The development of the Z4c formulation for numerical relativity

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with
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arXiv:{0912.2920, 1010.0523, 1107.5539, 1111.2177, 1212.2901},

Outline

Background

Single compact objects

Compact binary evolutions

Conclusions

Maxwell I

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Maxwell equations:

$$\partial_t A_i = -\pi_i + \partial_i \phi \,,$$
$$\partial_t \pi_i = -\Delta A_i + \partial^j \partial_i A_j \,,$$
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- Lorentz and other choices?

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What does this look like in GR?

Analogy with General Relativity

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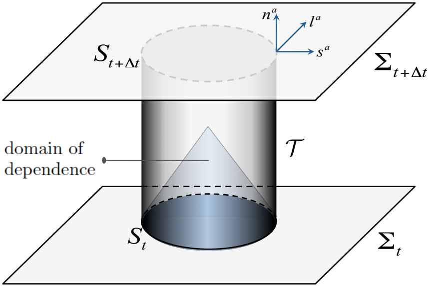
We've summarised the 'type' of equations to solve, but what about the domain?

Boundary conditions in numerical relativity

Applications in numerical relativity use a truncated domain, so boundary conditions needed for evolutions. Desirable properties?



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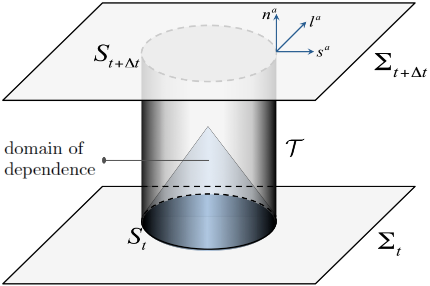

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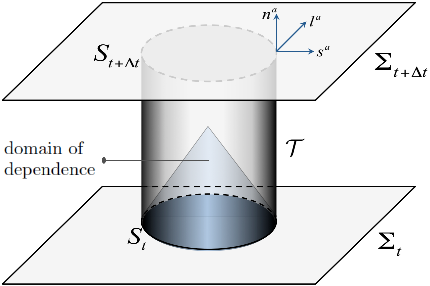

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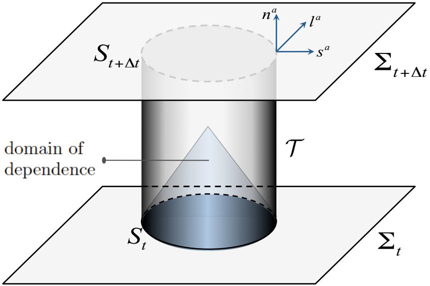

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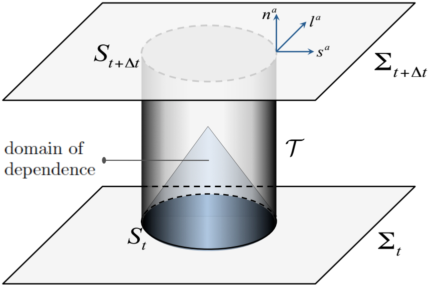

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To what extent is the two-body problem numerically solved?
Compare GHG and BSSNOK:

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Is there a formulation with the strengths of both?

Conformal decomposition of Z4

A natural choice seems to be a conformal decomposition (Bernuzzi & DH, '09) of the Z4 formulation (Bona et. al. 04).

$$\partial_t\chi = \frac{2}{3}\chi[\alpha(\hat{K}+2\Theta) - D_i\beta^i],$$

$$\partial_t\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} + \beta^k\tilde{\gamma}_{ij,k} + 2\tilde{\gamma}_{k(i}\beta^k_{,j)} - \frac{2}{3}\tilde{\gamma}_{ij}\beta^k_{,k},$$

$$\partial_t\hat{K} = -D^iD_i\alpha + \alpha[\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}(\hat{K}+2\Theta)^2] + 4\pi\alpha[S+\rho_{\mathrm{ADM}}] + \alpha\kappa_1(1-\kappa_2)\Theta + \beta^i\hat{K}_{,i}$$

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- Strongly hyperbolic with puncture gauge.
- Wave-like constraint subsystem.
- Constraint damping scheme.

Radiation controlling CPBCs

The trivial structure of the constraint subsystem was used to construct constraint absorbing preserving boundary conditions (Ruiz et. al. '10).

$$(l^a\partial_a)^L\Theta \hat{=} 0\,, \qquad (l^a\partial_a)^L\tilde{Z}^i \hat{=} 0\,.$$

With GW controlling condition:

$$(l^a\partial_a)^{L-1}\Psi_0 \hat{=} \partial_t^2 h_{\Psi_0}.$$

[Recall for incoming GWs, $\Psi_0 \sim \ddot{h}^+ + i\,\ddot{h}^\times$.]

Recipe for BCs implementation

Implementation?

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- Conformal decomposition.

$$\begin{aligned}
\partial_t\Theta \hat{=}\; & -\alpha\,(\partial_s\Theta + \tfrac{1}{r}\Theta) + \beta^i\,\partial_i\Theta\,,\\
\partial_t\tilde{A}_{ss} \hat{=}\; & -\alpha\,\chi\,\Big\{2\,\tilde{D}^i\tilde{A}_{is} - \frac{2}{3}\,\tilde{D}_s(2\,\hat{K}+\Theta) - \frac{2}{3}\,R_{ss}\\
& + \frac{2}{3}\chi\,\partial_s\left[\tilde{\Gamma}^s - (\tilde{\Gamma}_{\mathrm{d}})^s\right] - \frac{1}{3}\,\chi\,\partial_A\left[\tilde{\Gamma}^A - (\tilde{\Gamma}_{\mathrm{d}})^A\right]\\
& + \frac{1}{3}\,R_{qq} - 3\,\tilde{D}^i(\ln\chi)\tilde{A}_{is} - \kappa_1\,\left[\tilde{\Gamma}_s - (\tilde{\Gamma}_{\mathrm{d}})_s\right]\Big\}\\
& + \alpha\,\left[\tilde{A}_{ss}\,(\hat{K}+2\,\Theta) - 2\,\tilde{A}^i{}_s\,\tilde{A}_{is}\right] - \frac{2}{3}\,\chi\,D_sD_s\alpha\\
& + \frac{1}{3}\,\chi\,D^AD_A\alpha + \mathcal{L}_\beta\tilde{A}_{ss}\,,\\
\partial_t\tilde{A}^{\mathrm{TF}}_{AB} \hat{=}\; & -\alpha\Big[\tilde{D}_s\tilde{A}_{AB} - \tilde{D}_{(A}\tilde{A}_{B)s} + \frac{1}{2}\,\tilde{A}_{s(A}\tilde{D}_{B)}(\ln\chi)\\
& - \frac{1}{2}\,\tilde{A}_{AB}\tilde{D}_s(\ln\chi) + \tilde{A}^i{}_A\,\tilde{A}_{iB} - \frac{2}{3}\,\tilde{A}_{AB}\,(\hat{K}+2\Theta)\Big]^{\mathrm{TF}}\\
& - \chi\,D_AD_B^{\mathrm{TF}}\alpha + \mathcal{L}_\beta\tilde{A}^{\mathrm{TF}}_{AB}\,.
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Recipe for BCs implementation

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- Conformal decomposition.
- Populate ghostzones.

$$
\begin{aligned}
\partial_t\Theta \hat{=}& -\alpha\,(\partial_s\Theta + \tfrac{1}{r}\Theta) + \beta^i\,\partial_i\Theta\,,\\
\partial_t\tilde{A}_{ss} \hat{=}& -\alpha\,\chi\,\Big\{2\,\tilde{D}^i\tilde{A}_{is} - \frac{2}{3}\,\tilde{D}_s(2\,\hat{K}+\Theta) - \frac{2}{3}\,R_{ss}\\
&+\frac{2}{3}\chi\,\partial_s\left[\tilde{\Gamma}^s - (\tilde{\Gamma}_{\mathrm{d}})^s\right] - \frac{1}{3}\,\chi\,\partial_A\left[\tilde{\Gamma}^A - (\tilde{\Gamma}_{\mathrm{d}})^A\right]\\
&+\frac{1}{3}\,R_{qq} - 3\,\tilde{D}^i(\ln\chi)\tilde{A}_{is} - \kappa_1\,\left[\tilde{\Gamma}_s - (\tilde{\Gamma}_{\mathrm{d}})_s\right]\Big\}\\
&+\alpha\,\left[\tilde{A}_{ss}\,(\hat{K}+2\,\Theta) - 2\,\tilde{A}^i{}_s\,\tilde{A}_{is}\right] - \frac{2}{3}\,\chi\,D_sD_s\alpha\\
&+\frac{1}{3}\,\chi\,D^AD_A\alpha + \mathcal{L}_\beta\tilde{A}_{ss}\,,\\
\partial_t\tilde{A}^{\mathrm{TF}}_{AB} \hat{=}& -\alpha\Big[\tilde{D}_s\tilde{A}_{AB} - \tilde{D}_{(A}\tilde{A}_{B)s} + \frac{1}{2}\,\tilde{A}_{s(A}\tilde{D}_{B)}(\ln\chi)\\
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Recipe for BCs implementation

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Recipe for BCs implementation

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- Conformal decomposition.
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- Normal EoMs for metric.

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\end{aligned}
$$

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Recipe for BCs implementation

Implementation?

- Conformal decomposition.
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Do they work?

$$\partial_t\Theta \hat{=} -\alpha\,(\partial_s\Theta + \tfrac{1}{r}\Theta) + \beta^i\,\partial_i\Theta\,,$$

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Outline

Background

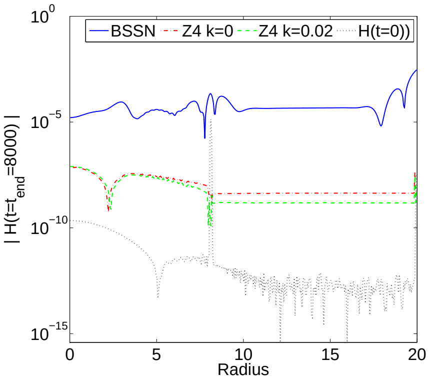
Single compact objects

Compact binary evolutions

Conclusions

Spherical symmetry I

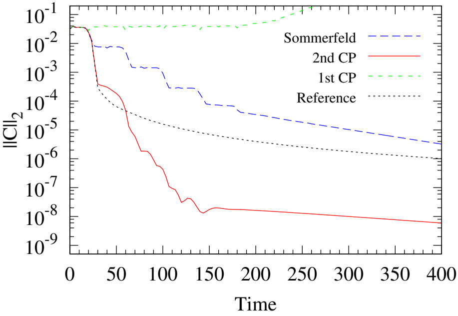
First applications:



- The zero speed mode in the BSSNOK constraint subsystem (Beyer & Sarbach '02, Gundlach & M Garcia '04) causes Hamiltonian constraint violation. Z4c does not suffer from the problem.

Spherical symmetry II

CPBCs with Spherical numerics:



Spherical symmetry II

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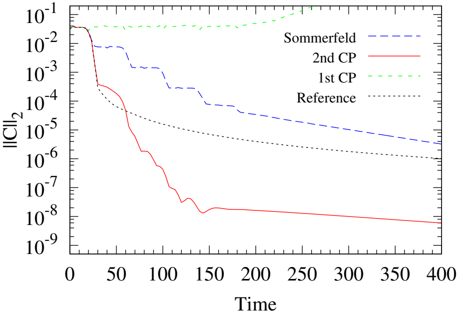

- A detailed examination of the constraint damping scheme was presented in spherical symmetry (Weyhausen et. al. '11).

Spherical symmetry II

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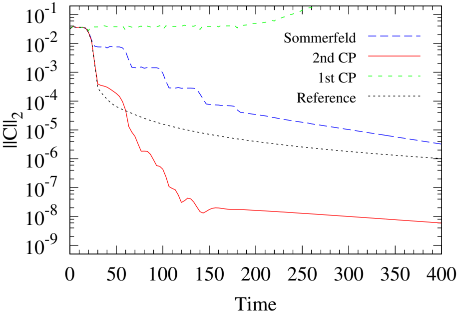
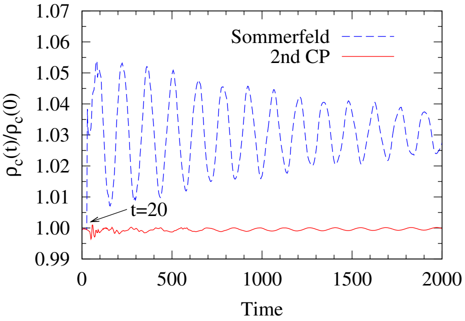


- A detailed examination of the constraint damping scheme was presented in spherical symmetry (Weyhausen et. al. '11).
- Evolutions of binary black holes were presented using a variation of the conformal decomposition called CCZ4 (Alic et. al. '11).

Boundary kick in spherical symmetry

Is it really worth making better boundary conditions?



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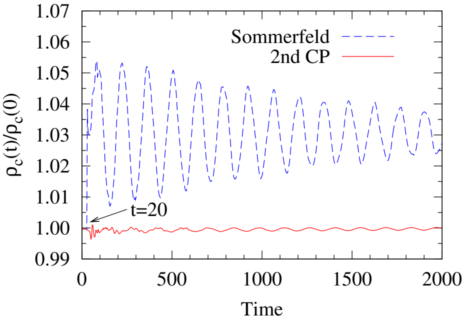

- Consider central density in evolution of single TOV star.

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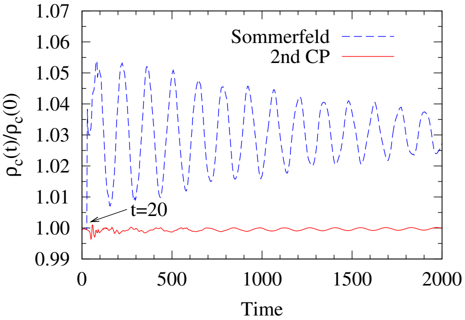

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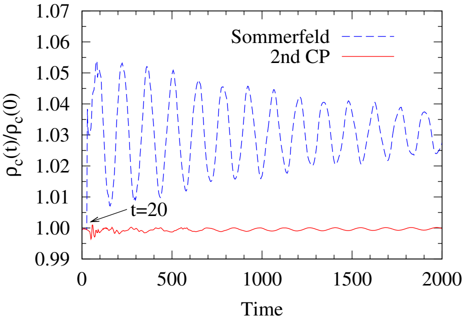
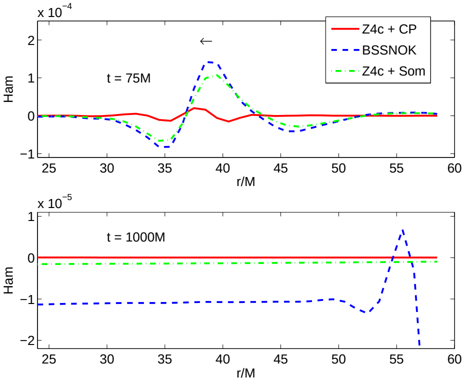


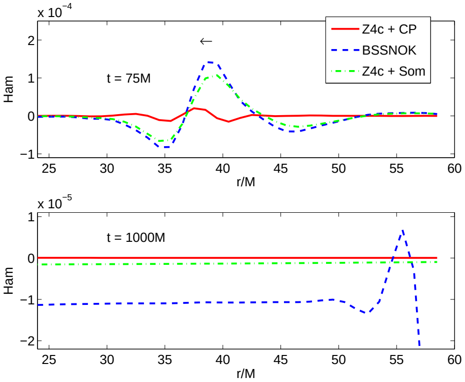
- Consider central density in evolution of single TOV star.
- Sommerfeld gives perturbation. Does not converge away with resolution. BCs can effect physics.
- Example maximises effect. Boundaries close.

Single puncture black hole



Single puncture black hole

- Incoming violation as Sommerfeld boundary causally connected.



Single puncture black hole

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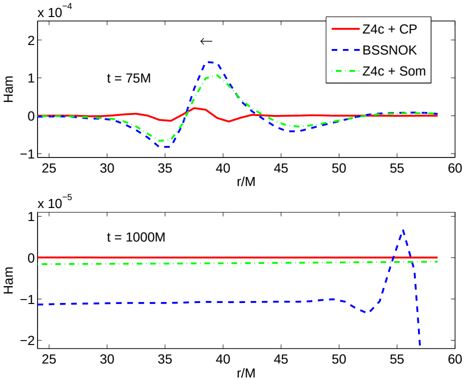
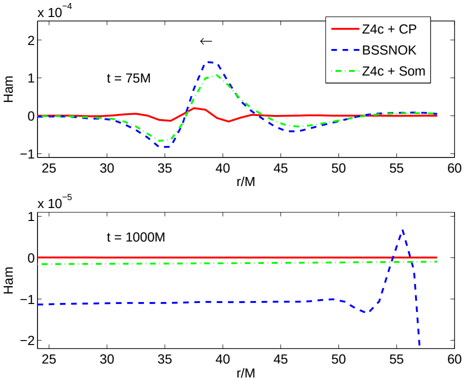
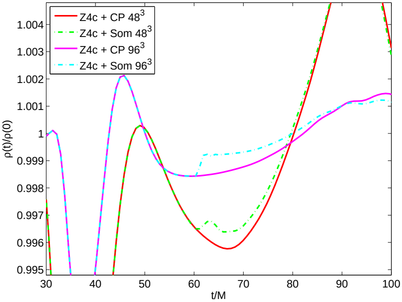


Single puncture black hole

- Incoming violation as Sommerfeld boundary causally connected.
- CPBCs reduce violation.
- At late times BSSNOK has large violation sitting at boundary.

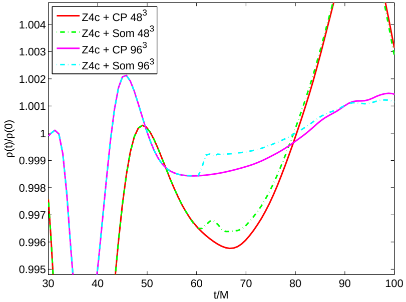


Boundary kick in 3D



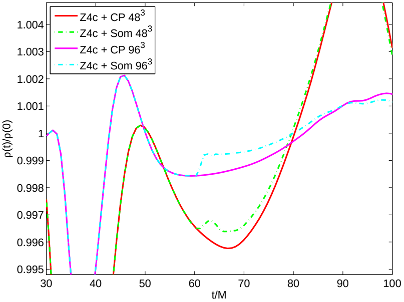
Boundary kick in 3D

- Incoming violation kicks star like in spherical symmetry.



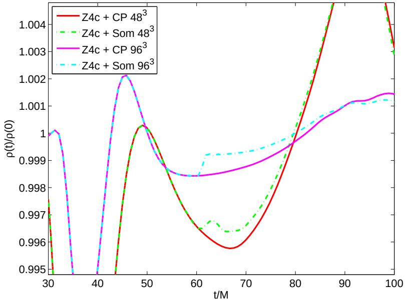
Boundary kick in 3D

- Incoming violation kicks star like in spherical symmetry.
- Effect tiny compared to that in spherical symmetry.



Boundary kick in 3D

- Incoming violation kicks star like in spherical symmetry.
- Effect tiny compared to that in spherical symmetry.
- Does not converge away.



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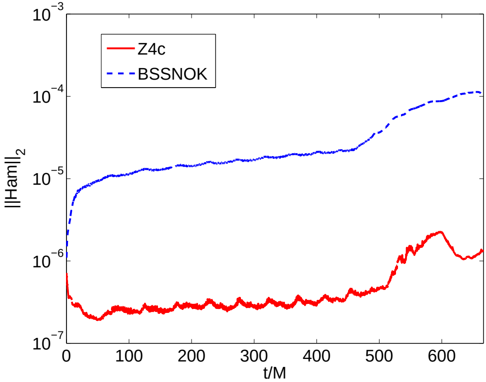
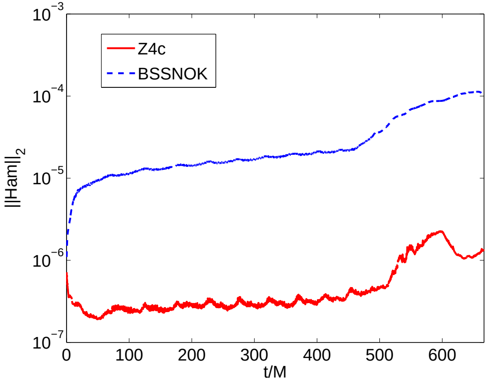
Binary Neutron Stars I



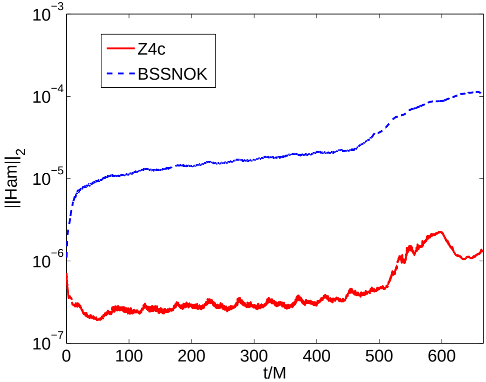
Binary Neutron Stars I

- Hamiltonian constraint violation reduced in Z4c data.



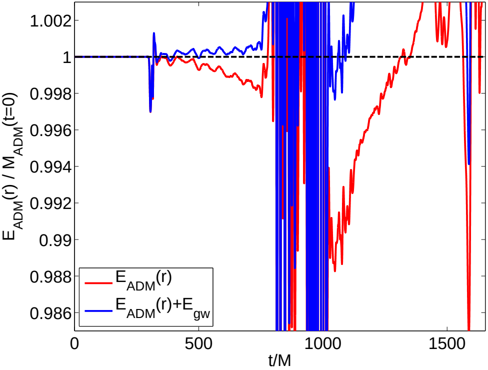
Binary Neutron Stars I

- Hamiltonian constraint violation reduced in Z4c data.
- Growth in constraint after merger with either formulation.



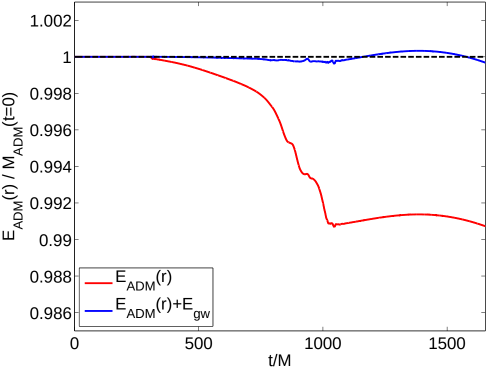
Binary Neutron Stars II

- ADM mass integral BSSNOK.
- Noise after outgoing signal reaches extraction sphere.



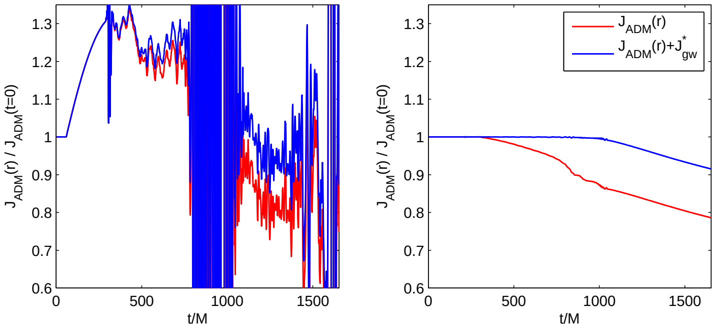
Binary Neutron Stars II

- ADM mass integral Z4c.
- Integral plus emitted GW energy constant.



Binary Neutron Stars III

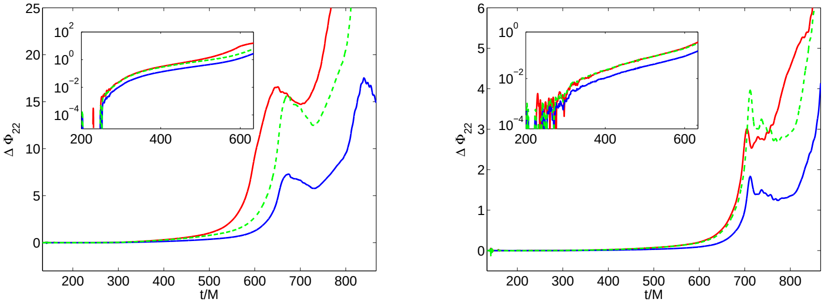
Angular momentum type integral:



Jump in BSSNOK when extraction sphere causally connected to outer boundary!

Binary Neutron Stars IV

Accuracy in GW phase



Convergence up to merger with Z4c for bns. Factor of two in amplitude and phase accuracy in bbh.

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- Large improvement over BSSNOK in terms of Hamiltonian constraint violation.
- CPBCs give improvement over BSSNOK with Sommerfeld in mass and angular momentum conservation.
- Improvement in GW error a factor between roughly 2 and 4.