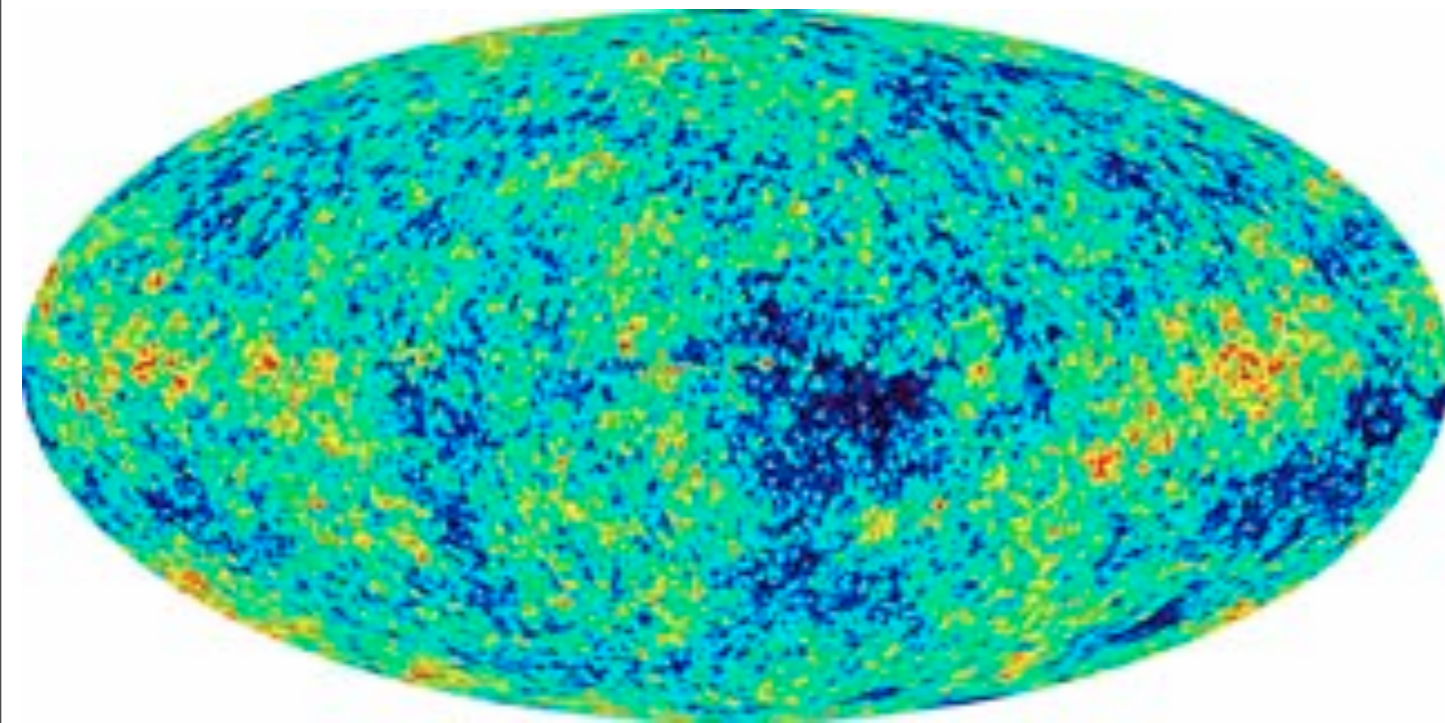


Testing Gravity with Cosmology (a new Golden Age?)

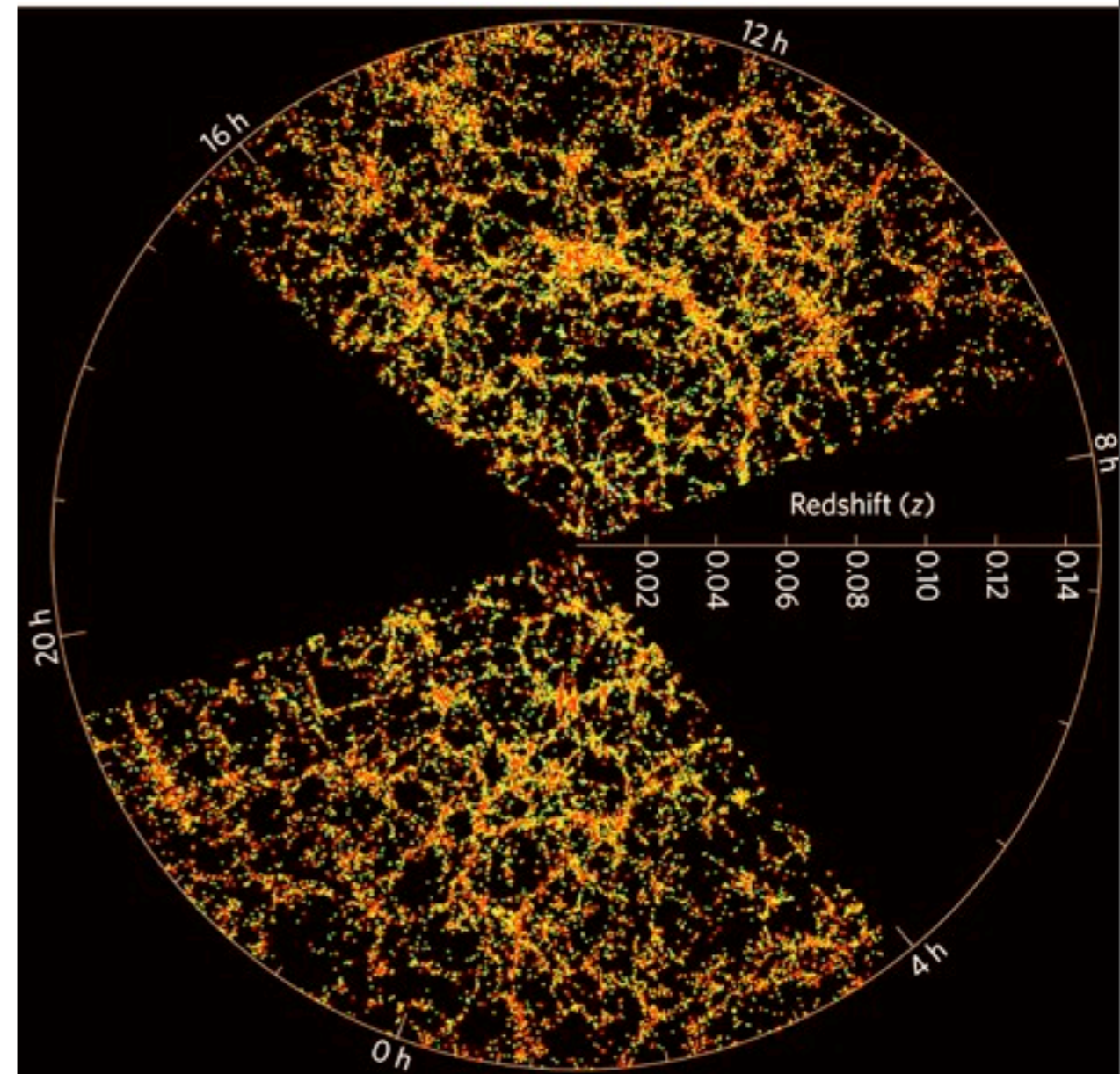
Pedro Ferreira
Oxford

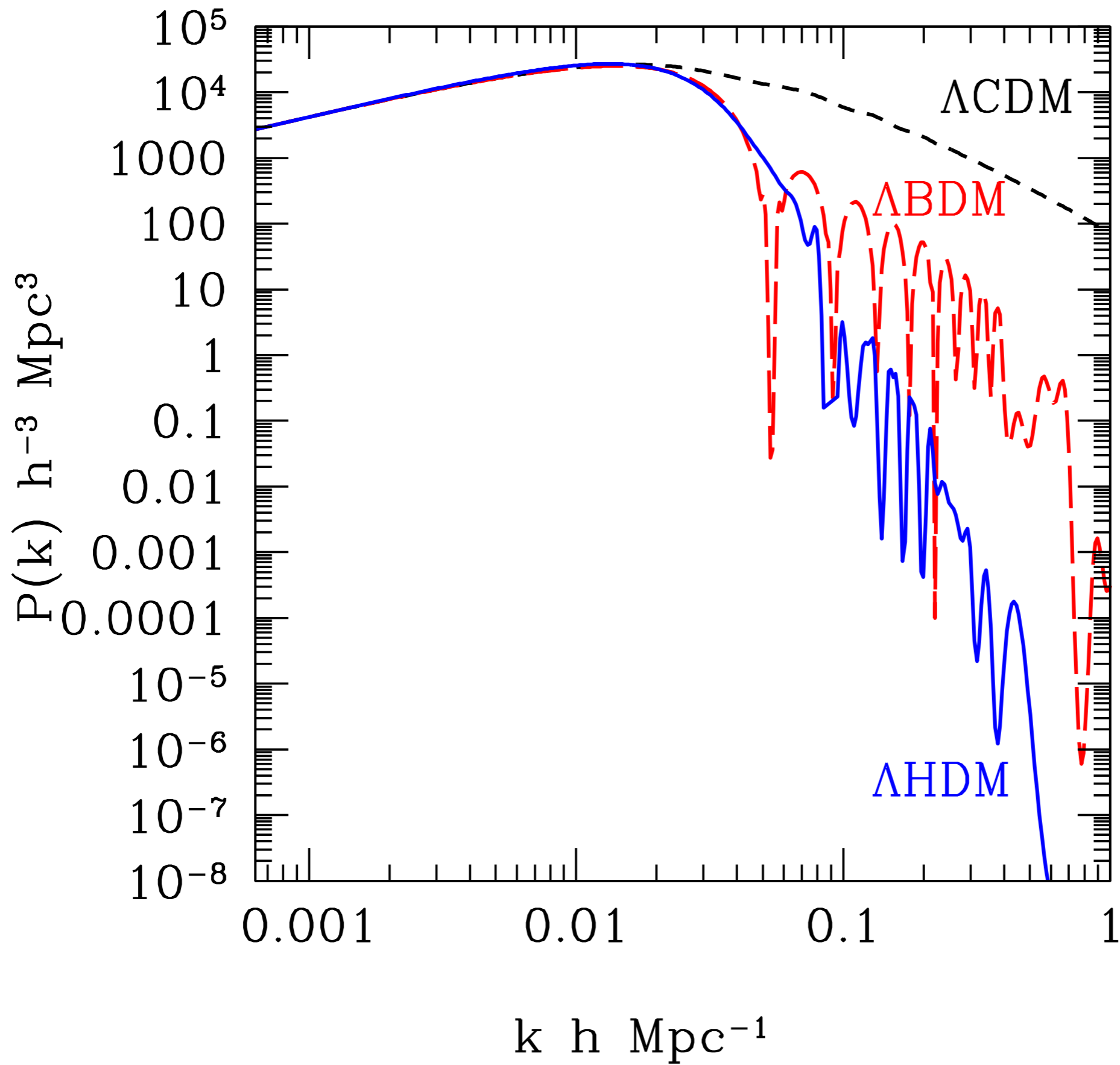
The Large Scale Structure of the Universe

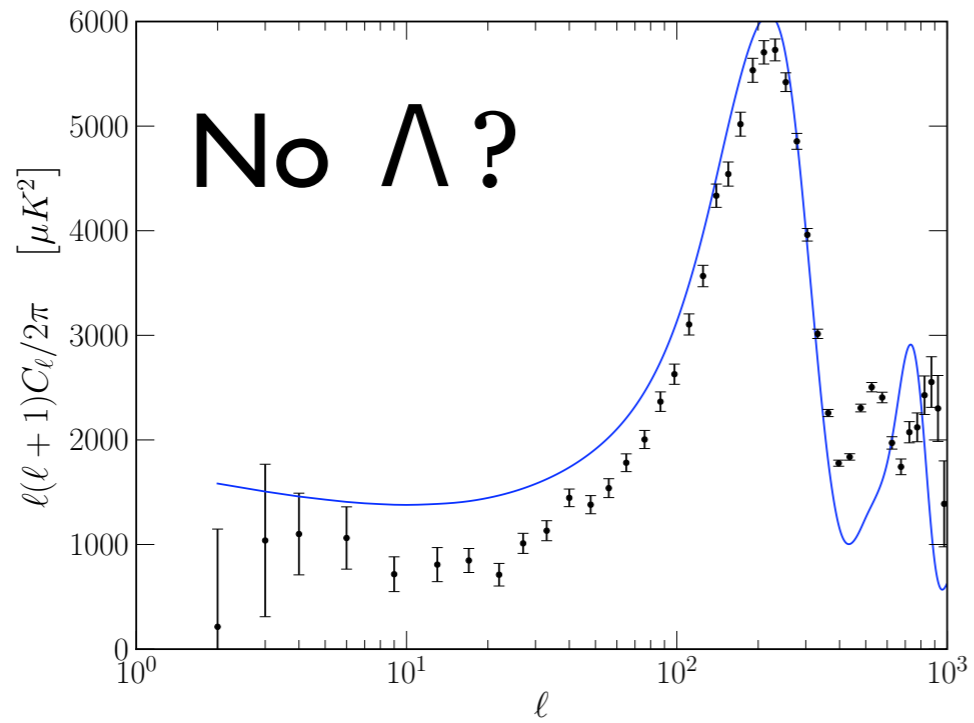
WMAP



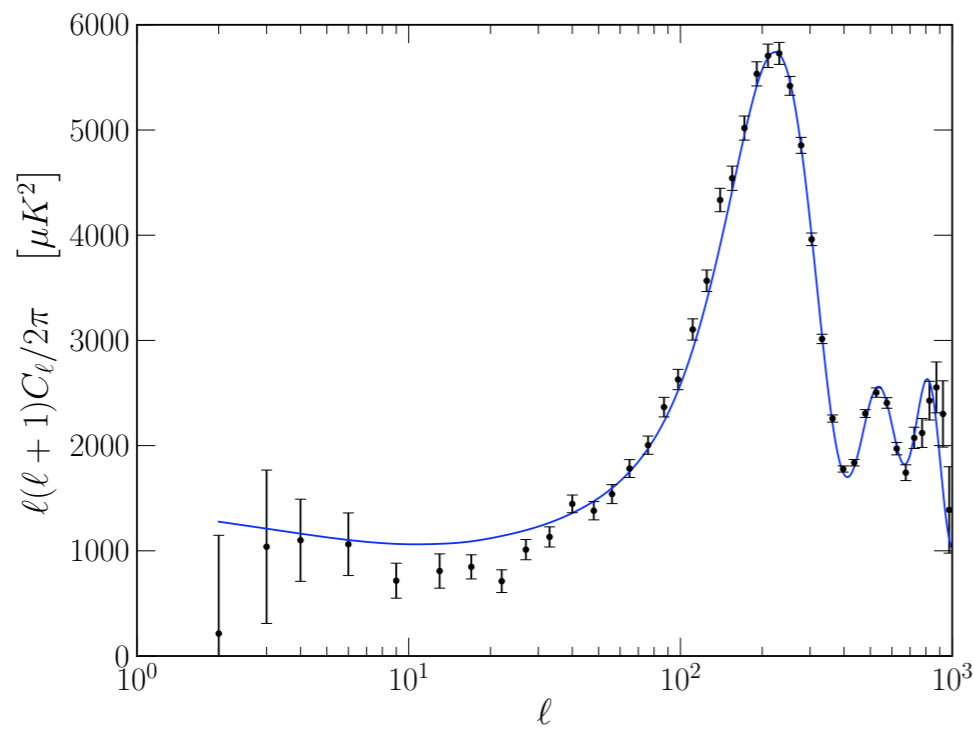
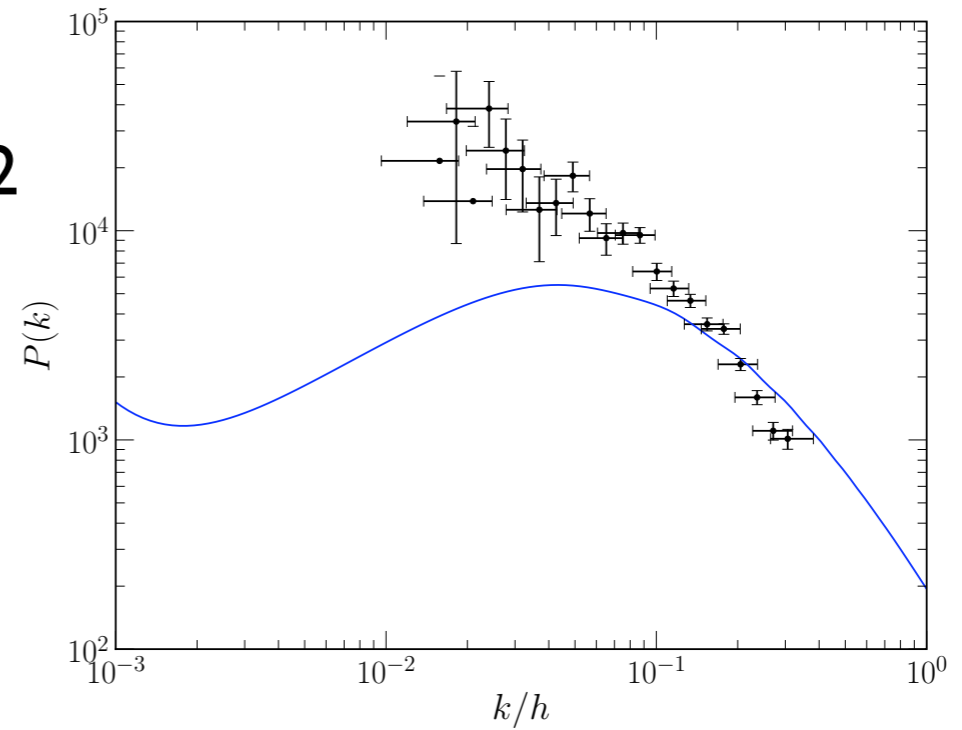
SDSS



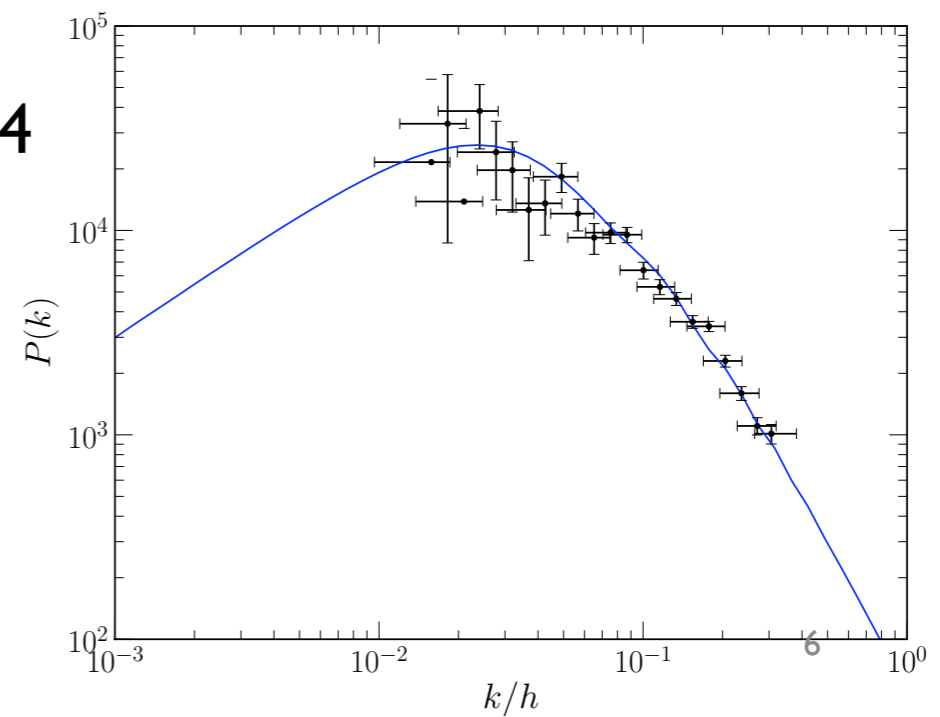




$H_0 = 72$



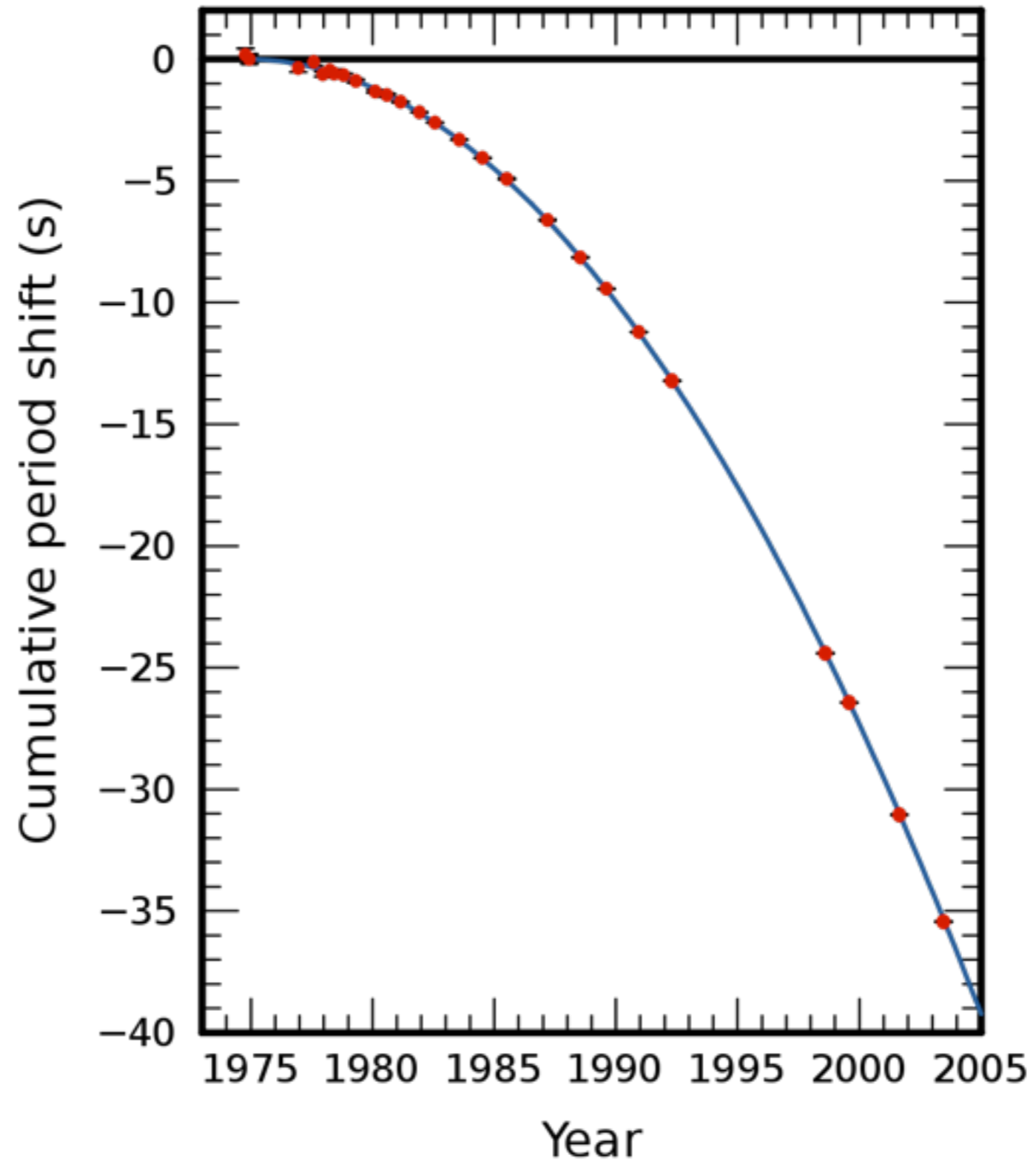
$H_0 = 44$



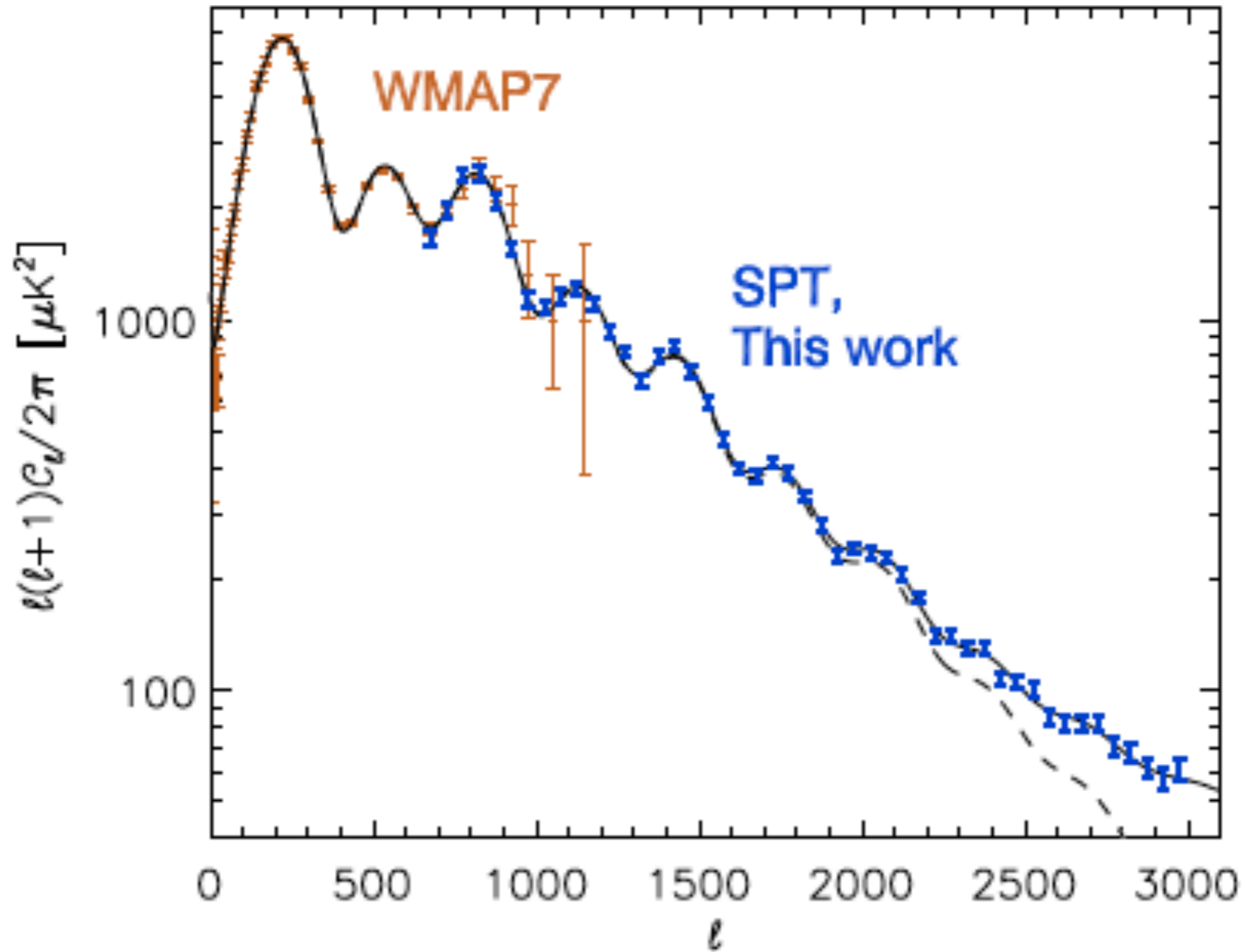
“The elegant logic of general relativity theory, and its precision tests, recommend GR as the first choice for a working model for cosmology. But the Hubble length is fifteen orders of magnitude larger than the length scale of the precision tests, at the astronomical unit and smaller, a spectacular extrapolation.”

Jim Peebles, IAU 2000

Spin-down of the Hulse - Taylor binary pulsar



The Angular Power Spectrum of the CMB



Keisler et al 2011

$$l \simeq \frac{180^\circ}{\theta}$$

Outline

- The panorama of gravitation
- Cosmological linear perturbations
- How to parametrize the space of theories
- How to measure the parameters
- The future

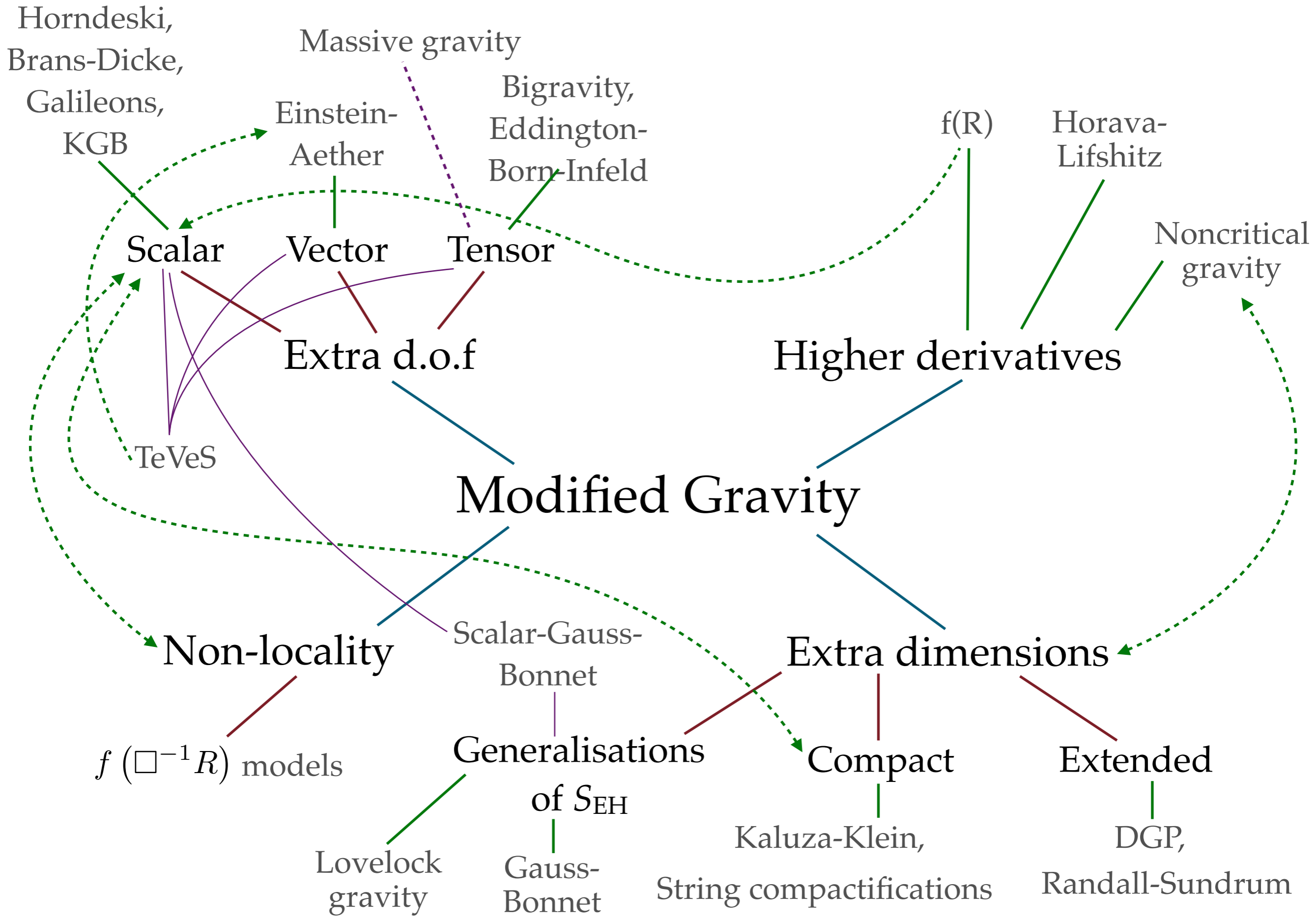
Einstein Gravity

Curvature

$$\frac{1}{16\pi G} \int d^4x \sqrt{-g} R(g) + \int d^4x \sqrt{-g} \mathcal{L}(g, \text{matter})$$

Metric of space time

Lovelock's theorem (1971) :*“The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant.”*



ArXiv:1106.2476

Linear Perturbation Theory $(10 - 10,000h^{-1} Mpc)$

$$(\hat{\Phi}, \hat{\Psi}) \quad \delta G_{\alpha\beta} = 8\pi G \delta T_{\alpha\beta} \quad + \text{E.M. Conservation}$$

Gauge invariant
Newtonian potentials

$$\hat{\Gamma} = \frac{1}{k} \left(\dot{\hat{\Phi}} + \mathcal{H} \hat{\Psi} \right)$$

$$E_{\Delta} = 2(\vec{\nabla}^2 + 3K)\hat{\Phi} - 6\mathcal{H}k\hat{\Gamma} - \frac{3}{2}\mathcal{H}EV = 8\pi Ga^2 \sum \rho_i \delta_i$$

$$E_{\Theta} = 2k\hat{\Gamma} + \frac{1}{2}EV = 8\pi Ga^2 \sum_i (\rho_i + P_i)\theta_i$$

$$E_P = 6k \frac{d\hat{\Gamma}}{d\tau} + 12\mathcal{H}k\hat{\Gamma} - 2(\vec{\nabla}^2 + 3K)(\hat{\Phi} - \hat{\Psi}) - 3E\hat{\Psi} + \frac{3}{2}(\dot{E}_R - 2\mathcal{H}E_R)V = 24\pi Ga^2 \sum_i \rho_i \Pi_i$$

$$E_{\Sigma} = \hat{\Phi} - \hat{\Psi} = 8\pi Ga^2 \sum_i (\rho_i + P_i)\Sigma_i$$

In fact- construct an algebraic equation: $(\nabla^2 + K)\hat{\Phi} = 4\pi Ga^2 \sum_i \rho_i \Delta_i$

Simplest Approach

Zhang, Liguori, Bean and Dodelson
Caldwell, Cooray and Melchiorri
Amendola, Kunz and Sapone
Bertschinger and Zúkin
Amin, Blandford and Wagoner
Pogosian, Silvestri, Koyama and Zhao
Bean and Tangmatitham

$$\text{Poisson} \quad -k^2 \Phi = 4\pi G a^2 \rho \Delta \quad (E_\Delta - 3\mathcal{H}E_\Theta)$$

$$\text{Slip} \quad \Phi - \Psi = 0 \quad (E_\Sigma)$$

$$-k^2 \Phi = 4\pi G a^2 G a^2 \rho \Delta + F_1 \quad F_1 = k^2 f_1 \Phi$$

$$\Phi - \Psi = F_2 \quad F_2 = f_2 \Phi$$

Rearrange and rename:

$$-k^2 \Phi = 4\pi G \mu a^2 \rho \Delta$$

$$\Psi = \gamma \Phi$$

$$-k^2 (\Phi + \Psi) = 8\pi G \Sigma a^2 \rho \Delta$$

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Poisson $-k^2\Phi = 4\pi G a^2 \rho \Delta$ $(E_\Delta - 3\mathcal{H}E_\Theta)$

Slip $\Phi - \Psi = 0$ (E_Σ)

$$-k^2\Phi = 4\pi G a^2 G a^2 \rho \Delta + F_1 \quad F_1 = k^2 f_1 \Phi$$

$$\Phi - \Psi = F_2 \quad F_2 = f_2 \Phi$$

Rearrange and rename:

$$-k^2\Phi = 4\pi G \mu a^2 \rho \Delta$$

$$\Psi = \gamma \Phi$$

Each function must be constrained on a grid of (a,k) using Principal Component Analysis .

- There are a few variants, e.g.

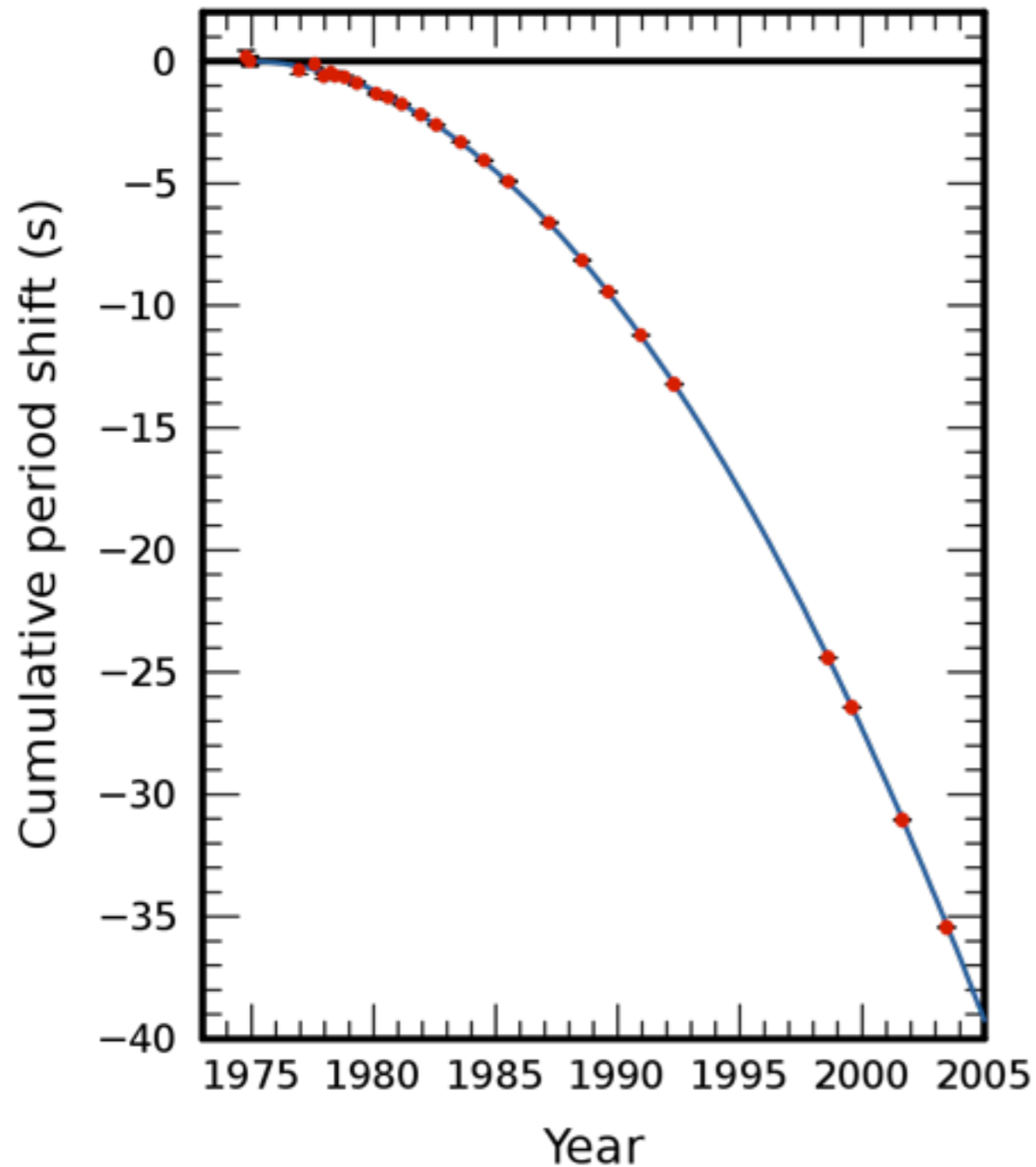
$$-k^2(\Phi + \Psi) = 8\pi G \Sigma a^2 \rho \Delta$$

Lessons from PPN

		γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Stratified	Vector-Tensor	Einstein (1916) GR	1	1	0	0	0	0	0	0	0
		Bergmann (1968), Wagoner (1970)		β	0	0	0	0	0	0	0
		Nordtvedt (1970), Bekenstein (1977)		β	0	0	0	0	0	0	0
	Tensor	Brans-Dicke (1961)		1	0	0	0	0	0	0	0
		Hellings-Nordtvedt (1973)	γ	β	0	α_1	α_2	0	0	0	0
	Bimetric	Will-Nordtvedt (1972)	1	1	0	0	α_2	0	0	0	0
		Rosen (1975)	1	1	0	0	$c_0/c_1 - 1$	0	0	0	0
		Rastall (1979)	1	1	0	0	α_2	0	0	0	0
		Lightman-Lee (1973)	γ	β	0	α_1	α_2	0	0	0	0
		Lee-Lightman-Ni (1974)	$a c_0 / c_1$	β	ξ	α_1	α_2	0	0	0	0
Scalar Field Theories	Ni (1973)	$a c_0 / c_1$	$b c_0$	0	α_1	α_2	0	0	0	0	
	Einstein (1912) {Not GR}	0	0	0	-4	0	-2	0	-1	0	
	Whitrow-Morduch (1965)	0	-1	0	-4	0	0	0	-3	0	
	Rosen (1971)	λ		0	$-4 - 4\lambda$	0	-4	0	-1	0	
	Papetrou (1954a, 1954b)	1	1	0	-8	-4	0	0	2	0	
	Ni (1972) (stratified)	1	1	0	-8	0	0	0	2	0	
	Yilmaz (1958, 1962)	1	1	0	-8	0	-4	0	-2	0	
	Page-Tupper (1968)	γ	β	0	$-4 - 4\gamma$	0	$-2 - 2\gamma$	0	ζ_2	0	
	Nordström (1912)	-1	β	0	0	0	0	0	0	0	
	Nordström (1913), Einstein-Fokker (1914)	-1	1	0	0	0	0	0	0	0	
	Ni (1972) (flat)	-1	$1 - q$	0	0	0	0	0	ζ_2	0	
	Whitrow-Morduch (1960)	-1	$1 - q$	0	0	0	0	0	q	0	
	Littlewood (1953), Bergman(1956)	-1	β	0	0	0	0	0	-1	0	

Lessons from PPN

Spin-down of the Hulse - Taylor binary pulsar



Parameter	Bound	Effects	Experiment
$\gamma - 1$	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
$\beta - 1$	2.3×10^{-4}	Nordtvedt effect, Perihelion shift	Nordtvedt effect
ξ	0.001	Earth tides	Gravimeter data
α_1	10^{-4}	Orbit polarization	Lunar laser ranging
α_2	4×10^{-7}	Spin precession	Solar alignment with ecliptic
α_3	4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	0.02	-	Combined PPN bounds
ζ_2	4×10^{-5}	Binary pulsar acceleration	PSR 1913+16
ζ_3	10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4	0.006	-	Usually not independent

Extending Einstein's equations

$$\delta G_{\mu\nu} = 8\pi G_N \delta T_{\mu\nu}^M + \boxed{\delta U_{\mu\nu}} \text{--- ?}$$

METRIC

NEW D.O.F.

$$\delta U_{\mu\nu} = \delta U_{\mu\nu}^{\hat{\Phi}} + \delta U_{\mu\nu}^{\hat{\delta\phi}} + \text{gauge-form invariance fixing term}$$



Built from Bardeen potentials ×
function of background



Fixed by
background
equations

$$\delta(\nabla^\mu G_{\mu\nu}) = 0 \rightarrow \delta(\nabla^\mu U_{\mu\nu}) = 0$$

ArXiv:1209.2117

Adding New Scalars

$$-a^2 \delta U_0^0 = U_\Delta = A_0 k^2 \hat{\Phi} + F_0 k^2 \hat{\Gamma} + \alpha_0 k^2 \hat{\chi} + \alpha_1 k \dot{\hat{\chi}} + M_\Delta k^3 V$$

$$U_\Theta = B_0 k \hat{\Phi} + I_0 k \hat{\Gamma} + \beta_0 k \hat{\chi} + \beta_1 \dot{\hat{\chi}} + M_\Theta k^2 V$$

$$a^2 \delta U_i^i = U_P = C_0 k^2 \hat{\Phi} + C_1 k \dot{\hat{\Phi}} + J_0 k^2 \hat{\Gamma} + J_1 k \dot{\hat{\Gamma}} \\ + \gamma_0 k^2 \hat{\chi} + \gamma_1 k \dot{\hat{\chi}} + \gamma_2 \ddot{\hat{\chi}} + M_P k^3 V$$

$$U_\Sigma = D_0 \hat{\Phi} + \frac{D_1}{k} \dot{\hat{\Phi}} + K_0 \hat{\Gamma} + \frac{K_1}{k} \dot{\hat{\Gamma}} \\ + \epsilon_0 \hat{\chi} + \frac{\epsilon_1}{k} \dot{\hat{\chi}} + \frac{\epsilon_2}{k^2} \ddot{\hat{\chi}}$$

Functions of
background
(a, k, φ_i ...)

Gauge form-fixing term, zero in CN gauge.

ArXiv:1209.2117

$$\nabla_i U_\Theta = -a^2 \delta U_i^0, \quad \left(\nabla^i \nabla_j - \frac{1}{3} \delta_j^i \nabla^2 \right) U_\Sigma = a^2 \delta U_j^i$$

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$$-a^2 \delta U_0^0 = U_\Delta = A_0 k^2 \hat{\Phi} + F_0 k^2 \hat{\Gamma} + \alpha_0 k^2 \hat{\chi} + \alpha_1 k \dot{\hat{\chi}} + M_\Delta k^3 V$$

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Adding New Scalars

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ArXiv:1209.2117

Category	Theory
Horndeski Theories	Scalar-Tensor theory (incl. Brans-Dicke)
	$f(R)$ gravity
	$f(\mathcal{G})$ theories
	Covariant Galileons
	The Fab Four
	K-inflation
	Generalized G-inflation
	Kinetic Gravity Braiding
	Quintessence (incl. universally coupled models)
	Effective dark fluid
Lorentz-Violating theories	Einstein-Aether theory
	Hořava-Lifschitz theory
> 2 new degrees of freedom	DGP (4D effective theory)
	EBI gravity
	TeV S

ArXiv:1209.2117

... and more to come.

All possible Coefficients

All theories

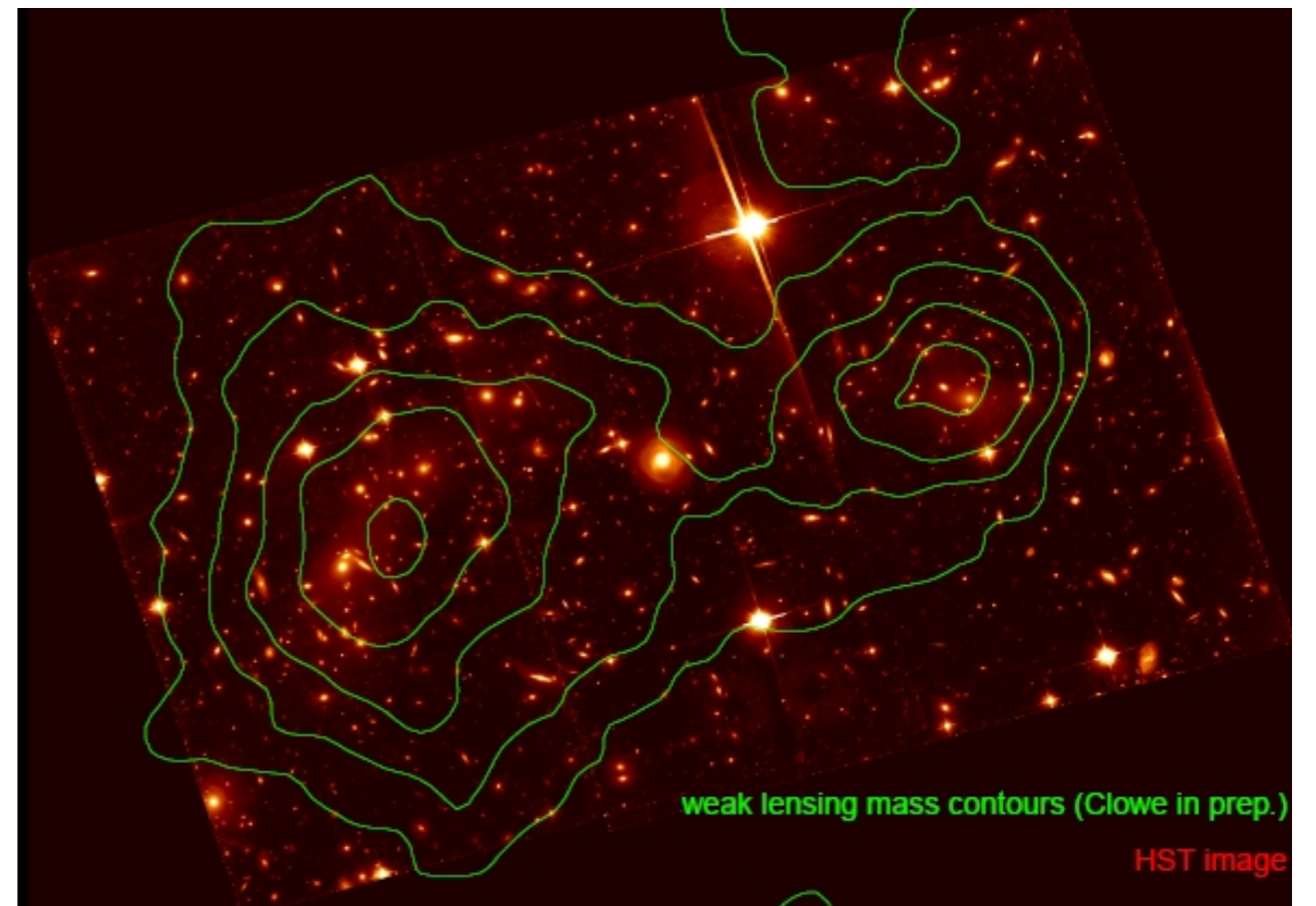
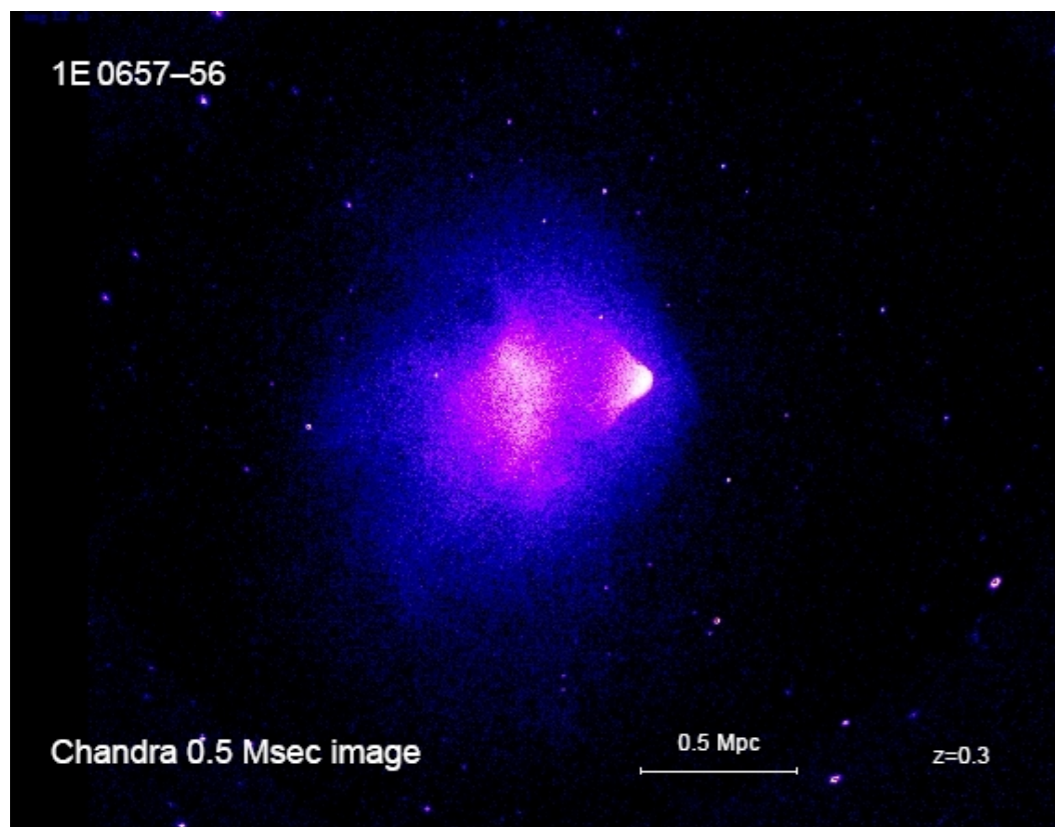
GR

All allowed
coefficients

What about the non-linear regime?

$$R \simeq 0.1 - 10 h^{-1} \text{Mpc}$$

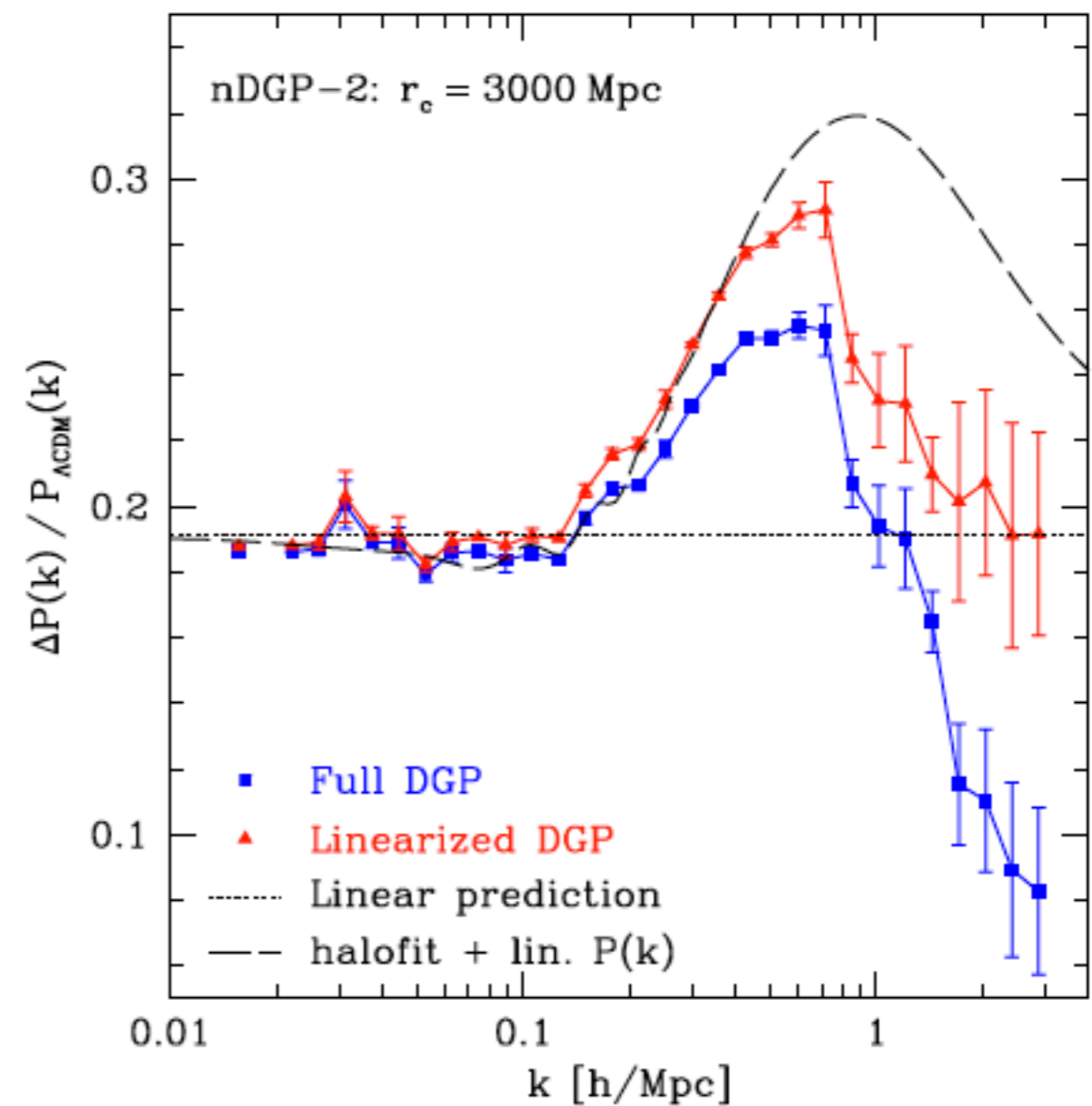
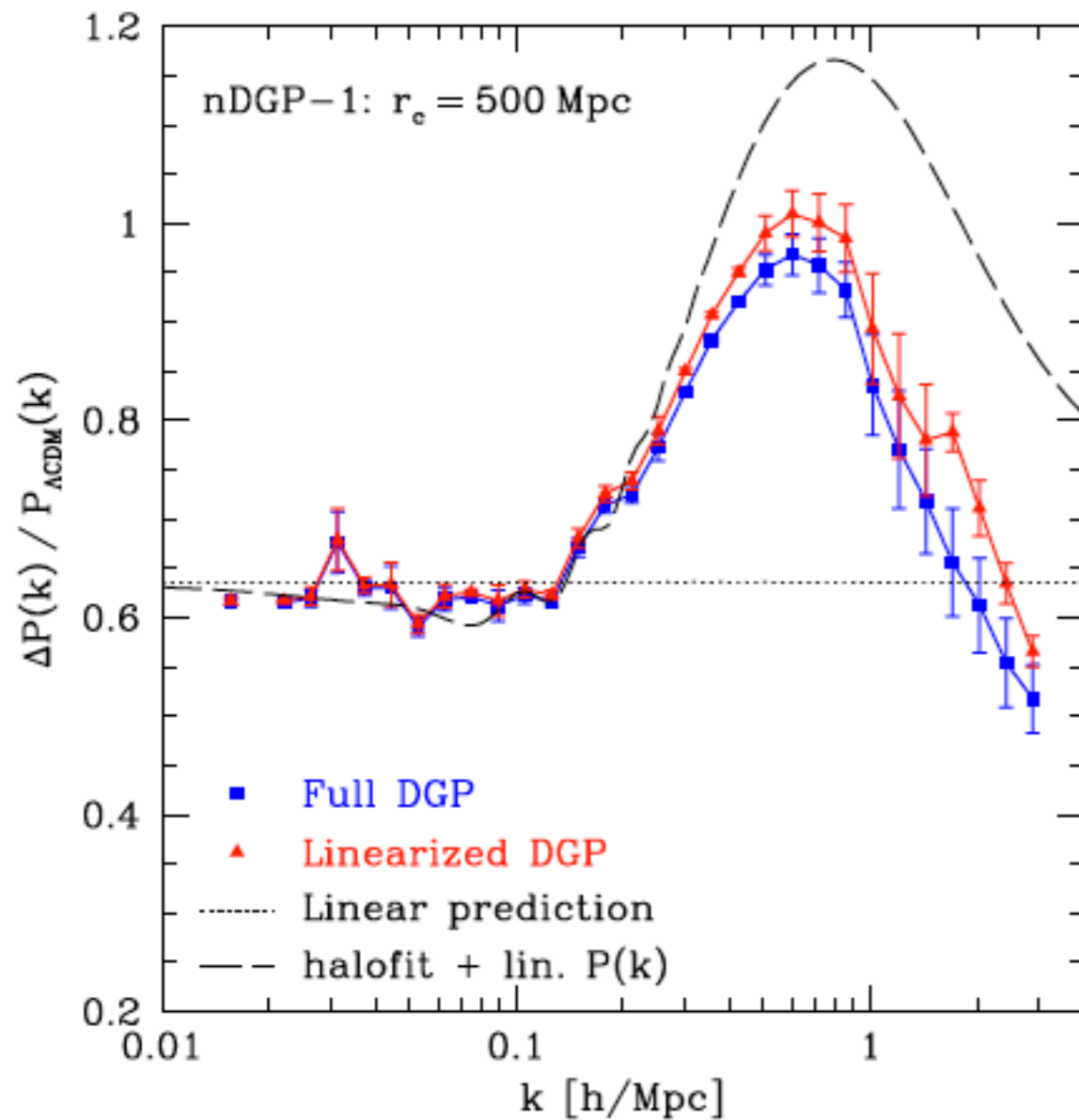
The Bullet Cluster



Clowe et al 2006

What about the non-linear regime?

$$R \simeq 0.1 - 10 h^{-1} \text{Mpc}$$



Schmidt 2009

Observables: Light vs. Matter

- For a perturbed line element of the form:

$$ds^2 = a^2(\tau) [-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\gamma_{ij}dx^i dx^j]$$

the equations of motion are:

$$\frac{1}{a} \frac{d(a\mathbf{v})}{d\tau} = -\nabla\Phi \quad (\text{non-relativistic particles})$$

$$\frac{d\mathbf{v}}{d\tau} = -\nabla_{\perp}(\Phi + \Psi) \quad (\text{relativistic particles})$$

Growth of structure,
RSDs and PVs.

ISW
Lensing (CMB, weak)

Growth of Structure

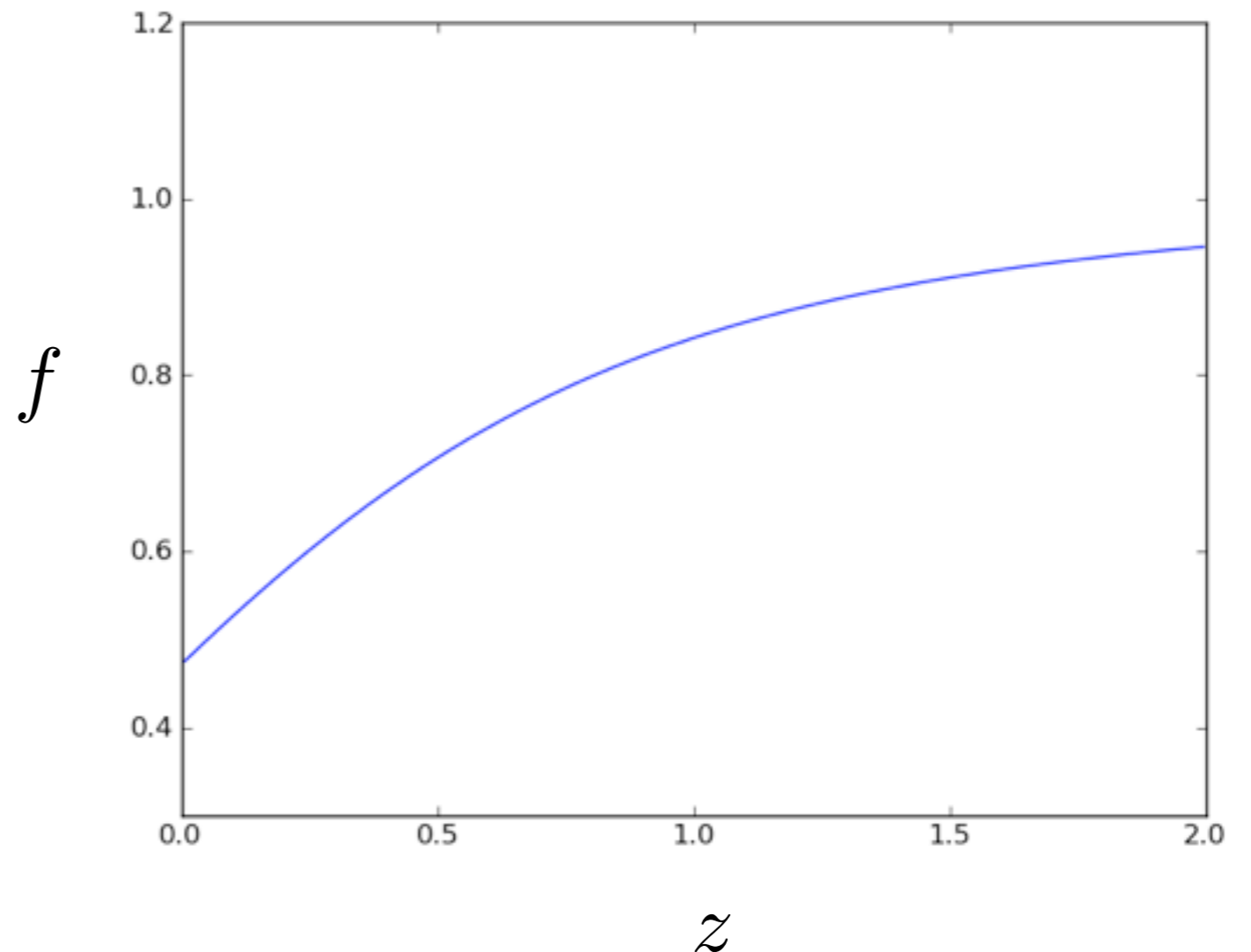
- Evolution of CDM energy density perturbations:

$$\ddot{\delta}_M + \mathcal{H}\dot{\delta}_M - 3\ddot{\Phi} - 3\mathcal{H}\dot{\Phi} + k^2\Psi = 0$$

- The growth rate of structure is quantified via f :

$$f(k, a) = \frac{d \ln \delta_M(k, a)}{d \ln a}$$

- In GR $\delta_M \propto a$ during matter domination, so $f=1$ (independent of k for linear scales).



Growth of Structure

- Evolution of CDM energy density perturbations:

$$\ddot{\delta}_M + \mathcal{H}\dot{\delta}_M - 3\ddot{\Phi} - 3\mathcal{H}\dot{\Phi} + k^2\Psi = 0$$

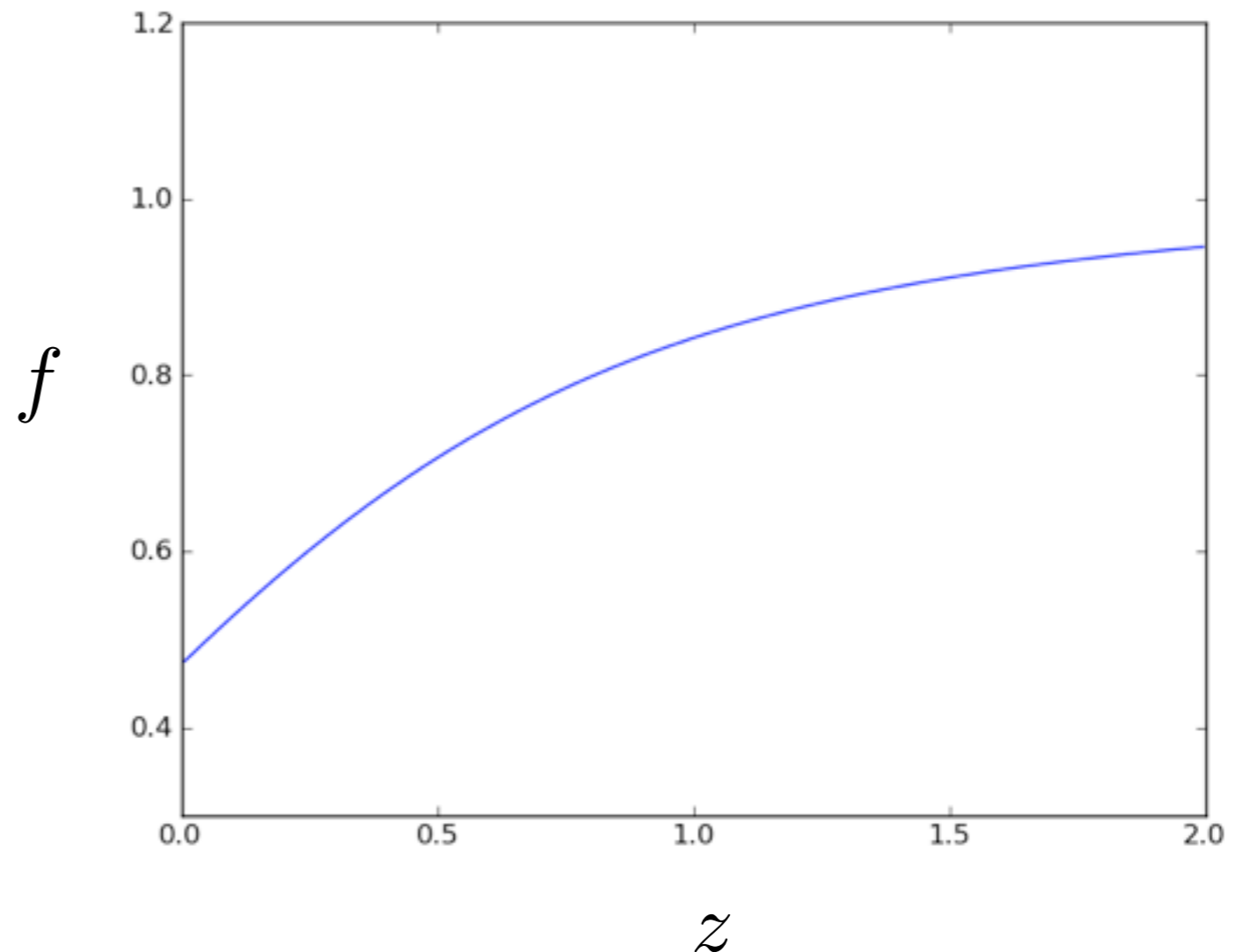
↓
Relation to δ_M has changed.

↓
Relation to Φ has changed.

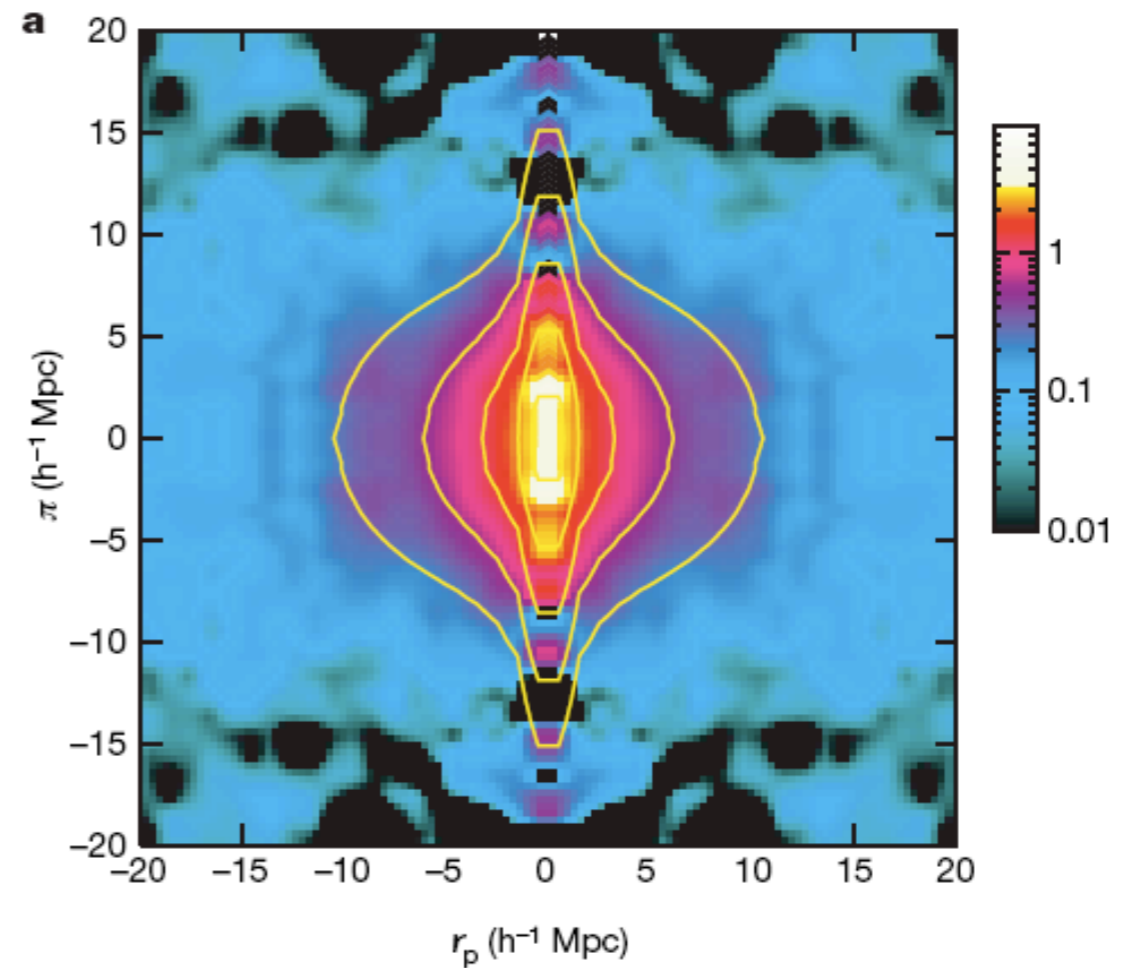
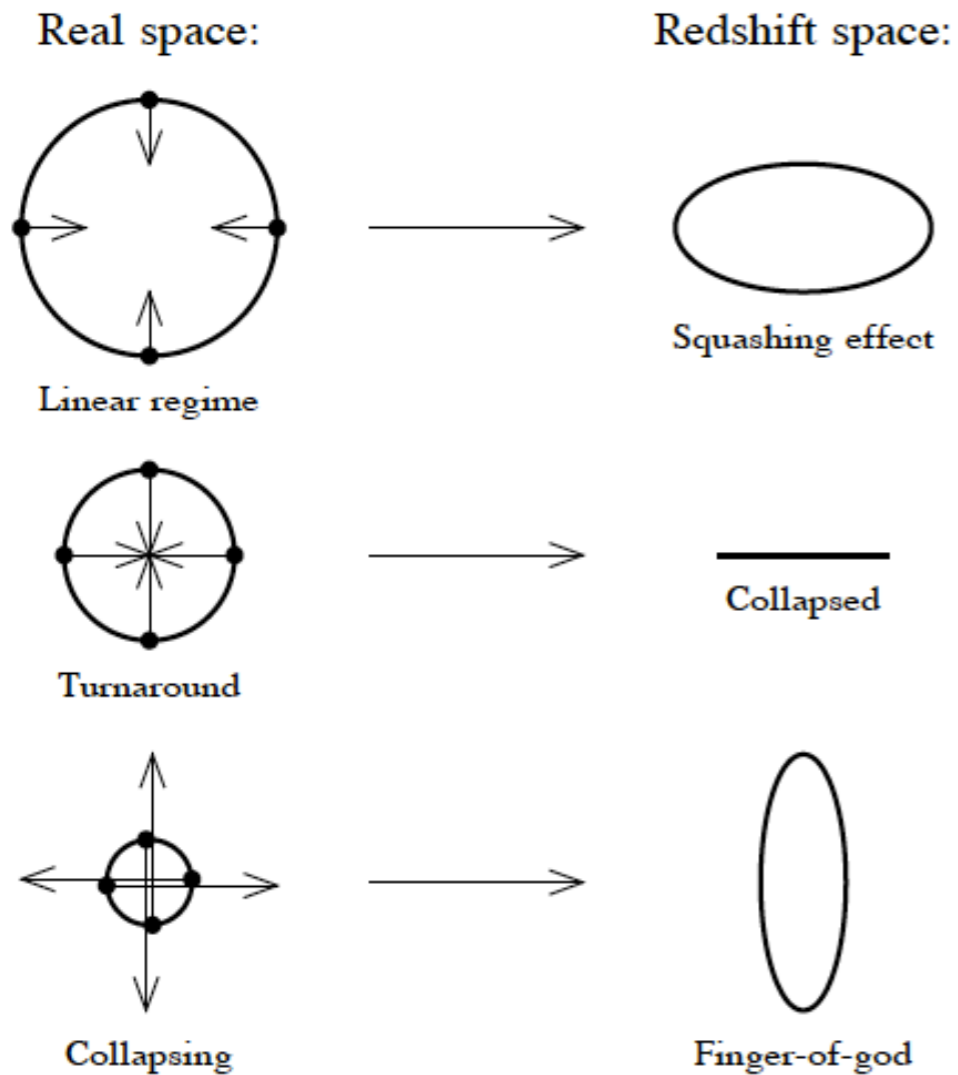
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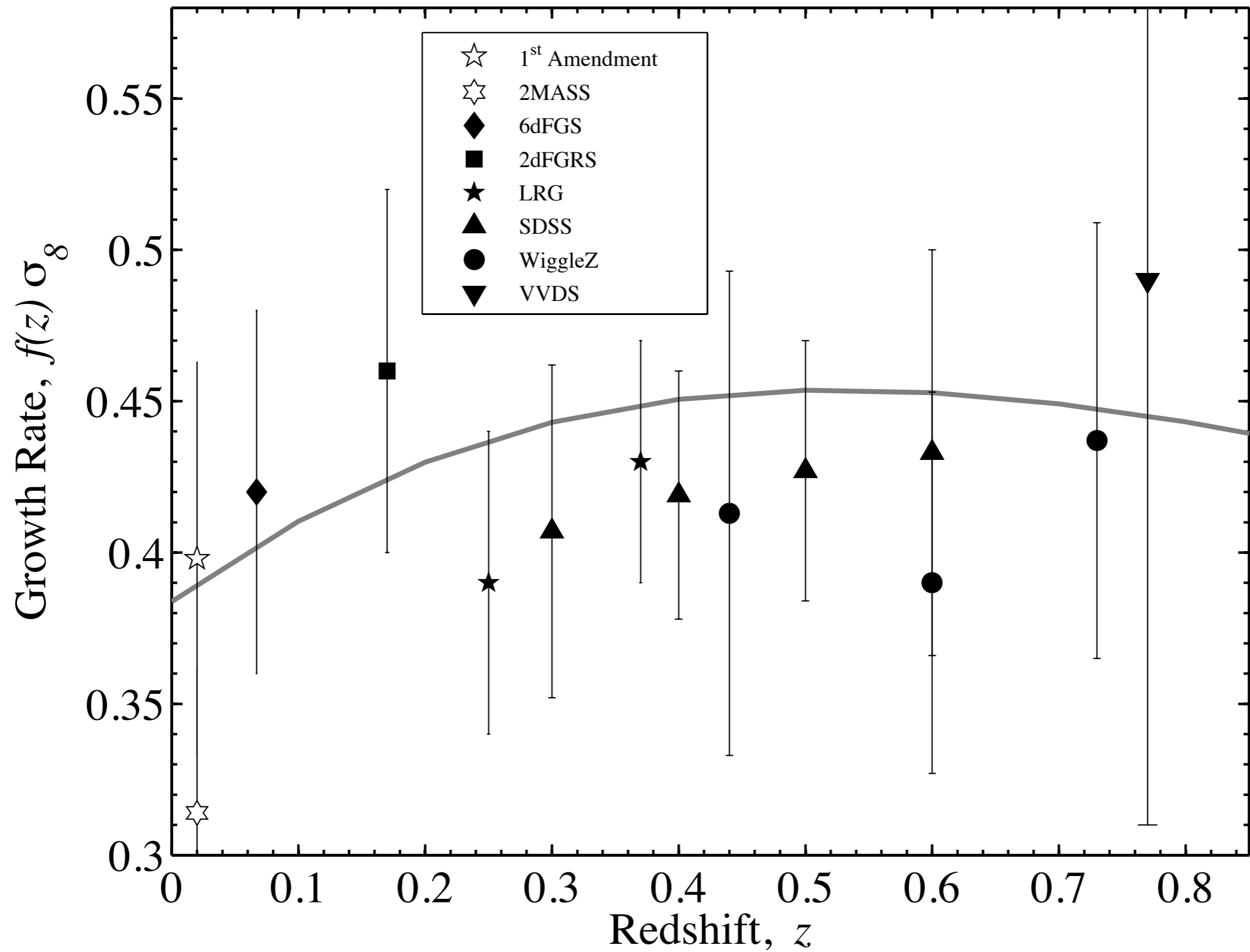
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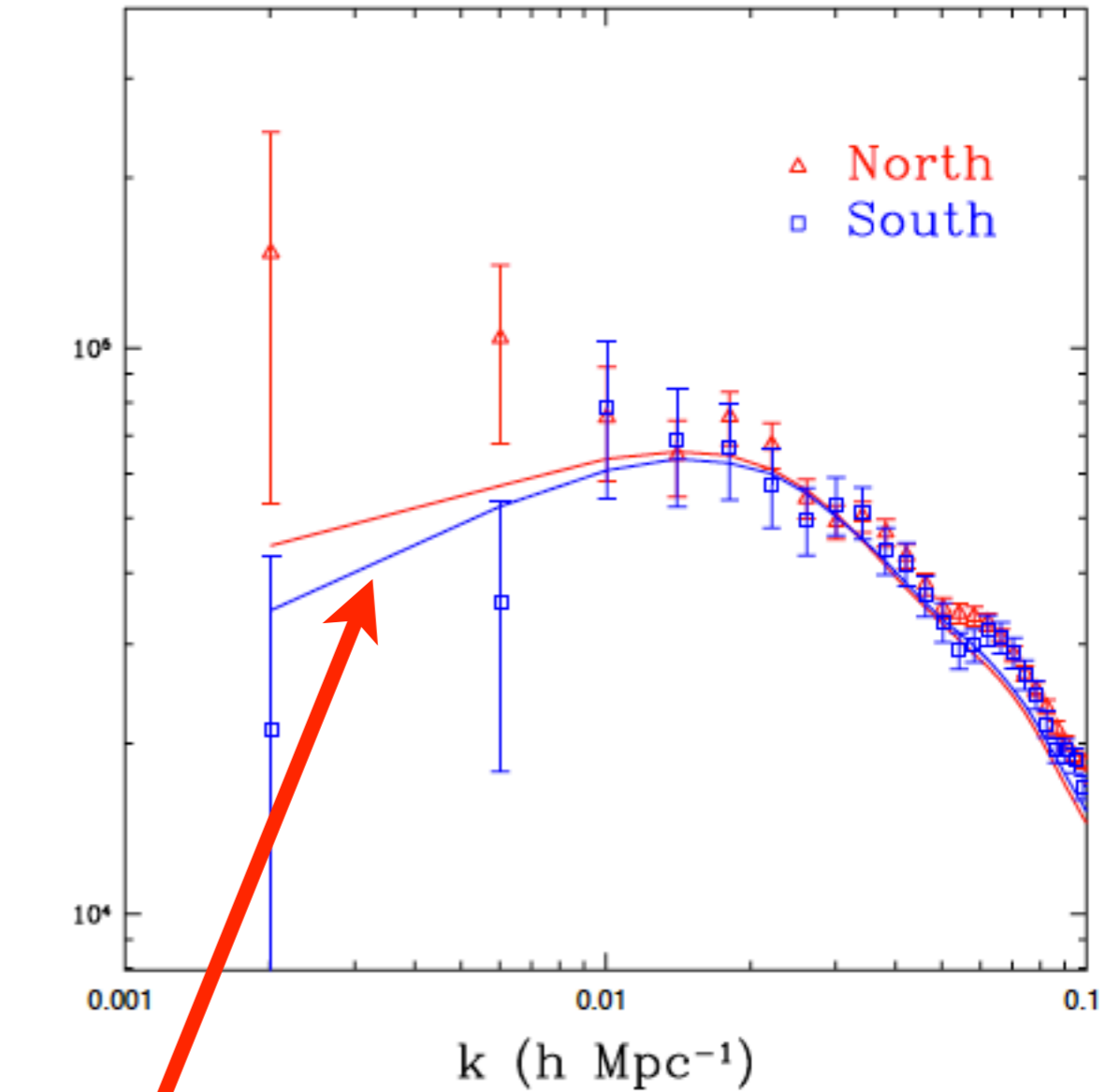
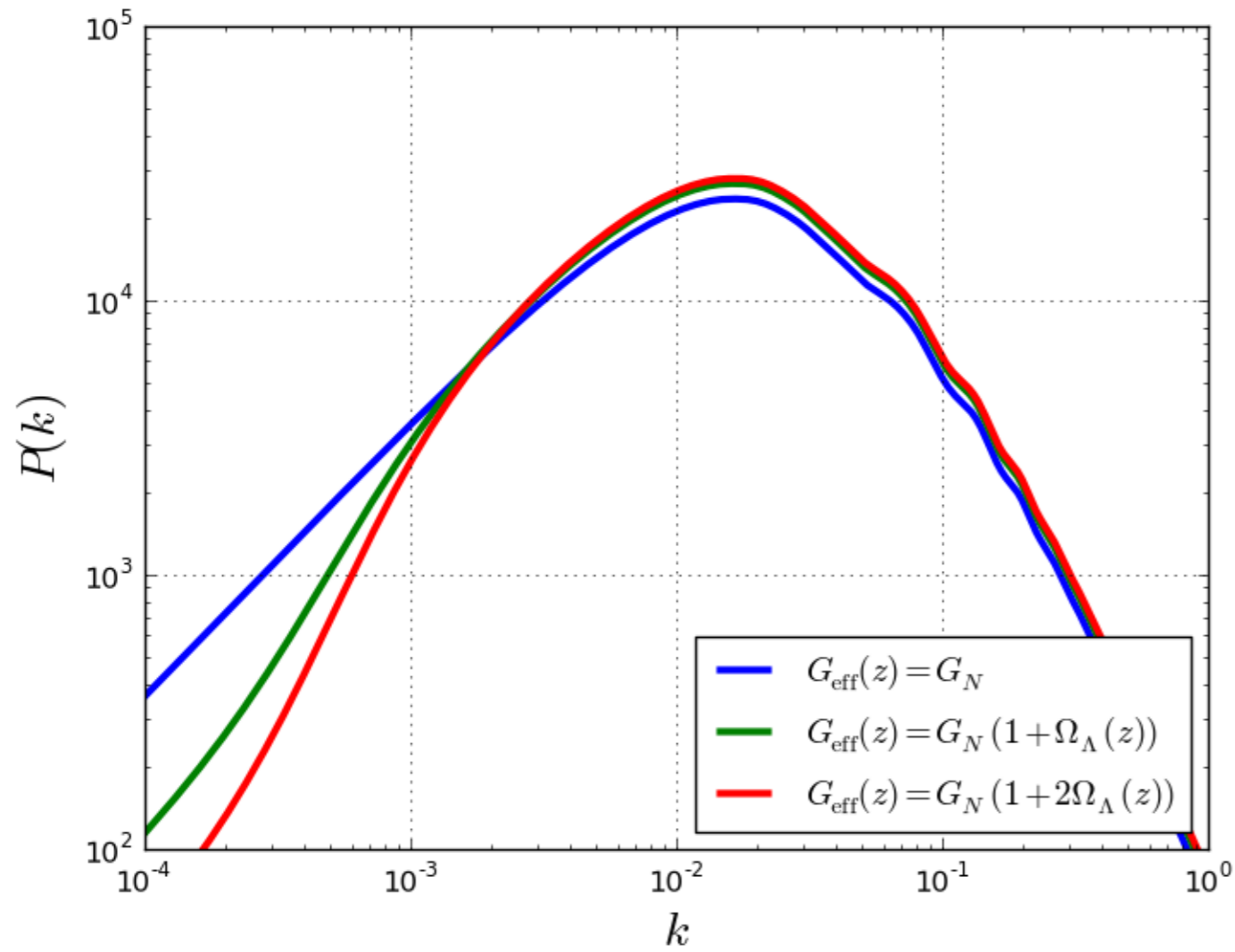
Redshift Space Distortions



Guzzo et al 2008

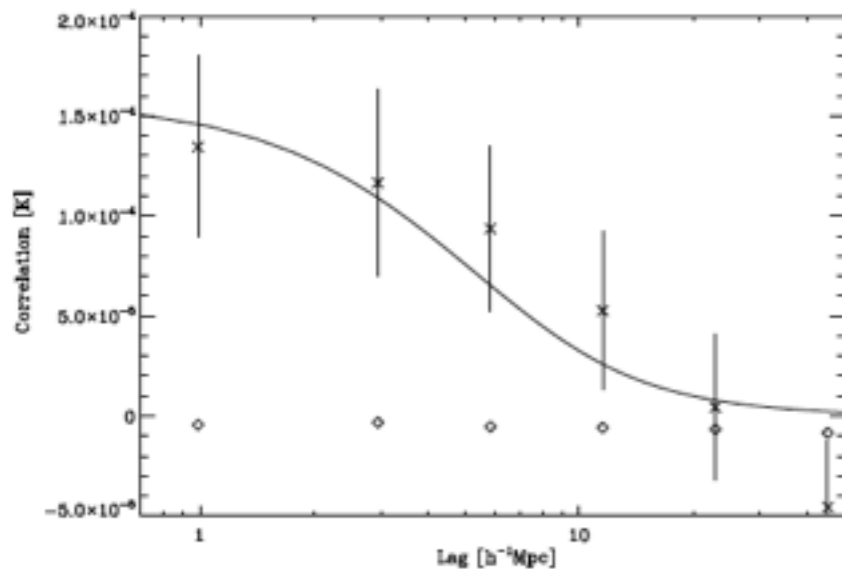
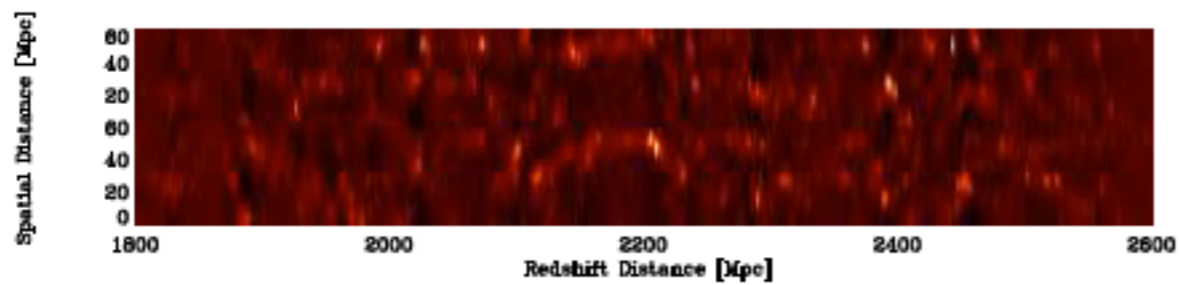
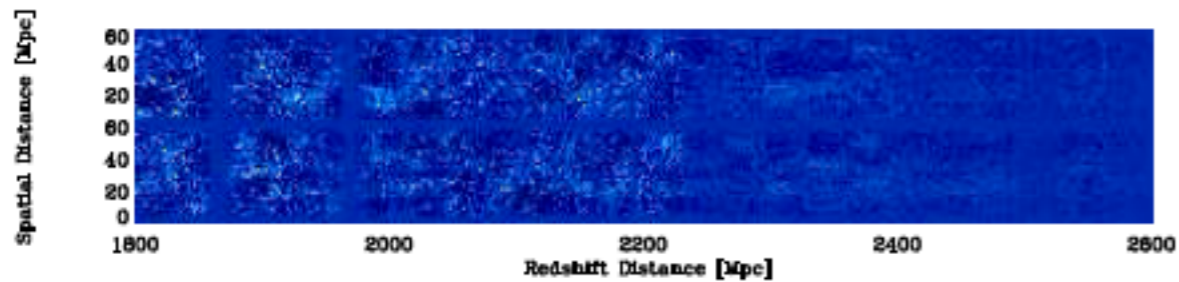


Ultra-large scale surveys

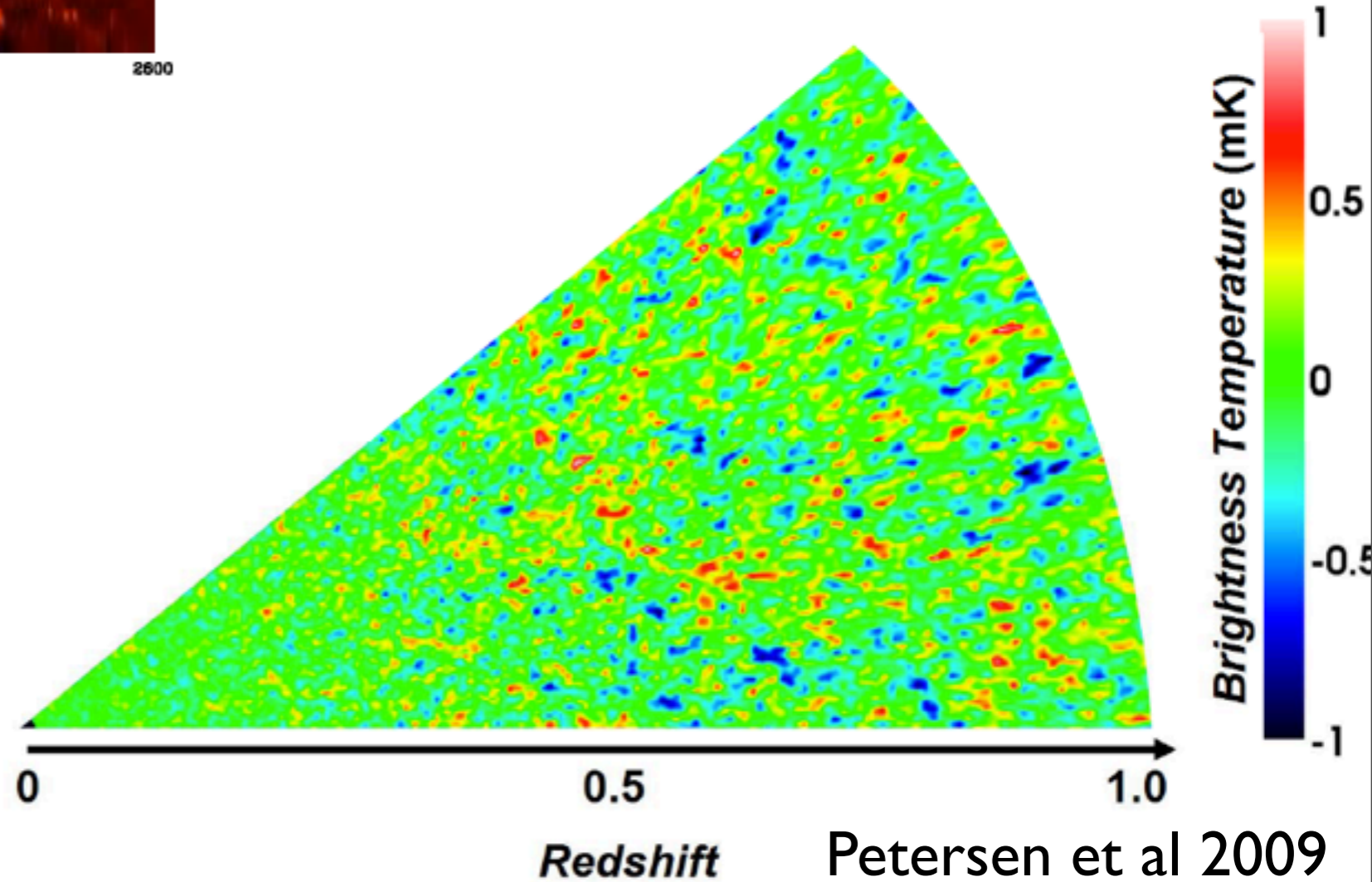


Ross et al (BOSS) 2012

Radio surveys and total intensity mapping

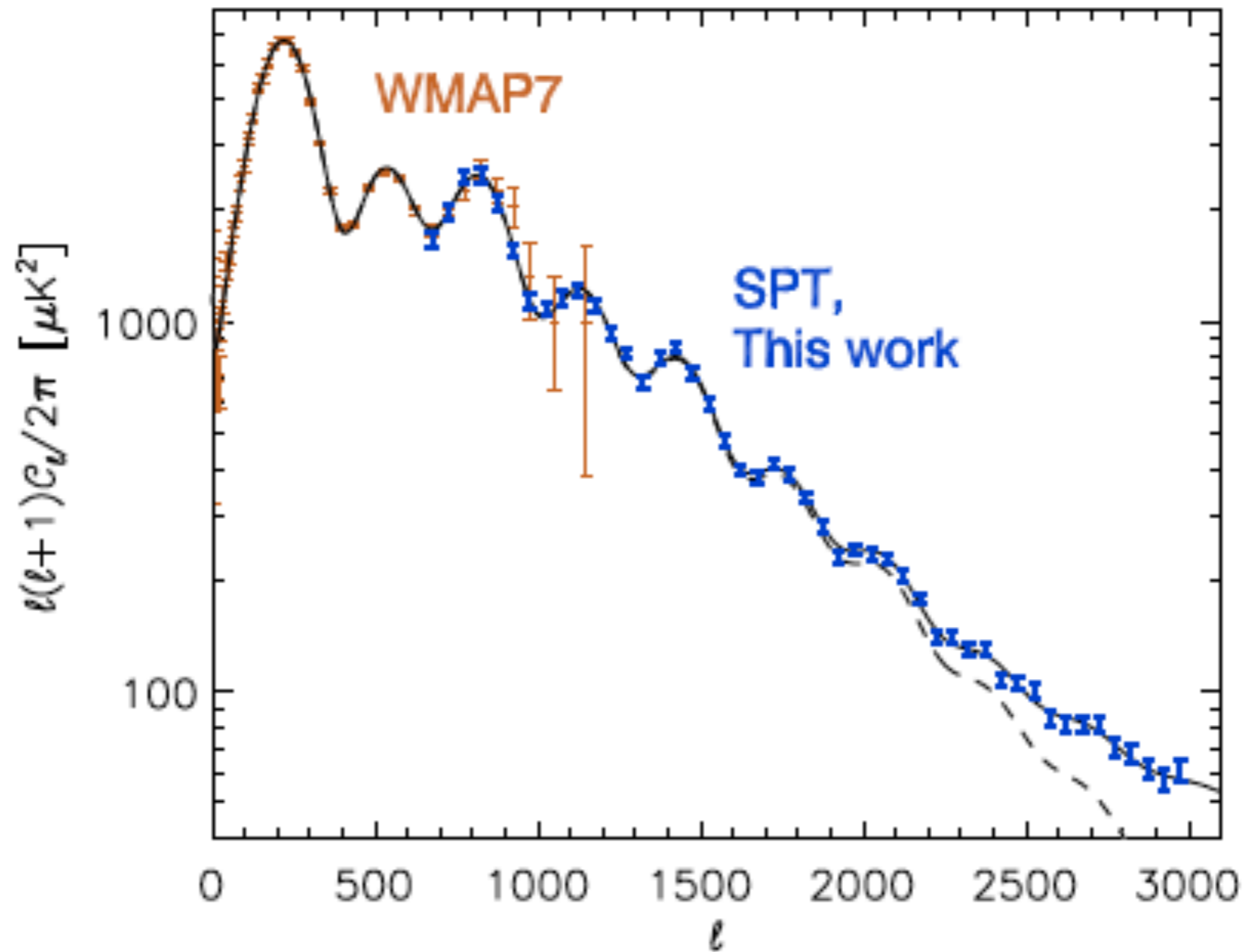


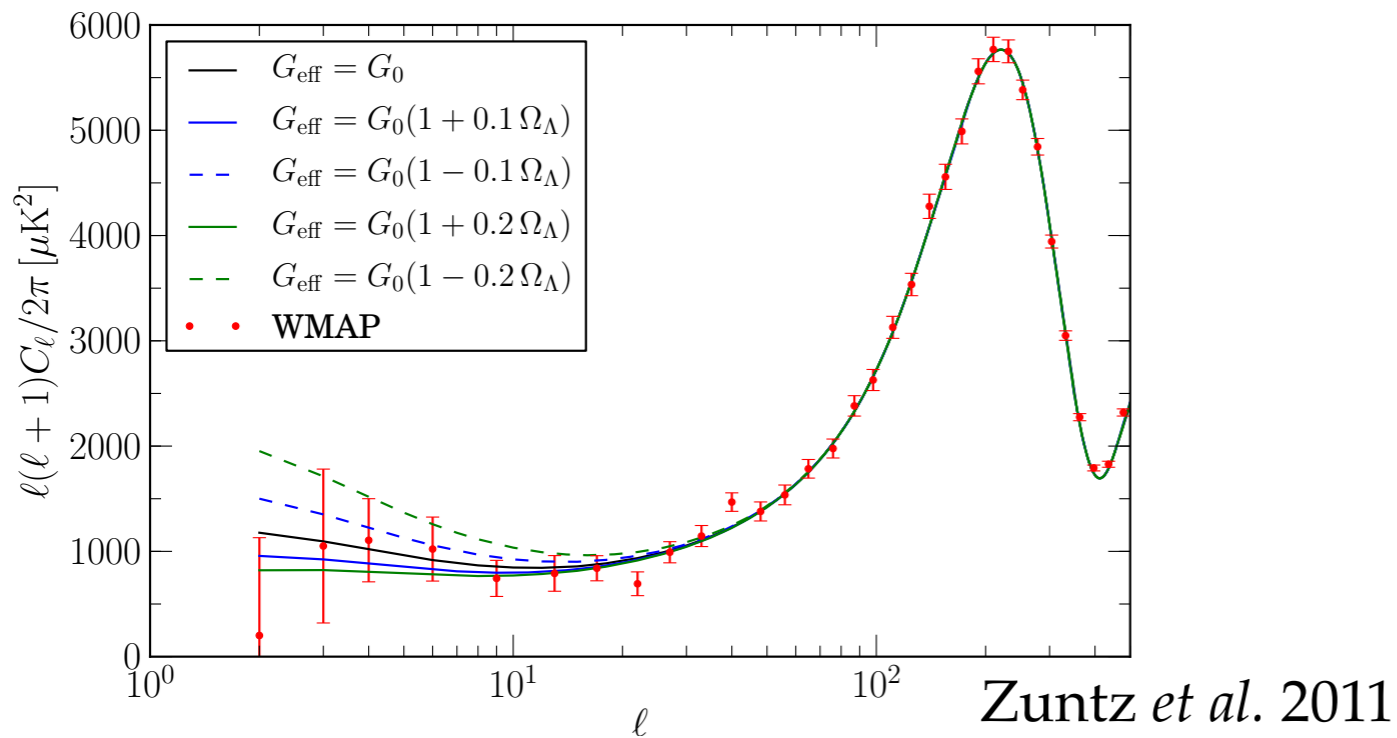
Chang et al 2010



Petersen et al 2009

The Angular Power Spectrum of the CMB



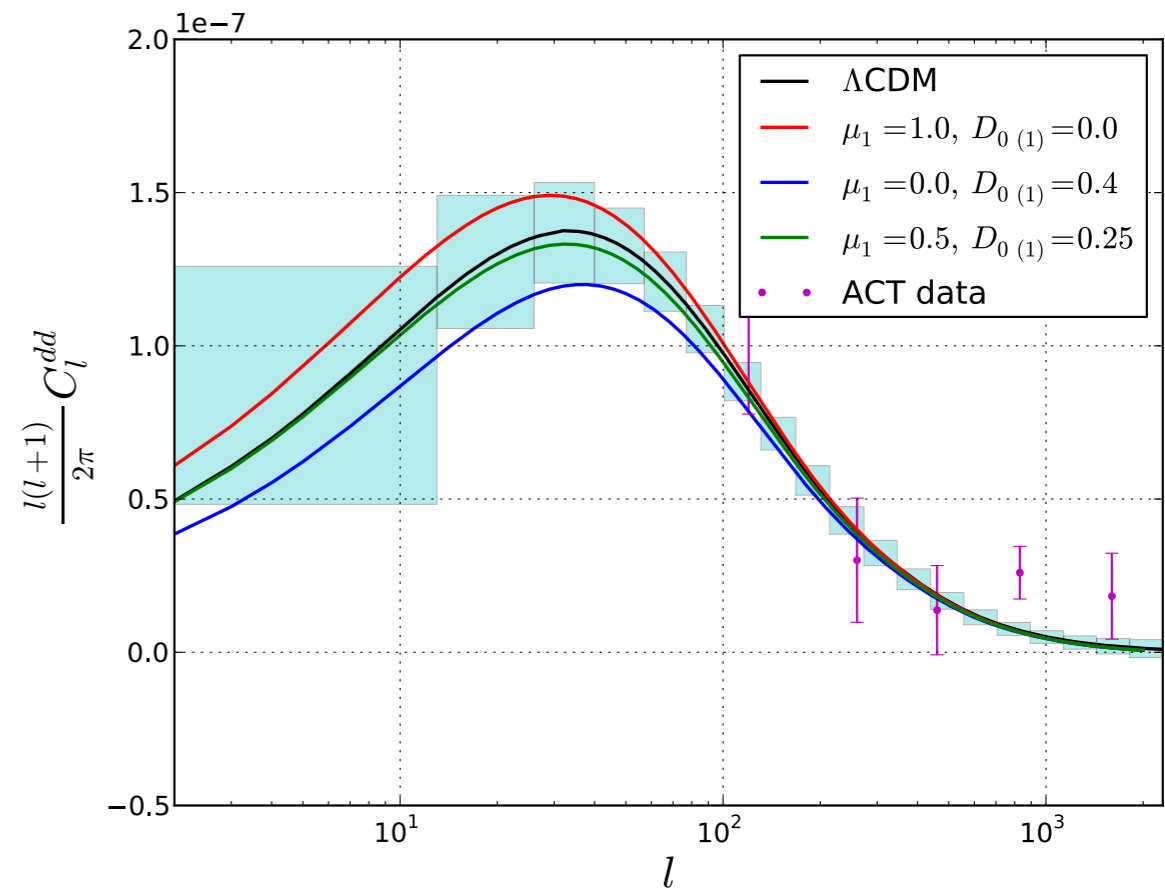


ISW- late time effects on large scales

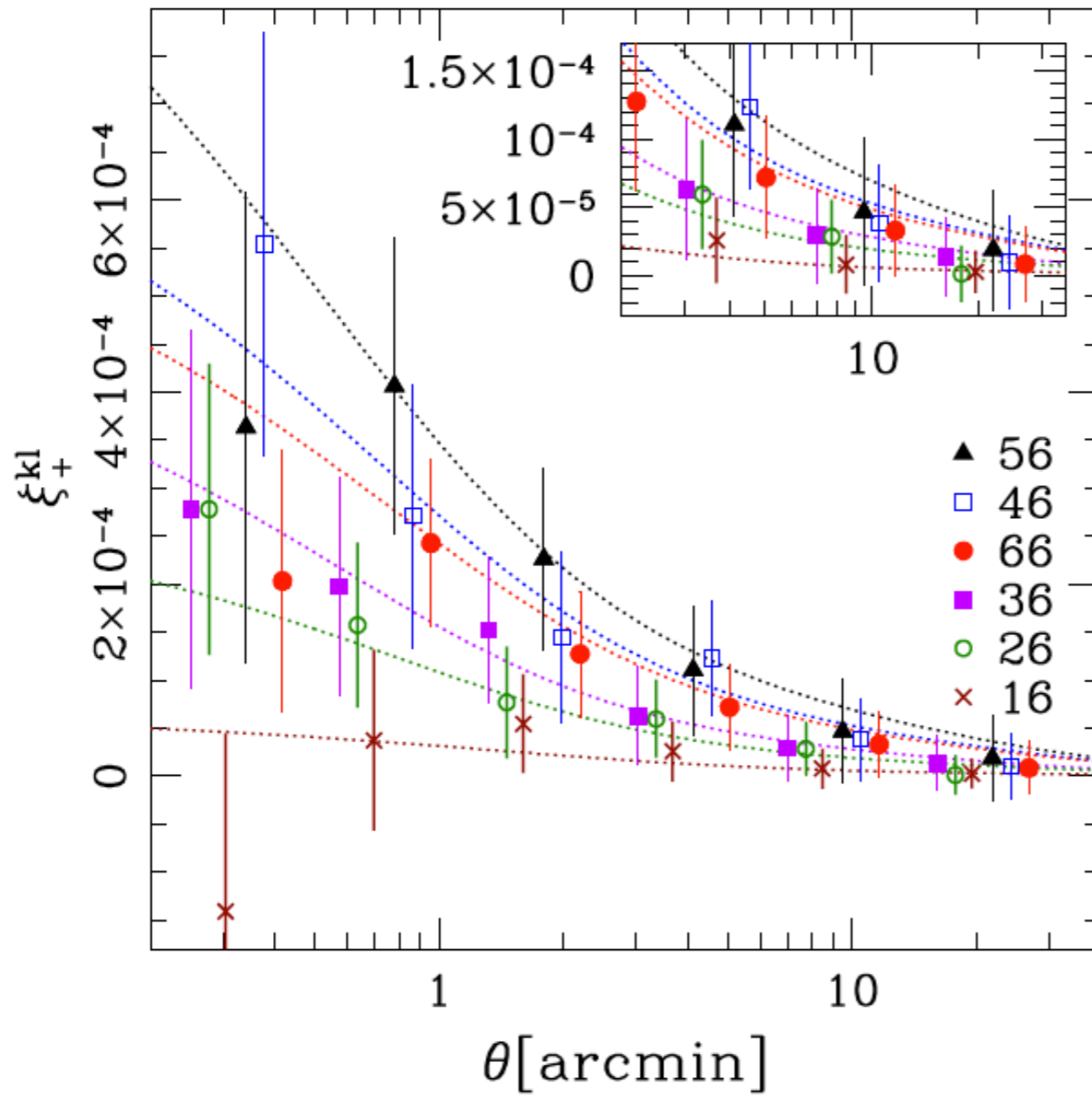
$$\propto \int (\dot{\Phi} + \dot{\Psi}) d\eta$$

Weak lensing on small scales

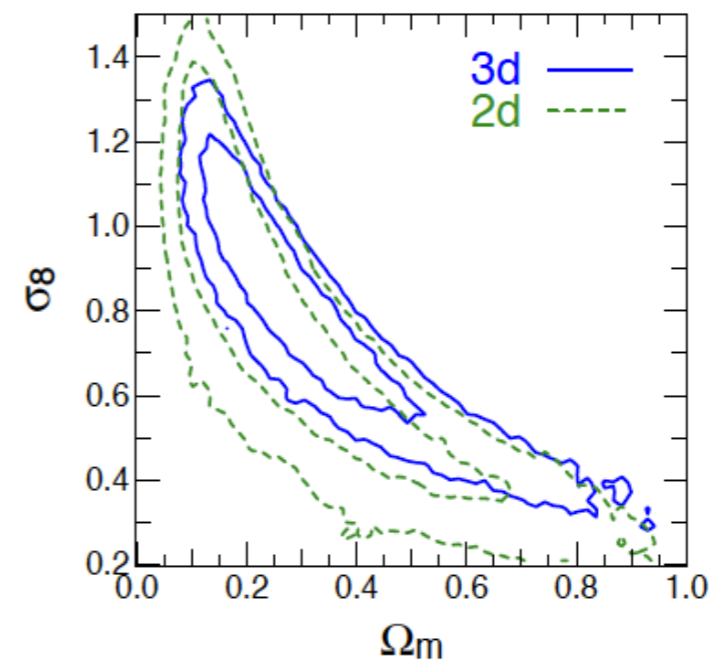
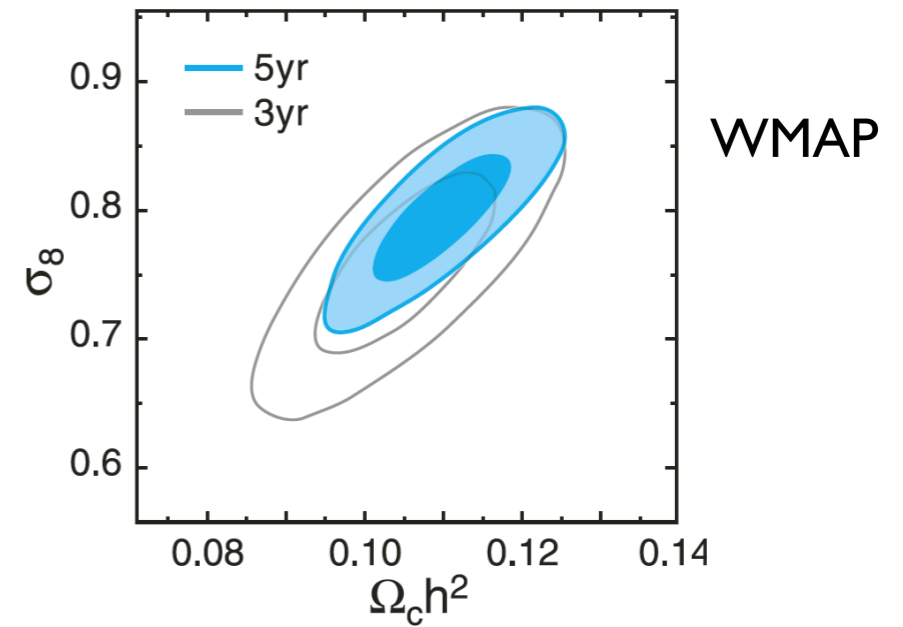
$$\propto \int (\Phi + \Psi) d\eta$$



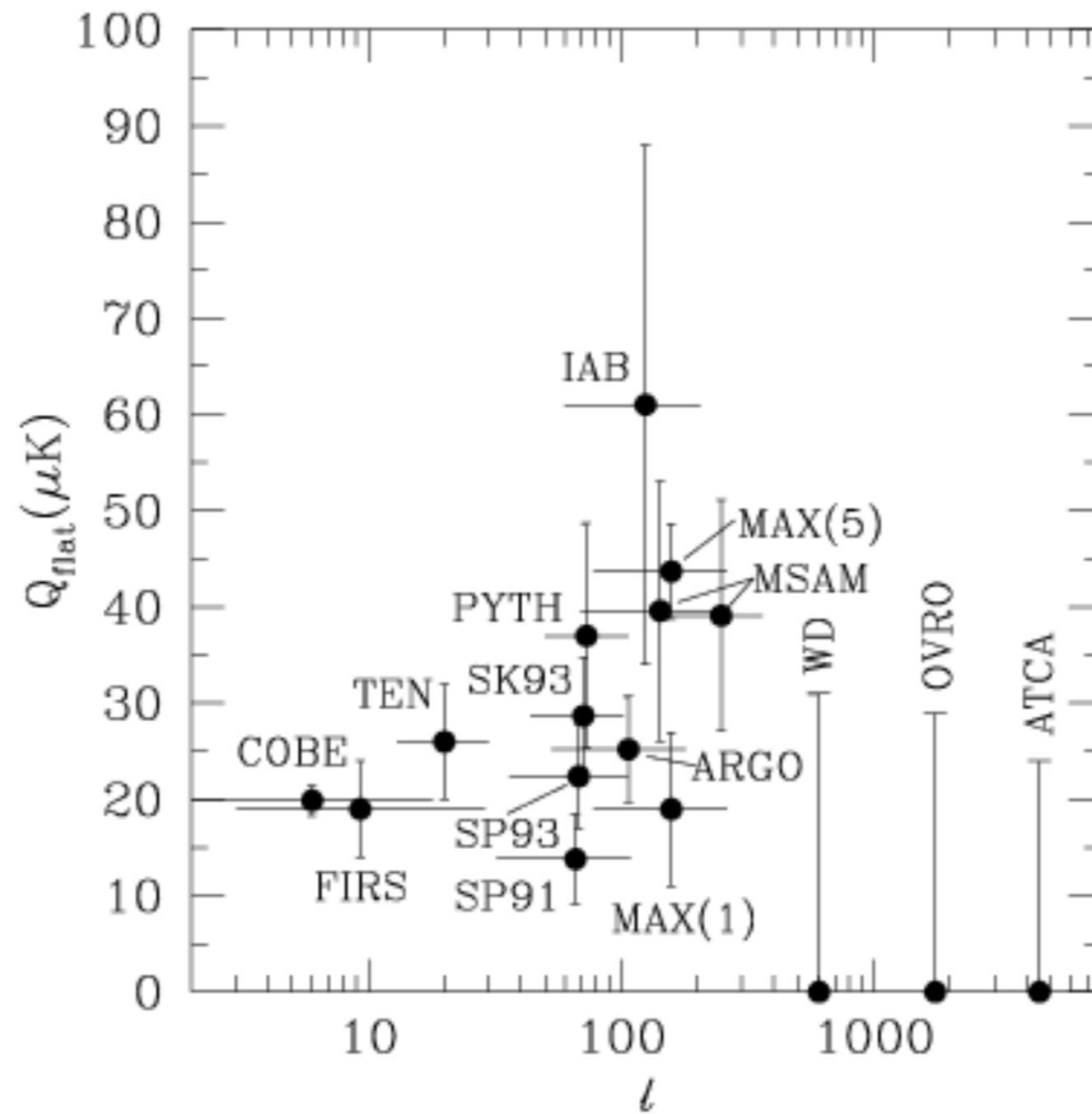
Weak Lensing: state of play



COSMOS: Schrabback et al 2010

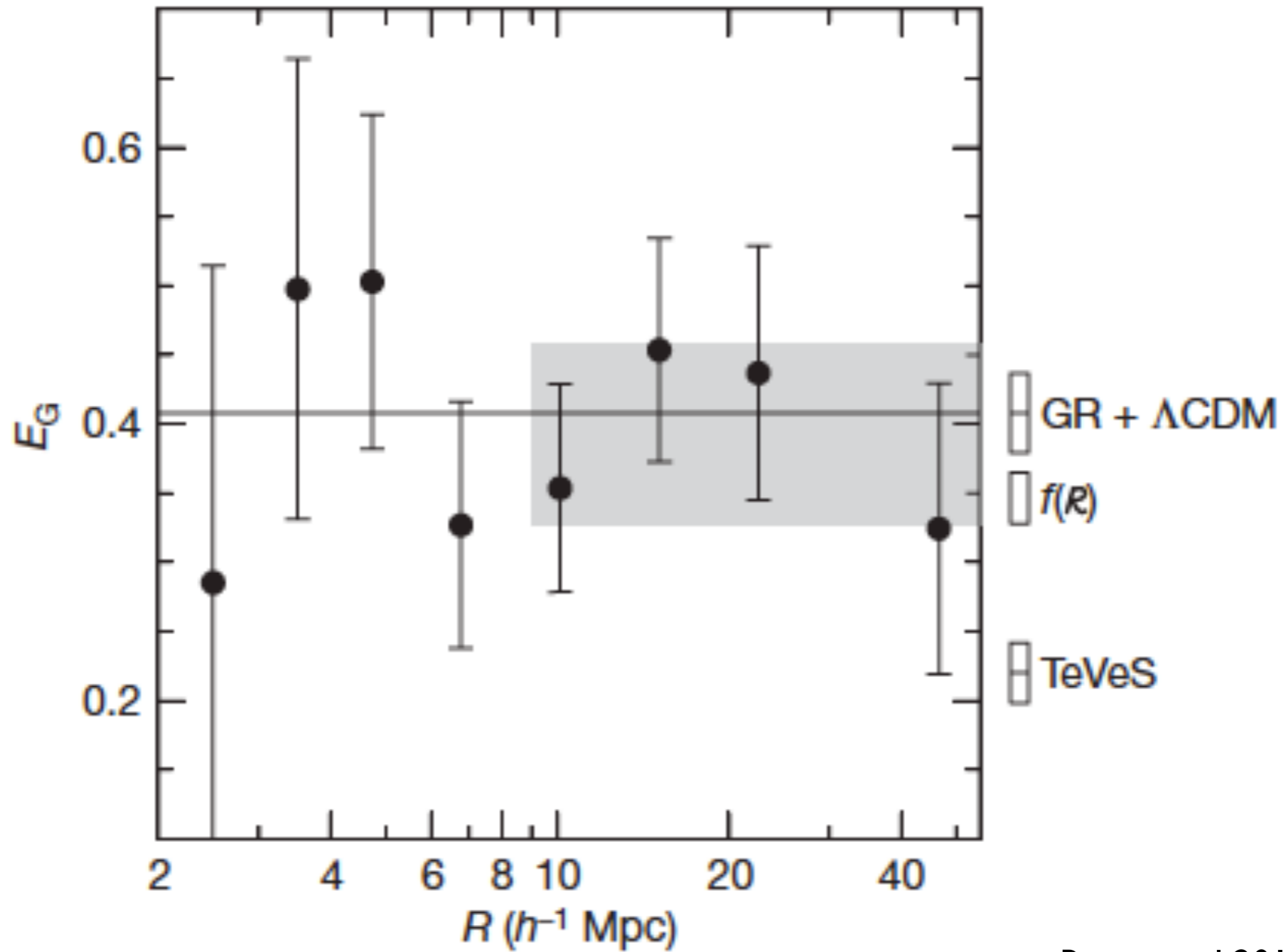


CMB circa 1995



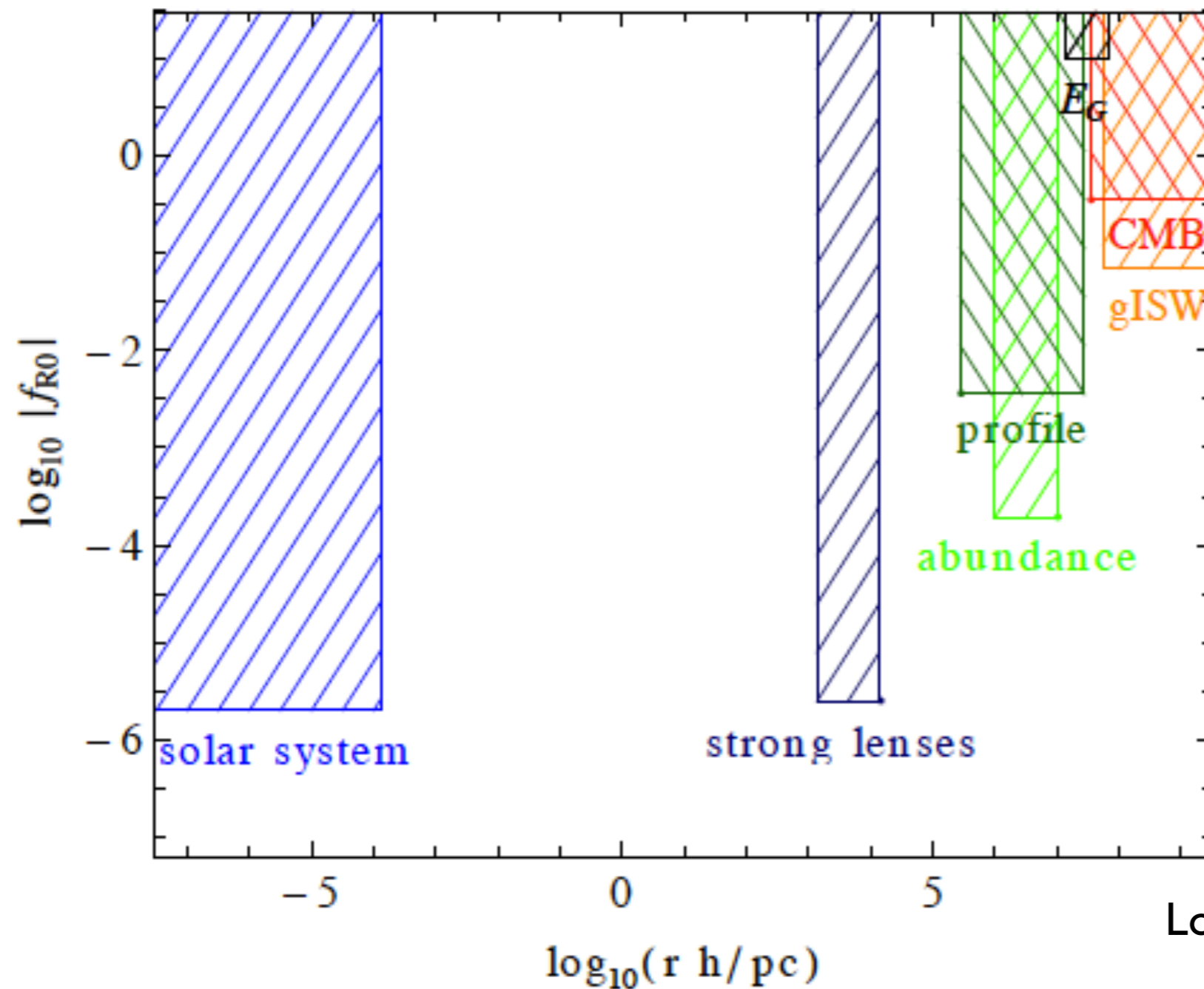
Scott 1995

Cross correlating data sets



Reyes et al 2010

Constraints on $F(R) = 1 + f(R)$



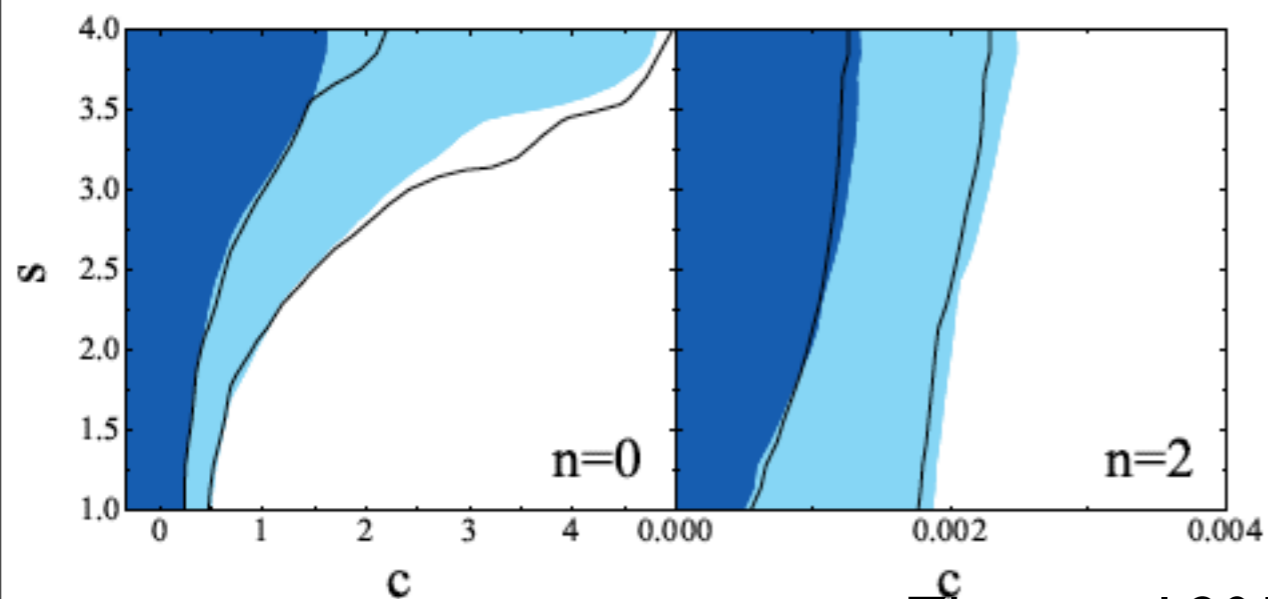
Lombriser et al 2012

Model Independent Constraints

$$-k^2\Phi = 4\pi G\mu a^2\rho\Delta$$

$$-k^2(\Phi + \Psi) = 8\pi G\Sigma a^2\rho\Delta$$

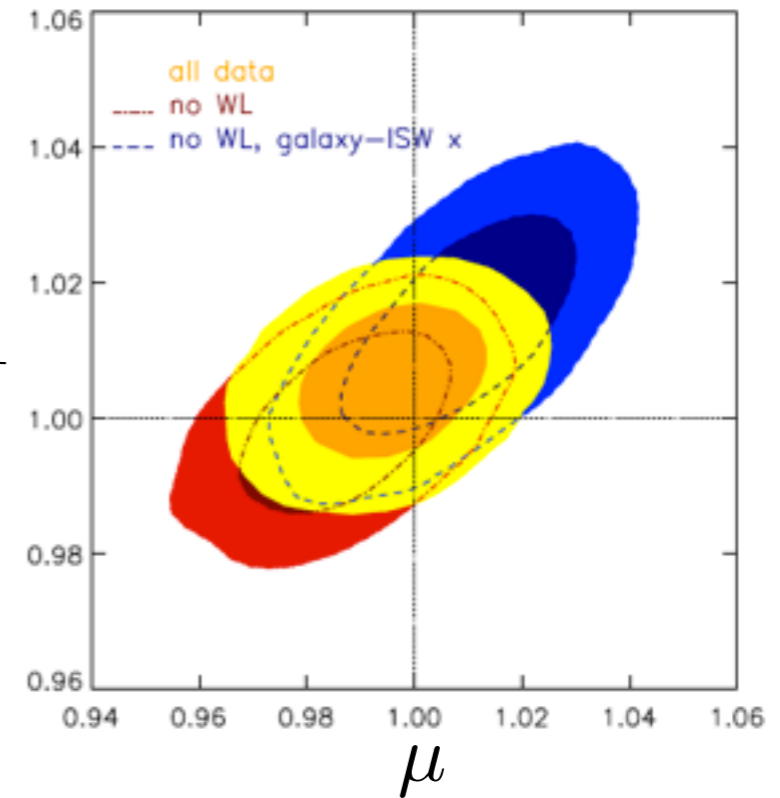
$$\mu = 1 + \frac{ca^s k_H^n}{1 + 3ca^s k_H^n}, \quad \Sigma = 1.$$



Zhao et al 2011

μ, Σ constant

$$\frac{2\Sigma}{\mu} - 1$$

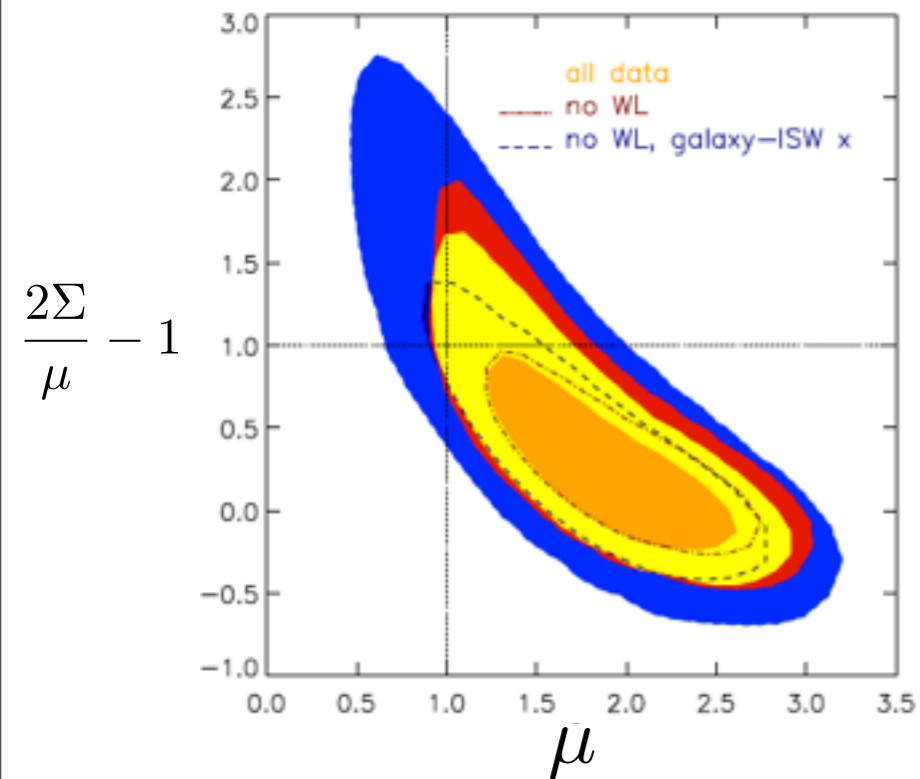


Bean and Tangmatitham 2011

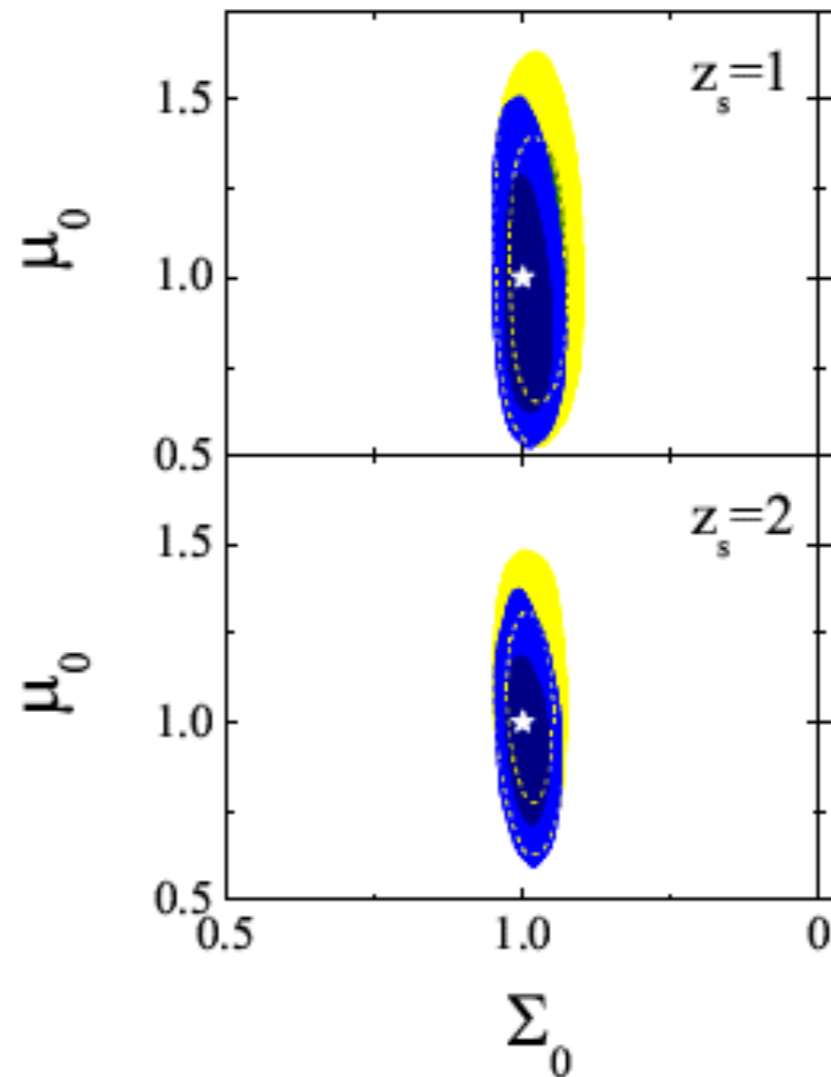
... are not that independent.

Model Independent Constraints

Time and spatial variations in μ , Σ



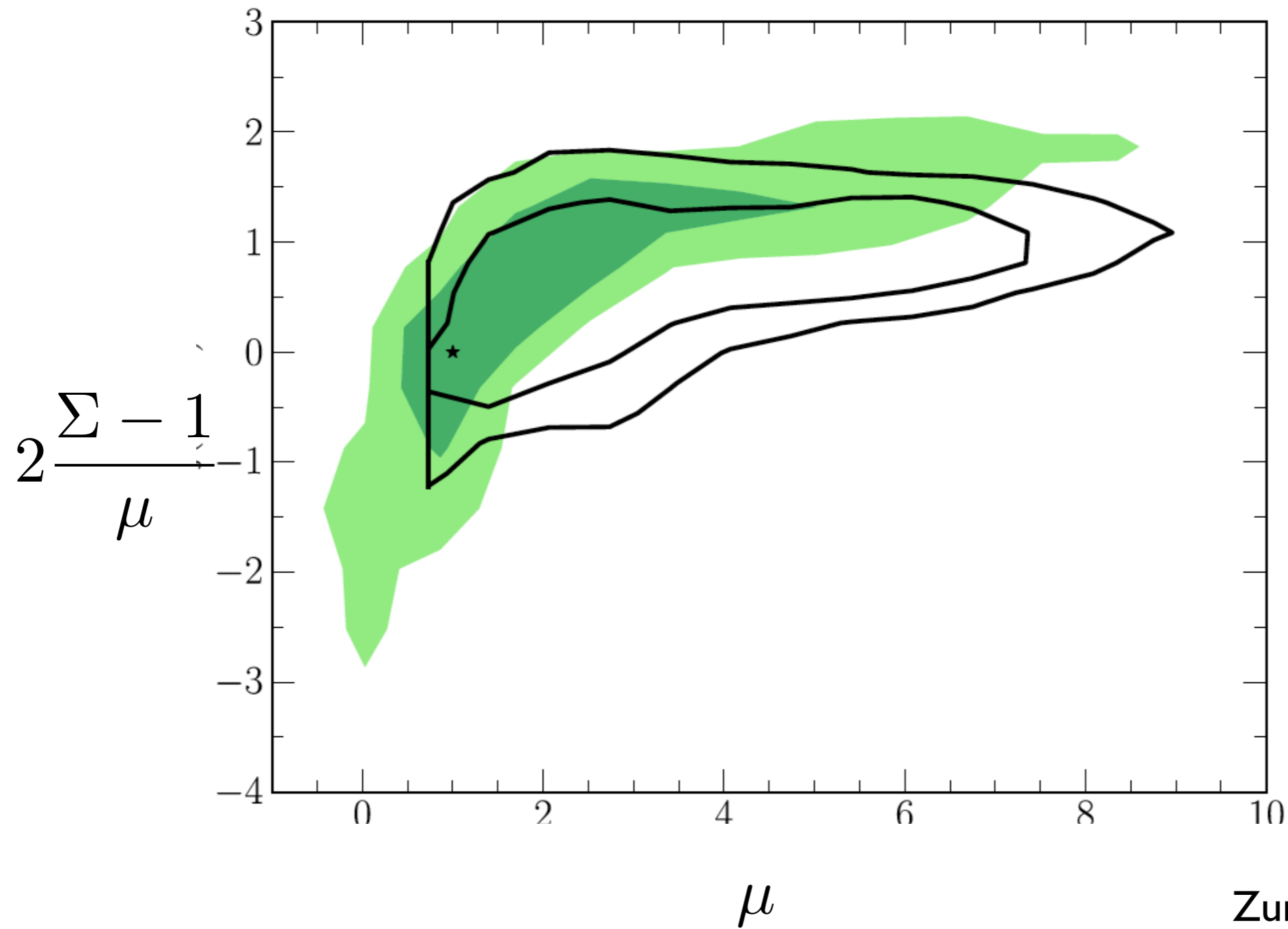
Bean and Tangmatitham 2011



Zhao et al 2011

Dependent on parametrization and combination of data sets.

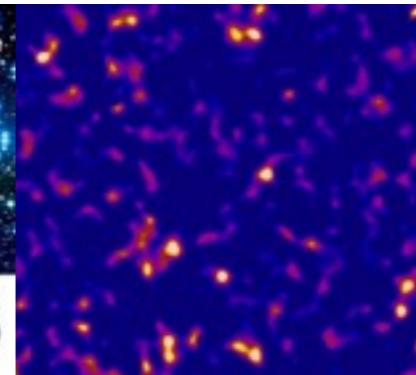
The State of Play



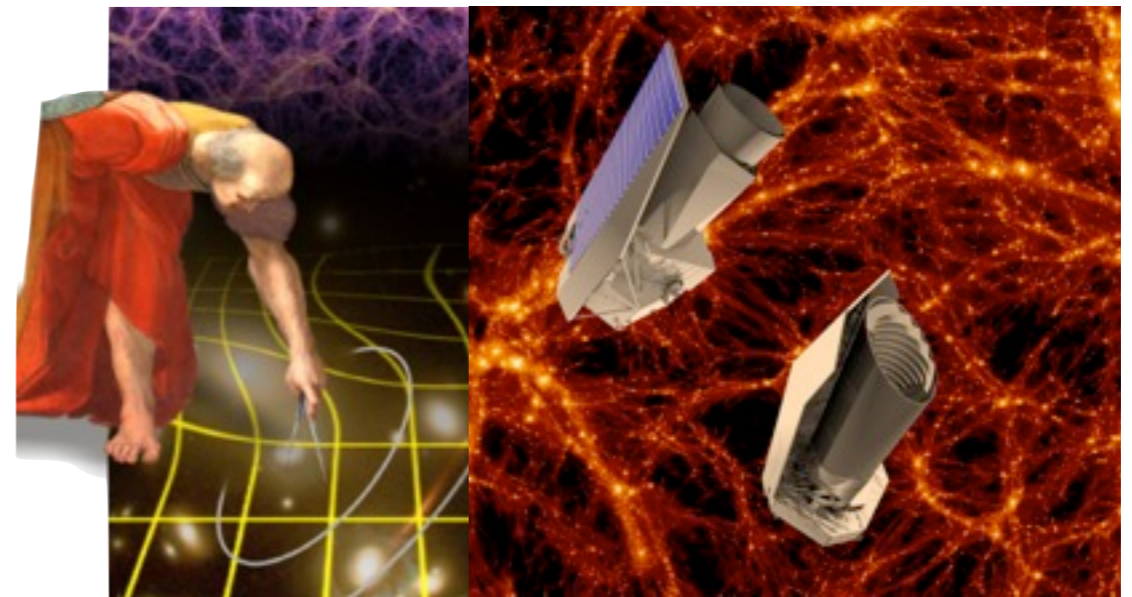
Zuntz et al 2011

The Future

Soon: ground-based galaxy weak lensing.

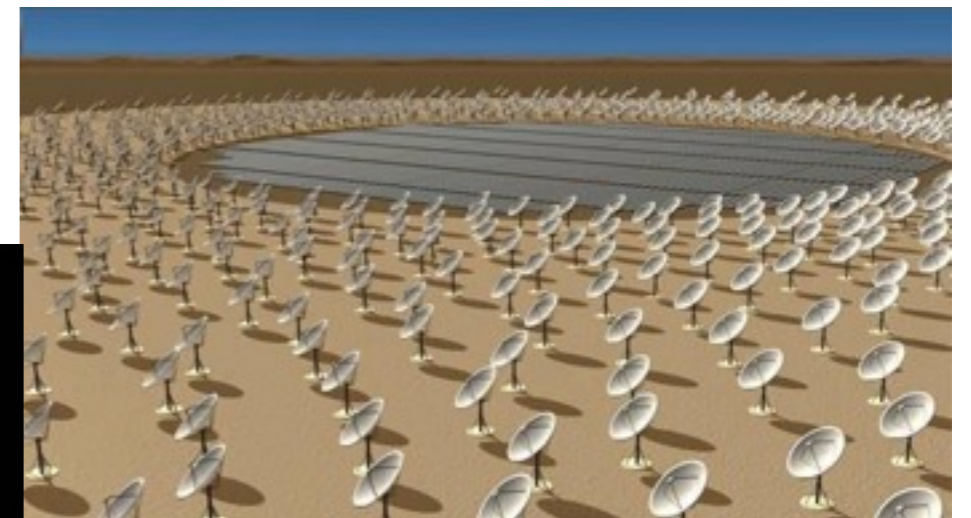


Soon-ish (2020): Euclid mission. Space-based weak lensing and redshift-space distortions.

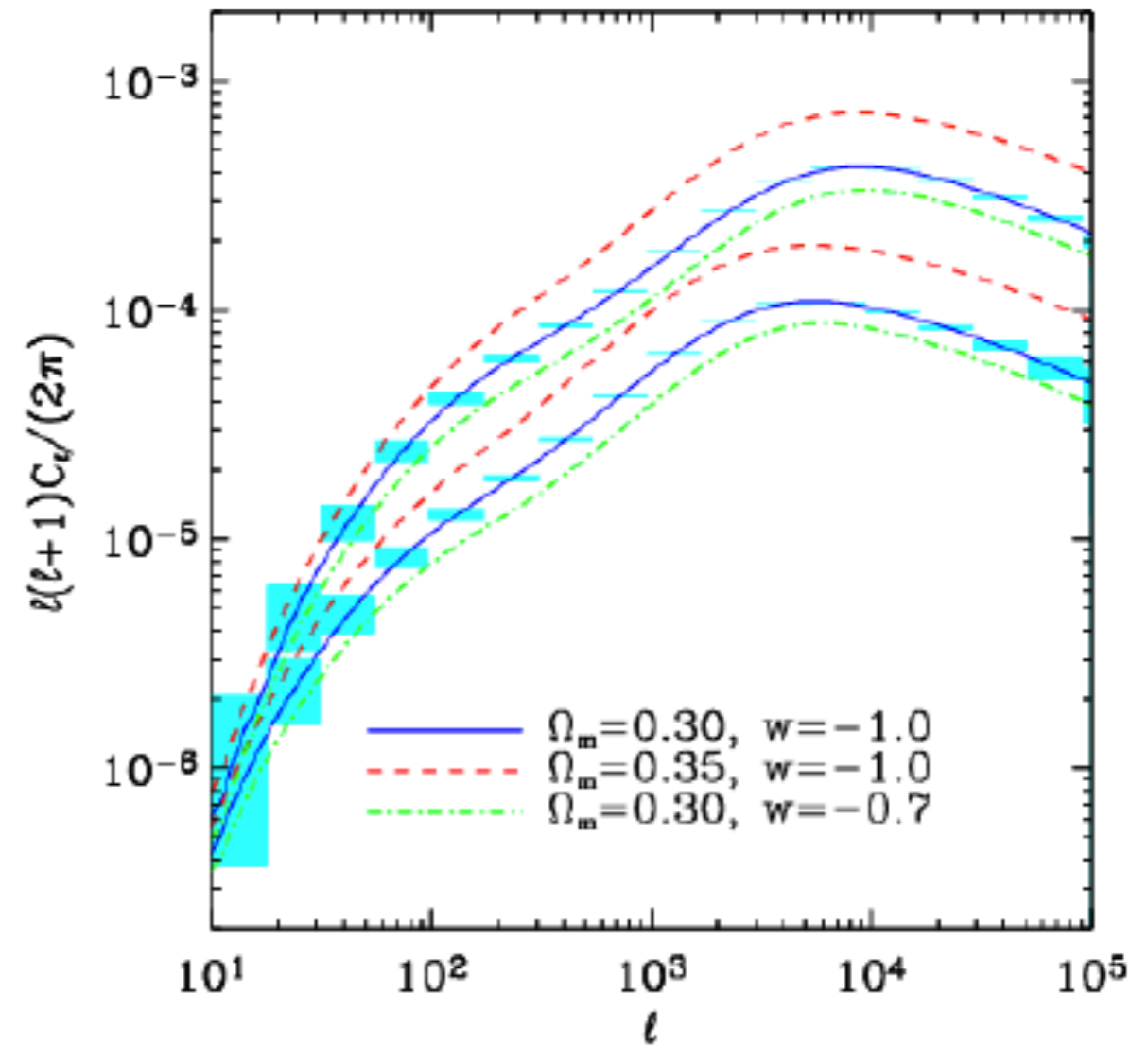
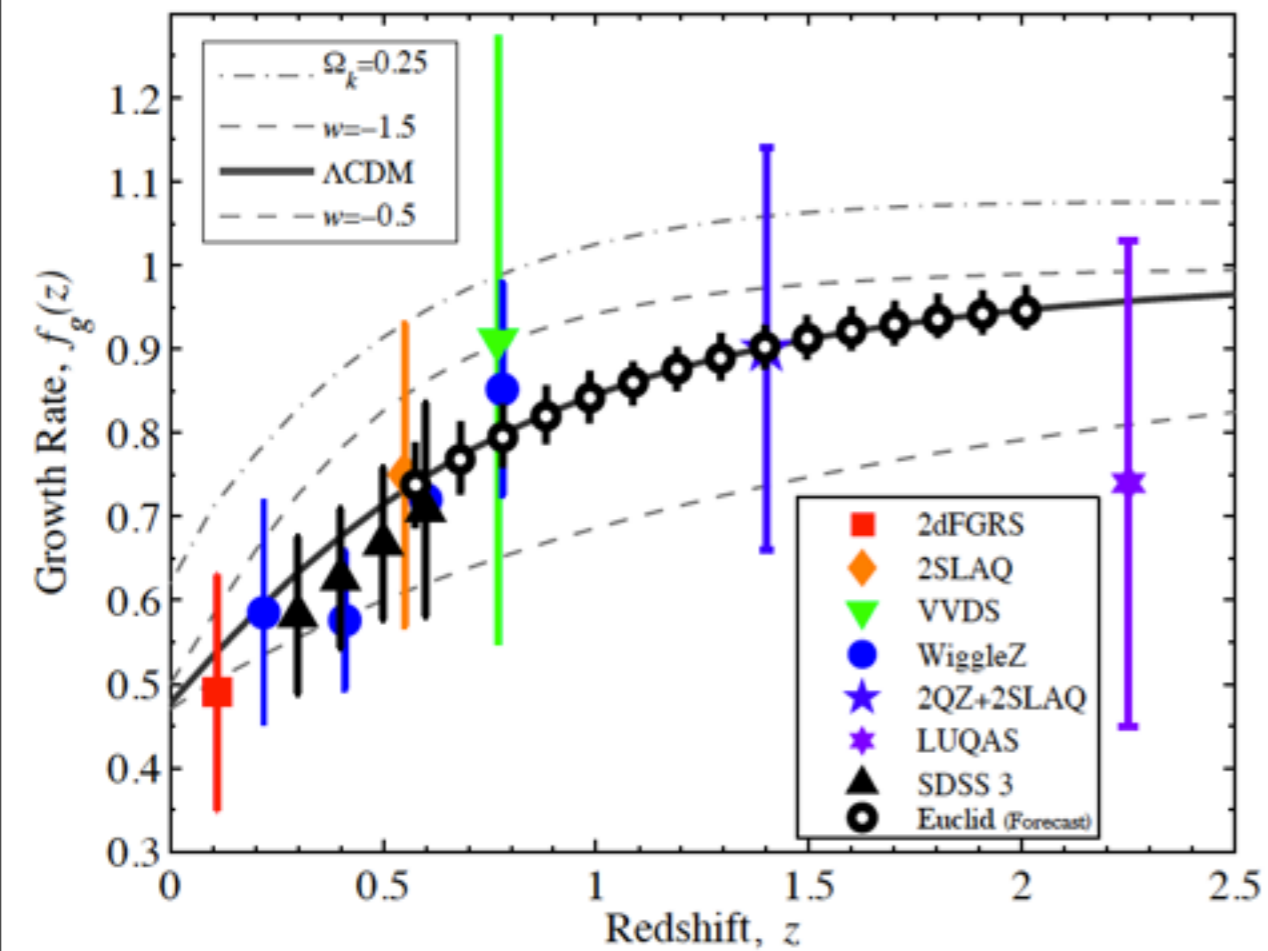


The future? The Square Kilometer Array (2020 onwards). An almighty survey of radio galaxies to high $z \Rightarrow$ RSDs,

peculiar velocities, BAO; also continuum mapping.



The Future



Summary

- The large scale structure of the Universe can be used to test gravity.
- There is a immense landscape of gravitational theories.
- We need a unified framework- “PPF”- for constraining such theories.
- We need to focus on linear scales (for now) although non-linear scales can be incredibly powerful.
- Signatures on large scales and growth rate.
- There are a plethora of new experiments to look forward to.

Collaborators

- Tessa Baker (Oxford)
- Tim Clifton (QMW)
- Tony Padilla (Nottingham)
- Constantinos Skordis (Nottingham)
- Mario Santos (IST/Lisbon)
- Joseph Zuntz (Oxford/UCL)