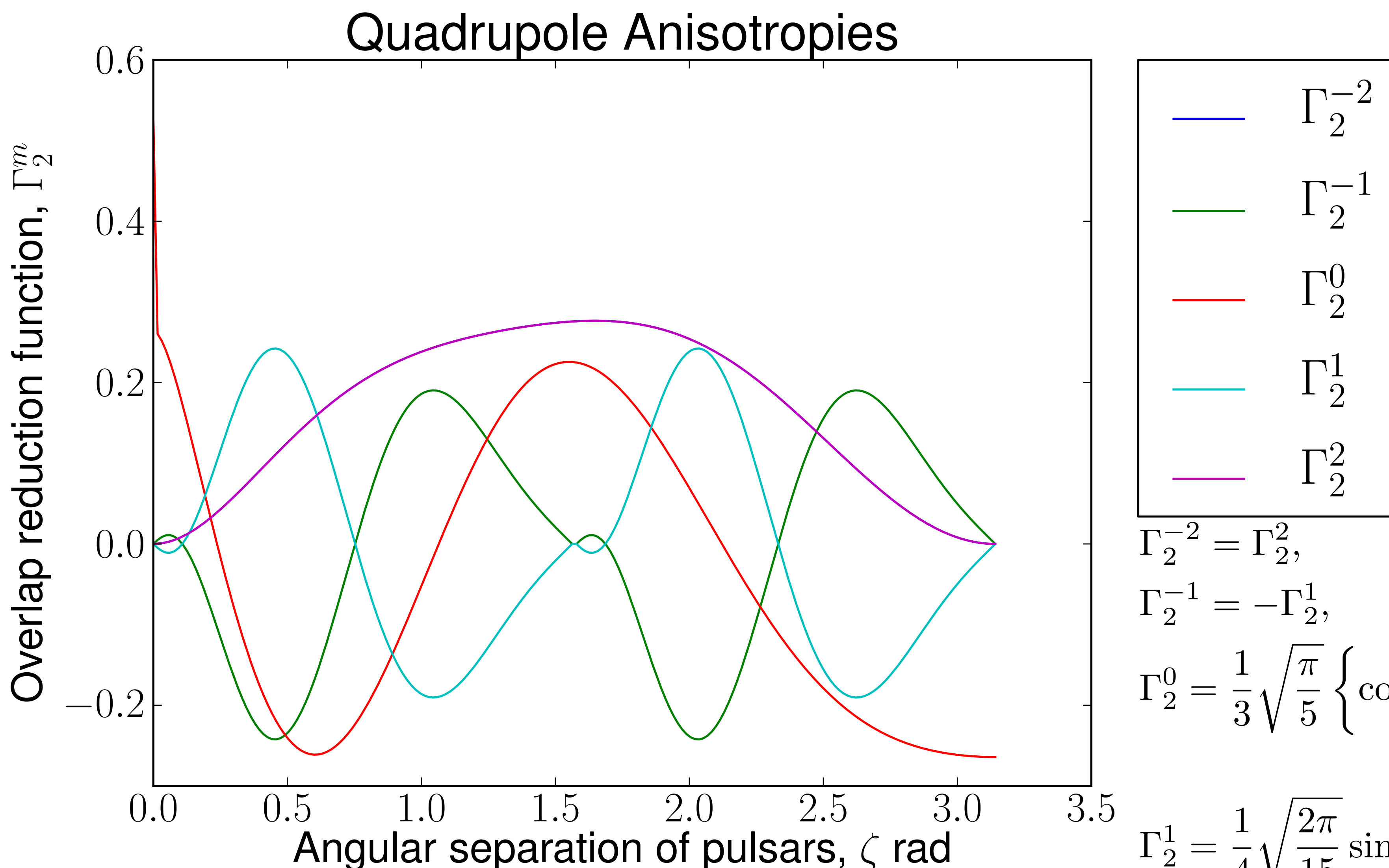
$$\Gamma_1^{-1} = -\frac{1}{2}\sqrt{\frac{\pi}{6}}\sin\zeta\left\{1+3b\left[1+\frac{4}{a}\ln\left(\sin\frac{\zeta}{2}\right)\right]\right\},$$

$$\Gamma_1^{0} = -\frac{1}{2}\sqrt{\frac{\pi}{3}}\left\{a+3b\left[a+4\ln\left(\sin\frac{\zeta}{2}\right)\right]\right\},$$

$$\Gamma_1^{1} = \frac{1}{2}\sqrt{\frac{\pi}{6}}\sin\zeta\left\{1+3b\left[1+\frac{4}{a}\ln\left(\sin\frac{\zeta}{2}\right)\right]\right\}.$$

Mingarelli *et al*, in prep (2013)

Quadrupole



$$\Gamma_2^{-2} = \Gamma_2^2,$$

$$\Gamma_2^{-1} = -\Gamma_2^1,$$

$$\Gamma_2^0 = \frac{1}{3}\sqrt{\frac{\pi}{5}}\left\{\cos\zeta + \frac{15b}{4}\left[a(\cos\zeta+3) + 8\ln\left(\sin\frac{\zeta}{2}\right)\right]\right\}, \tag{89a}$$

$$\Gamma_2^1 = \frac{1}{4}\sqrt{\frac{2\pi}{15}}\sin\zeta\left\{5\cos^2\zeta + 15\cos\zeta - 21 - 60\frac{b}{a}\ln\left(\sin\frac{\zeta}{2}\right)\right\}, \tag{89b}$$

$$\Gamma_2^2 = -\frac{1}{4}\sqrt{\frac{5\pi}{6}}\frac{b}{a}\left[a(\cos^2\zeta + 4\cos\zeta - 9) - 24b\ln\left(\sin\frac{\zeta}{2}\right)\right]. \tag{89c}$$

Mingarelli *et al*, in prep (2013)

State of Anisotropy work for EPTA

- only PTA to embed anisotropy in pipeline
- can currently inject and recover anisotropy up to quadrupole (Taylor, Mingarelli, Vecchio and Gair, in prep 2013)
- cross checking results across pipelines within EPTA before releasing anisotropic limits

From Anisotropy to Single Sources

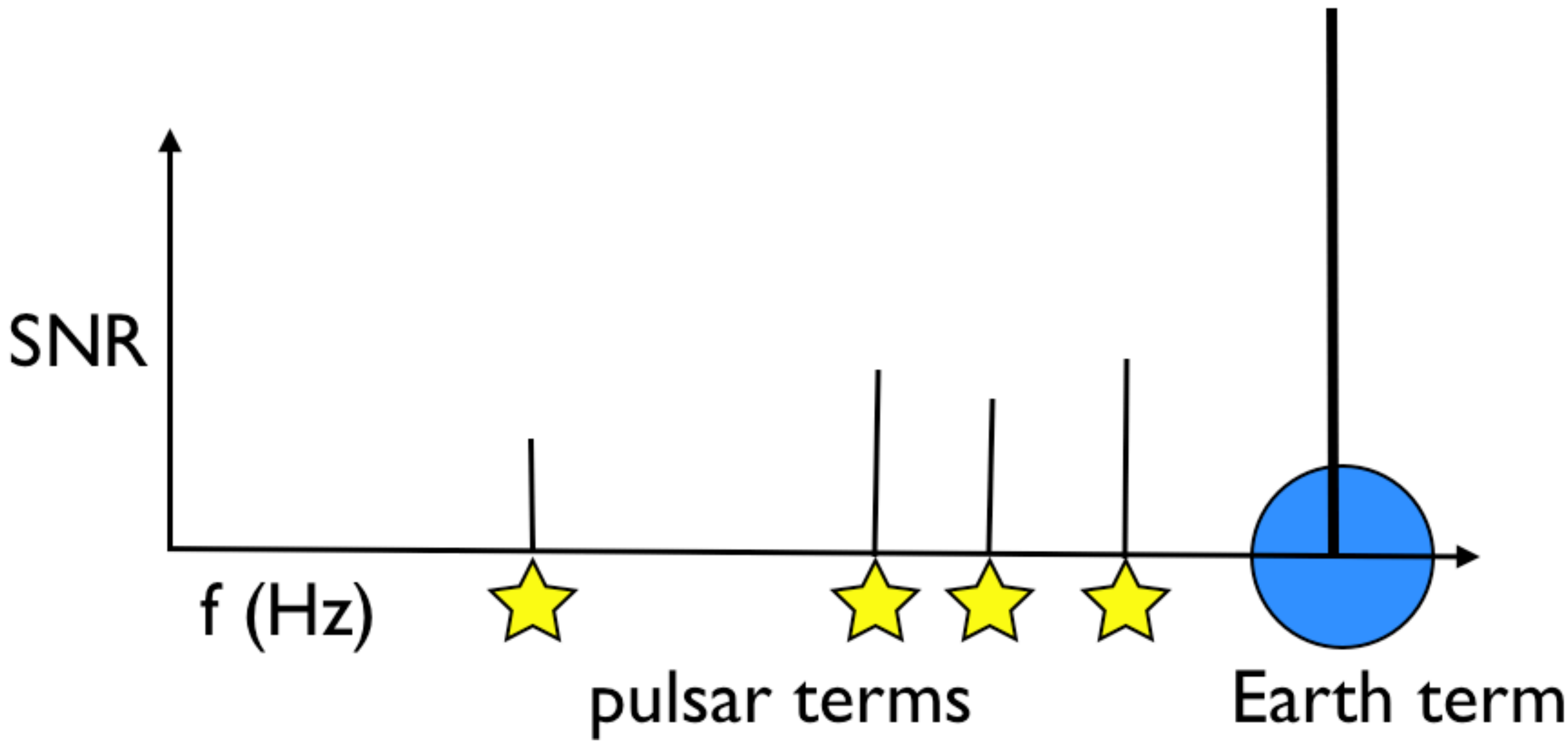
- Some sources may be sufficiently **close, high mass and high frequency** to rise above the background, and even the anisotropic background limit to become individually resolvable
- Becomes much more plausible in SKA era

PTAs as Time Machines

or Mingarelli *et al*, PRL **109**, 081104 (2012)

- IF the pulsar term, however, is in a different frequency bin AND is resolvable, PTAs can be used as time machines.
- In a PTA all the perturbations at the Earth add coherently (boosting SNR). Pulsar terms all at different frequencies.
- If pulsar term detected at SNR of 8, then the Earth term is at ~ $36\sqrt{N/20}$

Pulsar Terms



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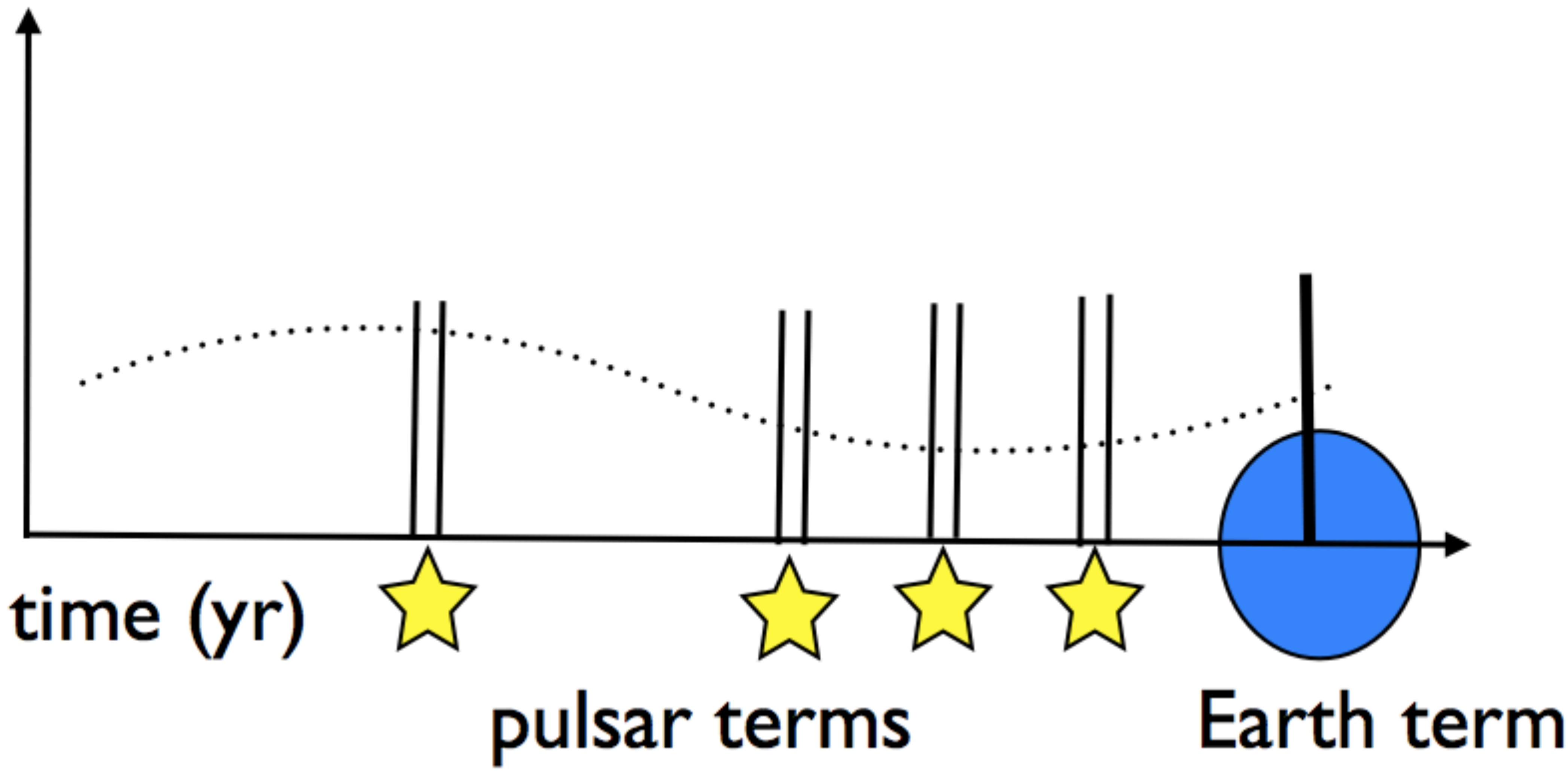
Connect the dots...



Typical parameters

- Expect to detect SMBHBs that are still in the weak field adiabatic inspiral regime, with an orbital velocity

$$v = 1.7 \times 10^{-2}\,(M/10^9\,M_\odot)^{2/3}\,(f/50\,\mathrm{nHz})^{2/3}$$

- Orbital timescale of SMBHB (in yrs):

$$f/\dot{f} = 1.6 \times 10^3(\mathcal{M}/10^9\,M_\odot)^{-5/3}(f/50\,\mathrm{nHz})^{-8/3}$$

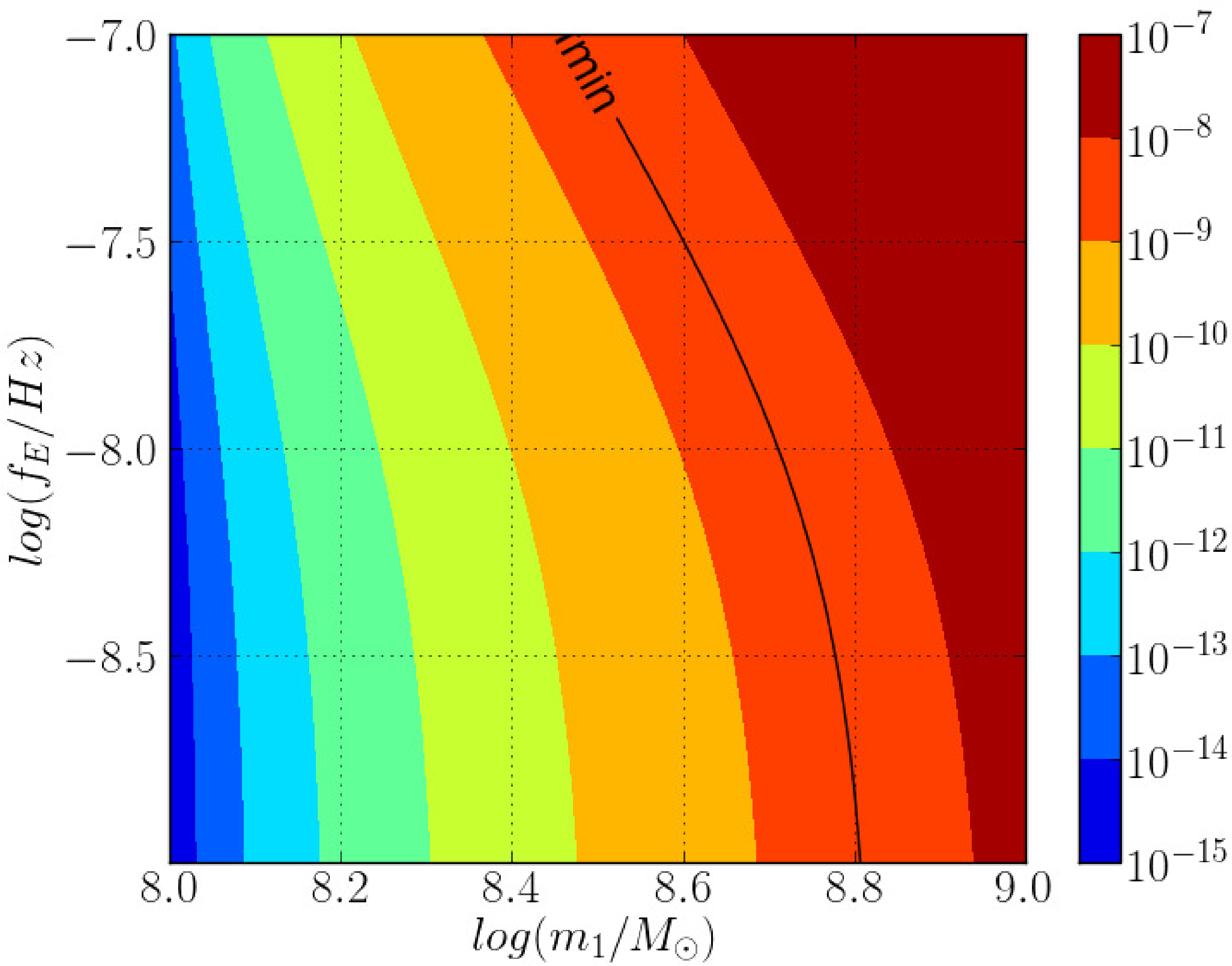
where $\mu = m_1 m_2/M$

$$\mathcal{M} = M^{2/5}\mu^{3/5}$$

Orbital period vs light travel time from PSR

- Orbital period of the binary evolves over the light travel time between Earth and pulsar $\sim 3.3 \times 10^3 \, (L_p(1 + \hat{\mathbf{\Omega}} \cdot \hat{\mathrm{p}})/\mathrm{kpc})$yr.
- Extended baseline is now comparable to orbital timescale.

Source detectability



In future, would need angular resolution of

$$\lesssim 3(100\,\mathrm{nHz}/f)(1\,\mathrm{kpc}/L_p)$$

arcsecs

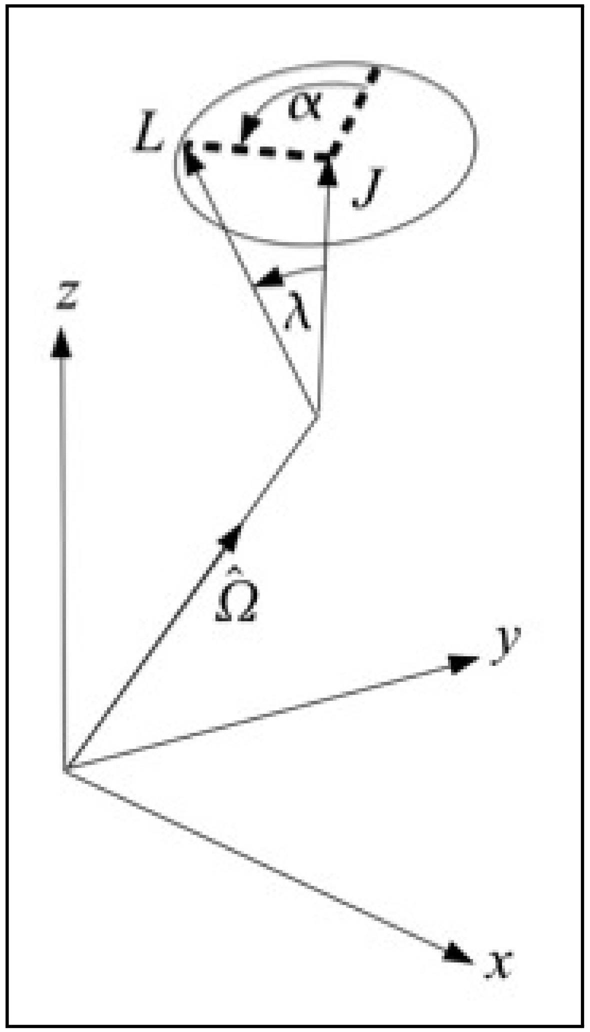
and

$$\Delta L_p < 0.01(f/100\mathrm{nHz})^{-1}$$ pc

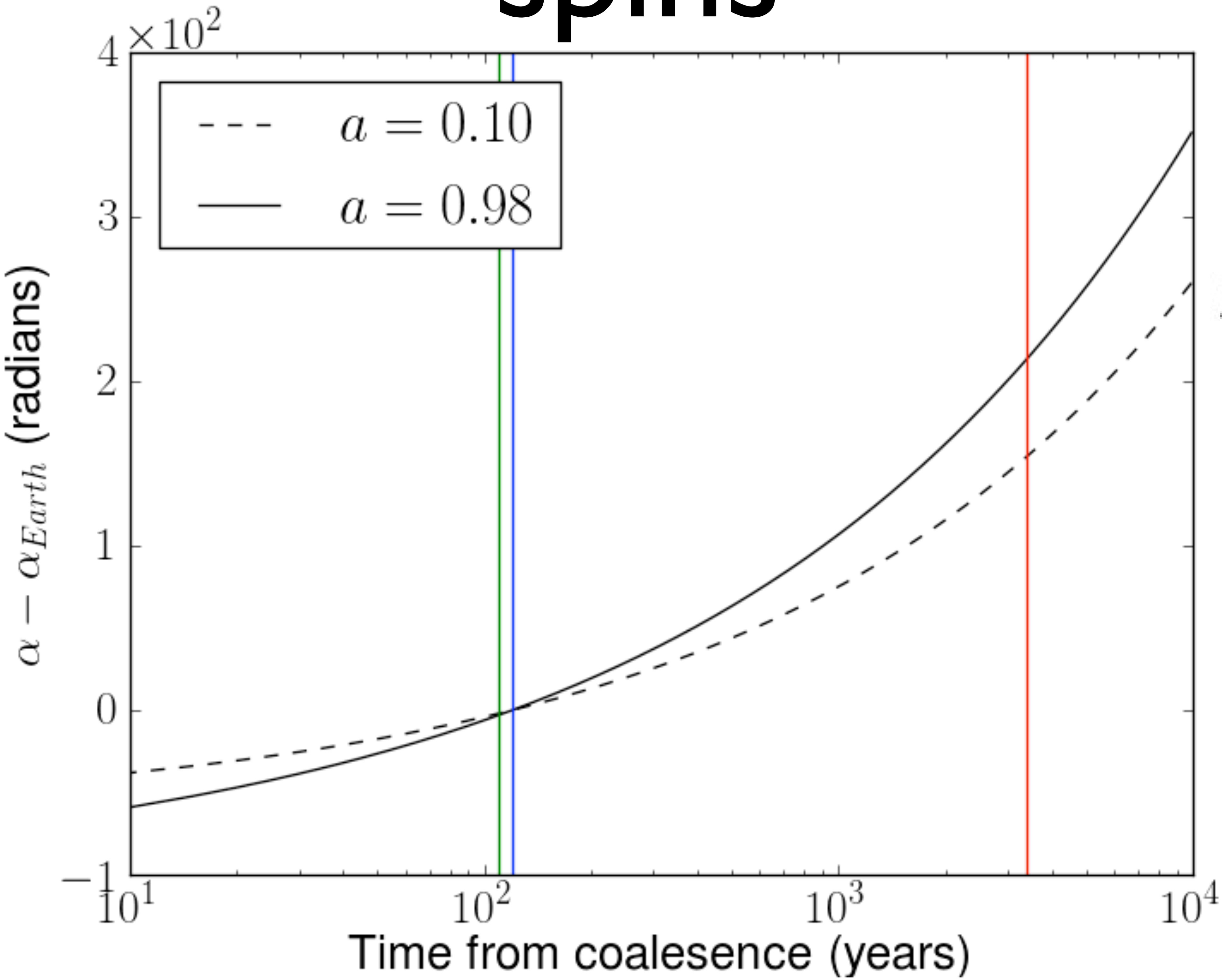
(SKA, R. Smits *et al.*, 2011)

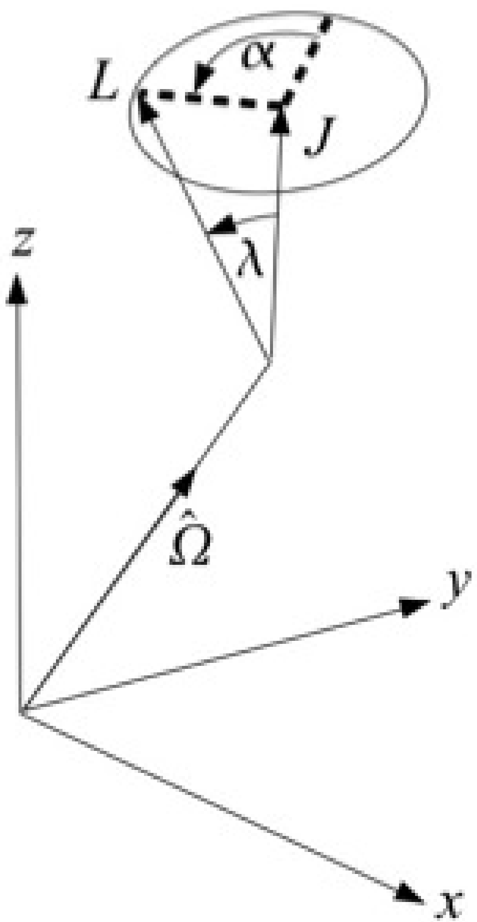
Use simple precession

- Use simple precession approximation to model spin-orbit coupling: $m_1 = m_2$
- total spin $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$ and $\mathbf{L}$, precesses around the (essentially) constant direction of the total angular momentum, $\hat{\mathbf{J}}$
- precesses at the same rate $d\alpha/dt = (2 + 3m_2/(2m_1))\,(|\mathbf{L} + \mathbf{S}|)/r(t)^3$, while preserving the angle of the precision cone, λ_L. In this case, $|S/L| \sim \mathcal{O}(0.1)$



Precession for varying spins





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Signals from precessing BHS

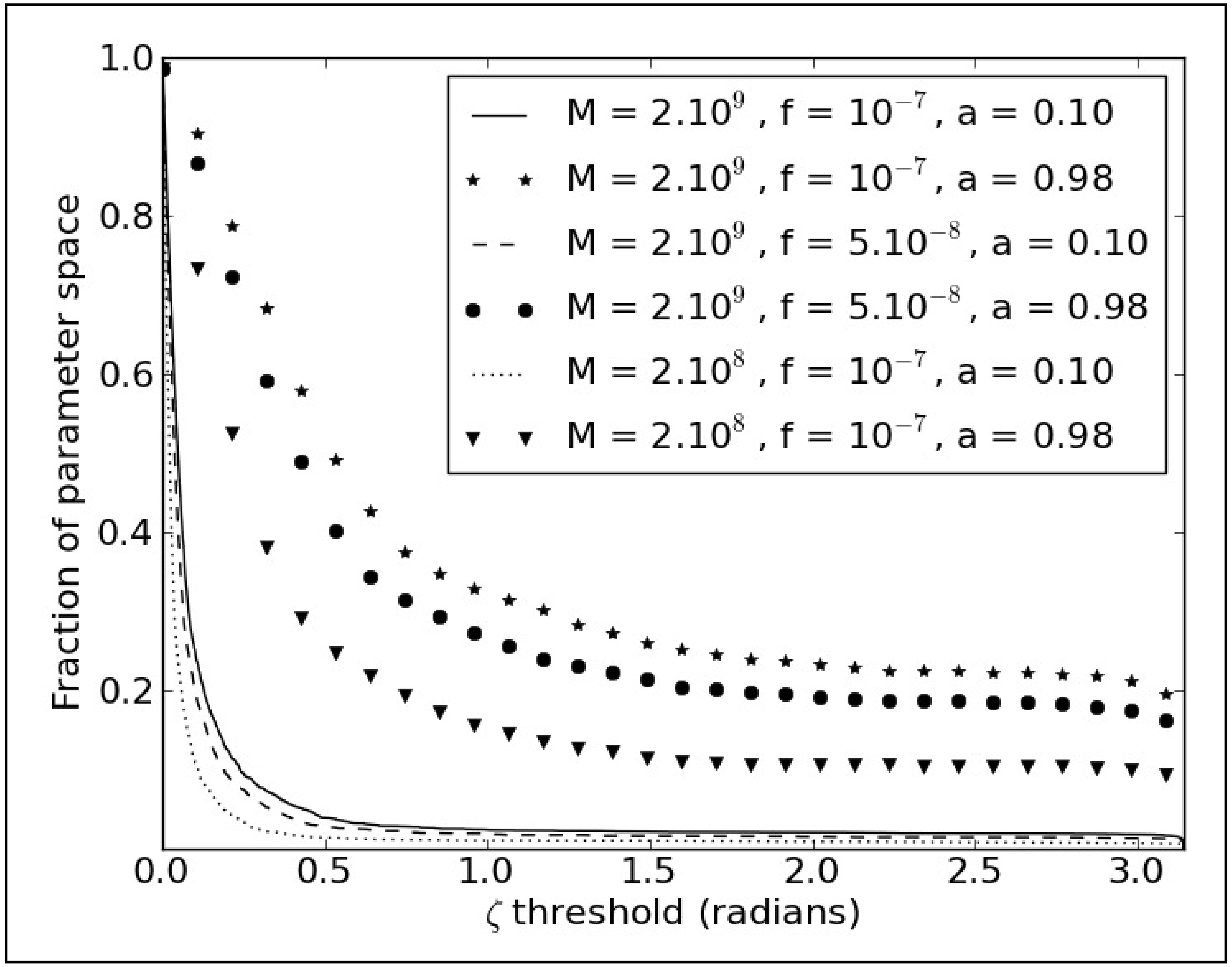
- model GW from a SMBHB using "restricted" pN approximation: amplitude is taken at leading Newtonian order, but we include the modulation effects produced by spin-orbit coupling and pN corrections are included only in the phase.
- strain for a single source given by $h(t) = -A_{\rm gw}(t)A_{\rm p}(t)\cos[\Phi(t) + \zeta(t) + \varphi(t)]$, where $A_{\rm gw}(t)$ is the lowest order Newtonian GW amplitude.
- The physical parameters leave different observational signatures in $h(t)$ and therefore in the TOAs.

Spin leaves 3 distinctive imprints in waveform

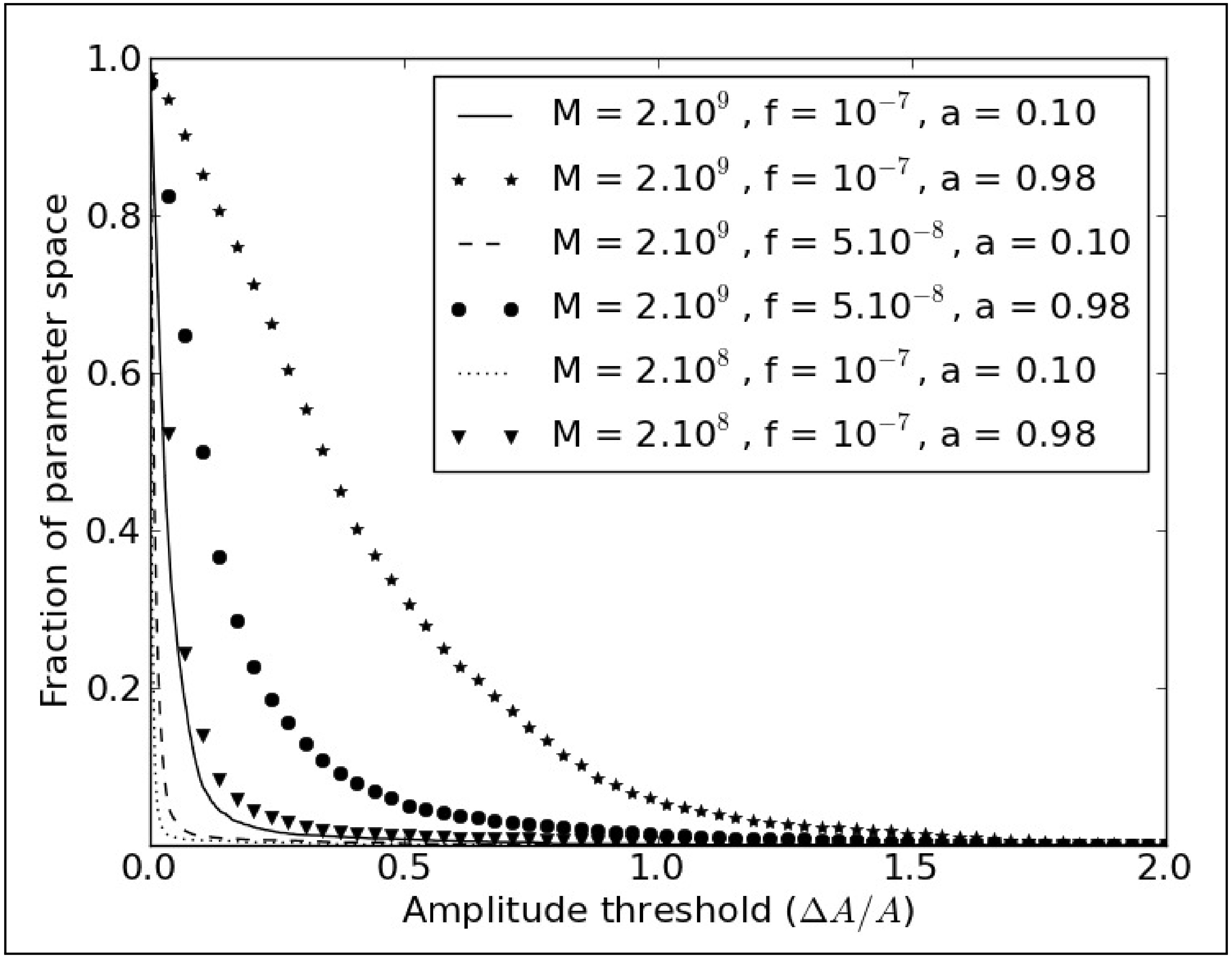
1. Alter the phase evolution through spin-orbit coupling (at p$^{1.5}$N order, proportional to the parameter $\beta = (1/12)\sum_{i=1}^{2}\left[113(m_i/M)^2 + 75\eta\right]\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_i$) and at p^{2}N via spin-spin coupling $\sigma = (\eta/48)[721(\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_1)(\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_2) - 247(\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2])$
2. they cause the orbital plane to precess due to (at lowest order) spin-orbit coupling and therefore induce amplitude and phase modulations in the waveform through $A_p(t)$ and $\zeta(t)$
3. through spin-orbit precession they introduce $\varphi(t)$, analogous to Thomas precession, to the waveform phase.

Phase Modulations

Imprint of precession is in $A_p(t)$, $\zeta(t)$ and $\varphi(t)$ whose size depends on λ_L, (maximised) $\hat{\mathbf{p}}$ and $\hat{\mathbf{\Omega}}$



Amplitude Modulations



The pN expansion

TABLE I. Frequency change Δf, total number of GW cycles, and individual contributions from the leading order terms in the pN expansion over the two relevant time scales—a 10 yr period starting at Earth and the time period $L_p(1 + \hat{\mathbf{\Omega}} \cdot \hat{\mathbf{p}})$ between the Earth and pulsar terms (hence the negative sign)—for selected values of $m_{1,2}$ and f_E.

m_1 ($M_\odot$)	m_2 ($M_\odot$)	f_E (nHz)	($v/c \times 10^{-2}$)	Time span	Δf (nHz)	Total	Newtonian	p^1N	$p^{1.5}$N	Spin orbit/β	p^2N
10^9	10^9	100	14.6	10 yr	3.22	32.1	31.7	0.9	−0.7	0.06	0.04
			9.6	−1 kpc	71.2	4305.1	4267.8	77.3	−45.8	3.6	2.2
		50	11.6	10 yr	0.24	15.8	15.7	0.3	−0.2	0.01	<0.01
			9.4	−1 kpc	23.1	3533.1	3504.8	53.5	−28.7	2.3	1.2
10^8	10^8	100	6.8	10 yr	0.07	31.6	31.4	0.2	−0.07	<0.01	<0.01
			6.4	−1 kpc	15.8	9396.3	9355.7	58.3	−19.9	1.6	0.5
		50	5.4	10 yr	0.005	15.8	15.7	0.06	−0.02	<0.01	<0.01
			5.3	−1 kpc	1.62	5061.4	5045.8	20.8	−5.8	0.5	0.1

directly from Mingarelli *et al*, PRL **109**, 081104 (2012)

Conclusions

pulsar term can be used to map the evolution of a SMBHB over the Earth-pulsar baseline. This will only be possible with future detectors as:

- the pulsar term needs to be detectable
- can measure the distance to the pulsar to better than $\Delta L_p < 0.01(f/100\mathrm{nHz})^{-1}$ pc and might need angular resolution of $3(100\,\mathrm{nHz}/f)(1\,\mathrm{kpc}/L_p)$ arcsec
- the resolvable SMBHB is sufficiently massive and high frequency
- need to use the full precession equations, not just simple precession.

All is not lost!

- If we can do this, PTAs can yield non-trivial information about the non-linear pN behaviour of gravity.
- Can do parameter estimation done on the map made by using the pulsar terms, which will improve with the number of pulsars, will enable us to estimate the mass and spin of the SMBHB.

Thank you!

Extra slides

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Model pulse phase

- Following accumulation of many TOAs, Taylor expand rotation frequency $\Omega = \frac{2\pi}{P}$ about a model value Ω_0, at some reference epoch $\mathcal{T}_0$. The model pulse phase is:

$$\phi(\mathcal{T}) = \phi_0 + (\mathcal{T} - \mathcal{T}_0)\Omega_0 + \frac{1}{2}(\mathcal{T} - \mathcal{T}_0)^2\dot{\Omega}_0 + \ldots,$$

- here T is the SSB time, ϕ_0 is the pulse phase at $\mathcal{T}_0$.

- Use SSB for inertial reference frame.

Lorimer, D, "Living Rev. Relativity", **11** (2008), 8