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# COSMOLOGY WITH GRAVITATIONAL WAVES

# Outline

- Gravitational waves (GW) as standard sirens
- General Bayesian approach
- Measurements with 2nd generation IFOs
- Measurements with 3rd generation IFOs
- Outlook

# EM observations

- All the information we have about the Universe is mediated by electromagnetic radiation
- The Universe is NOT transparent
  - absorption - emission processes
  - radiation hydrodynamics
- Our instruments have limited sensitivity

# EM observations

- Measuring distances requires:
  - identification of ‘standard candles’
  - cross-calibration of various candles
  - “iterate and hope it converges” (Shore, S.N., The Tapestry of Modern Astrophysics, 2002)
- This procedure defines the “cosmic distance scale ladder”

# GW as standard sirens

- The Universe is transparent to GWs
- GWs allow a direct measurement of the luminosity distance $D_L$ to a source
  - No "cosmic distance scale ladder" required
- Tens of detections with $D_L$ < *few* x 100 Mpc could plausibly measure $H_0$ within a few percent (Schutz 1986)
- In general, no redshift information can be extracted and must be obtained some other way

# GW as standard sirens

- Extensive literature on the subject, focused on 2 methods:
  1. with counterparts:
     - e.g.: Dalal et al. 2006, Sathyaprakash et al. 2010, Nissanke et al. 2010, Zhao et al. 2011, Del Pozzo 2012
  2. without counterparts:
     - e.g.: Chernoff & Finn 1993, Taylor et al. 2012, Taylor & Gair 2013

# GW as standard sirens

**With Counterparts**

- In this case one searches for an electromagnetic counterpart to get the redshift
  - GRBs
  - host galaxy identification
- The redshift information is certain:
  - the probability distribution for the redshift is either a delta function (e.g. Nissanke et al. 2010) or a sum of deltas (Del Pozzo 2012)

**Without counterparts**

- One relies on the knowledge of some intrinsic property of the system
  - equation of state (EOS) of neutron stars (Messenger, Read 2012)
  - mass function (e.g. Taylor et al. 2012)
- The redshift information is statistical:
  - we obtain a probability distribution for the value of the redshift for each source

# A general Bayesian approach

Del Pozzo, Physical Review D, vol. 86, Issue 4, 2012

# General solution

- Given a set of *n* GW events $\mathcal{E} \equiv \epsilon_1, \ldots, \epsilon_n$ and a cosmological model $\mathcal{H}$ , we want to estimate the value of the cosmological parameters $\vec{\Omega}$
- From Bayes theorem:

$$p(\vec{\Omega}|\mathcal{E},\mathcal{H},\mathcal{I}) = p(\vec{\Omega}|\mathcal{H},\mathcal{I})\frac{p(\mathcal{E}|\vec{\Omega},\mathcal{H},\mathcal{I})}{p(\mathcal{E}|\mathcal{H},\mathcal{I})}$$

# General solution

- We can consider the events independent from each other, thus

$$p(\vec{\Omega}|\mathcal{E},\mathcal{H},\mathcal{I}) = p(\vec{\Omega}|\mathcal{H},\mathcal{I}) \prod_{i=1}^{n} \frac{p(\epsilon_i|\vec{\Omega},\mathcal{H},\mathcal{I})}{p(\epsilon_i|\mathcal{H},\mathcal{I})}$$

# General solution

- Indicate with $\vec{\theta}$ the set of parameters on which the GW signal depends
  - for non-spinning signals:

$$\vec{\theta} \equiv (m_1, m_2, \phi_c, \tau_c, \alpha, \delta, \psi, \iota, z, D_L)$$

- The (quasi-)likelihood is given by:

$$p(\epsilon_i|\vec{\Omega}, \mathcal{H}, \mathcal{I}) = \int \mathrm{d}\vec{\theta}\, p(\vec{\theta}|\vec{\Omega}, \mathcal{H}, \mathcal{I})p(\epsilon_i|\vec{\Omega}, \vec{\theta}, \mathcal{H}, \mathcal{I})$$

# Priors

- Expand the prior probability distribution for the GW parameters:

$$p(\vec{\theta}|\vec{\Omega},\mathcal{H},\mathcal{I}) = p(m_1, m_2, \phi_c, t_c, \psi, \iota, \alpha, \delta, z|\vec{\Omega},\mathcal{H},\mathcal{I})p(D_L|z,\vec{\Omega},\mathcal{H},\mathcal{I})$$

- In general:

$$p(D_L|z,\vec{\Omega},\mathcal{H},\mathcal{I}) = \delta(D_L - D(\vec{\Omega},z))$$

because of the explicit dependence on the cosmological model $\mathcal{H}$

# Priors

- In a FRWL universe:

$$D(\vec{\Omega}, z) = \begin{cases} \frac{c(1+z)}{H_0} \frac{1}{\sqrt{\Omega_k}} \sinh[\sqrt{\Omega_k} \int_0^z \frac{dz'}{E(z')}] & \text{for} \quad \Omega_k > 0 \\ \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')} & \text{for} \quad \Omega_k = 0 \\ \frac{c(1+z)}{H_0} \frac{1}{\sqrt{|\Omega_k|}} \sin[\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{E(z')}] & \text{for} \quad \Omega_k < 0 \end{cases}$$

$$E(z') = \sqrt{\Omega_m (1+z')^3 + \Omega_k (1+z')^2 + \Omega_\Lambda} \, .$$

# Priors

- Other terms in the prior probability distribution are set by the information $\mathcal{I}$ we have for the event under consideration, e.g.:
  - for GRBs:

$$p(\alpha, \delta, z|\mathcal{I}_{GRB}) = \delta(z - z_{GRB})\delta(\alpha - \alpha_{GRB})\delta(\delta - \delta_{GRB})$$

  - knowledge of the mass function:

$$p(m_1, m_2, z|\mathcal{I}_{MF}) = p(m_1, m_2|\mathcal{I}_{MF})p(z|m_1, m_2, \mathcal{I}_{MF})$$

# Likelihood

- The likelihood assumes the standard form; for a network of K detectors:

$$p(\epsilon_i|\vec{\Omega},\vec{\theta},\mathcal{H},\mathcal{I}) = \prod_{k=1}^{K} p(\epsilon_i^{(k)}|\vec{\Omega},\vec{\theta},\mathcal{H},\mathcal{I})$$

- And in each detector:

$$p(\epsilon_i^{(k)}|\vec{\Omega},\vec{\theta},\mathcal{H},\mathcal{I}) = e^{-(s_i^{(k)}-h^{(k)}(\vec{\Omega},\vec{\theta})|s_i^{(k)}-h^{(k)}(\vec{\Omega},\vec{\theta}))/2}$$

- where I introduced the scalar product:

$$(\mathrm{f}|\mathrm{g}) = 2\int_0^\infty df\, \frac{\tilde{\mathrm{f}}^*\tilde{\mathrm{g}} + \tilde{\mathrm{f}}\tilde{\mathrm{g}}^*}{S_n^{(k)}(f)}$$

# Measuring $H_0$ with $2^{nd}$ generation IFOs

# Inference of $H_0$

- I applied the aforementioned formalism to compact binary coalescences observed by three networks of 2nd generation instruments

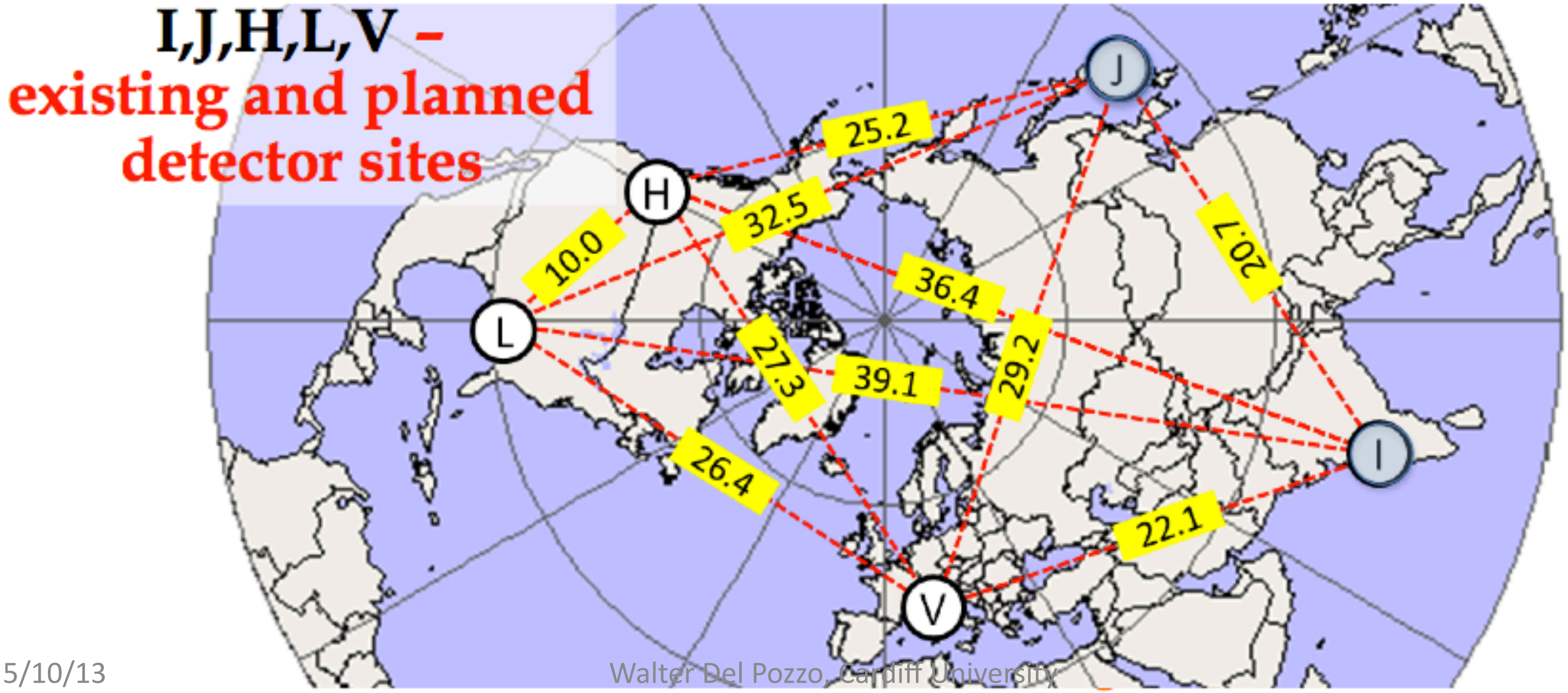



figure from Klimenko

# Inference of $H_0$

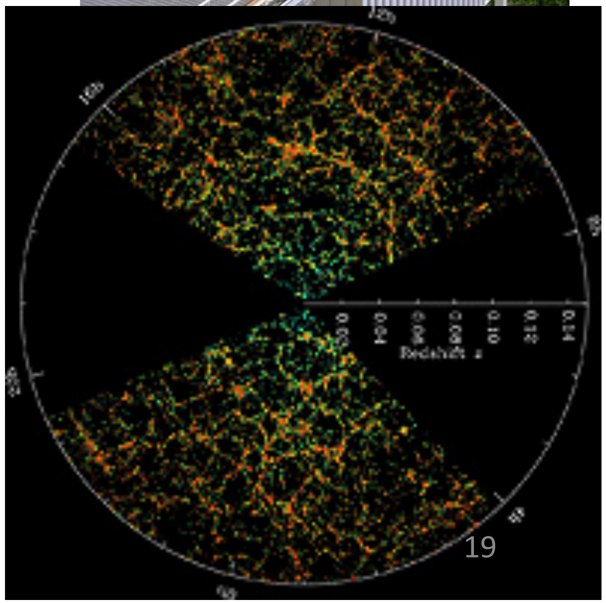
- If we assume that each GW event happens within a galaxy, then the sky position and redshift priors:

$$p(\alpha, \delta, z|\mathcal{I}) \propto \sum_{i=1}^{N} p_i \delta(z - z_i)\delta(\alpha - \alpha_i)\delta(\delta - \delta_i)$$

  where *i* labels each galaxy and the $p_i$ are some weights which might depend on the morphology, SFR, environment, luminosity, etc.

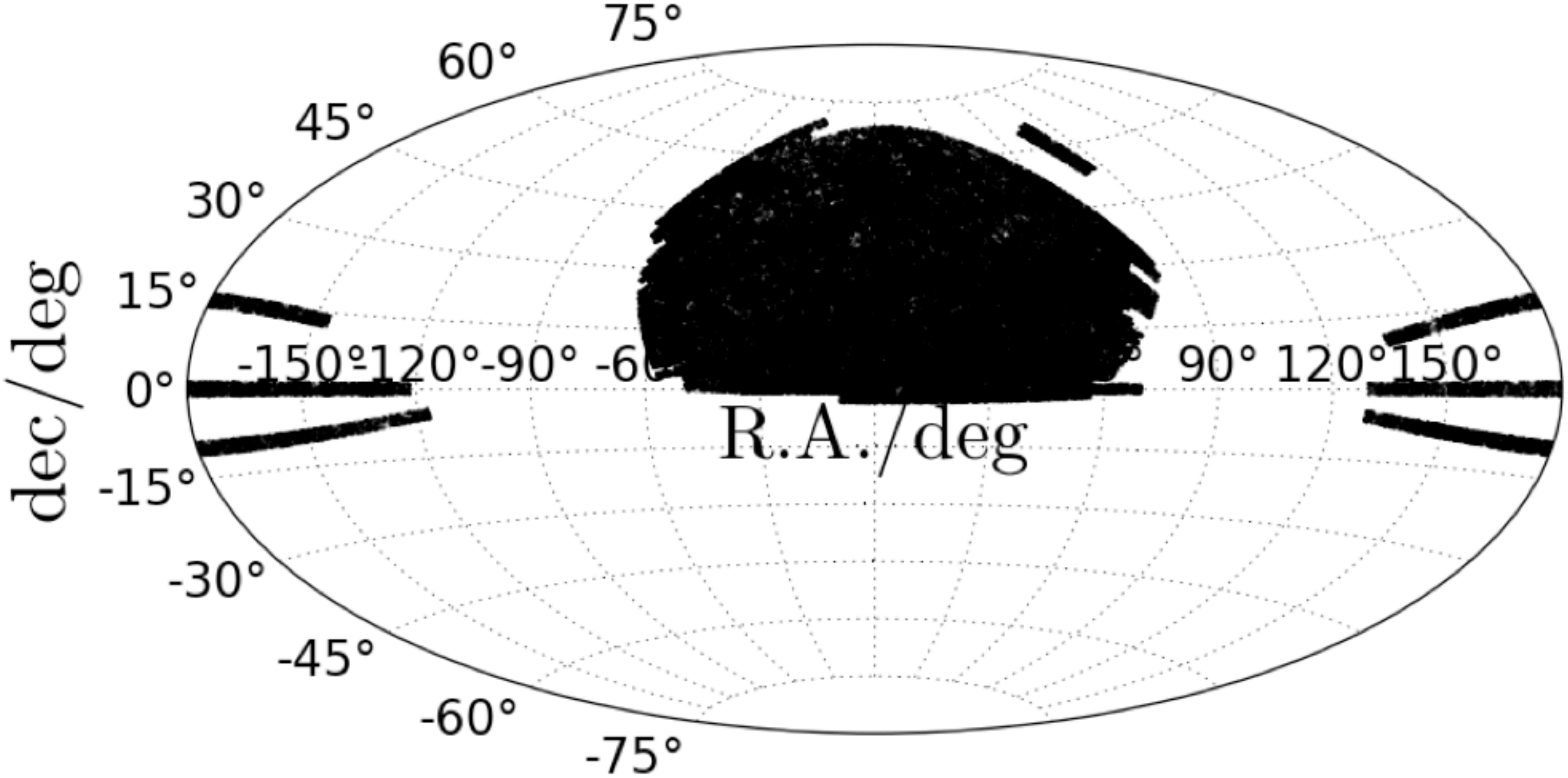
# Sloan Digital Sky Survey

- SDSS (http://www.sdss.org/) is an imaging and spectroscopic survey initiated in 2001 and whose operation is still continuing.
- The SDSS II (DR8) dataset includes 230 million celestial objects detected in 8,400 square degrees of imaging and spectra of 930,000 galaxies, 120,000 quasars, and 225,000 stars.



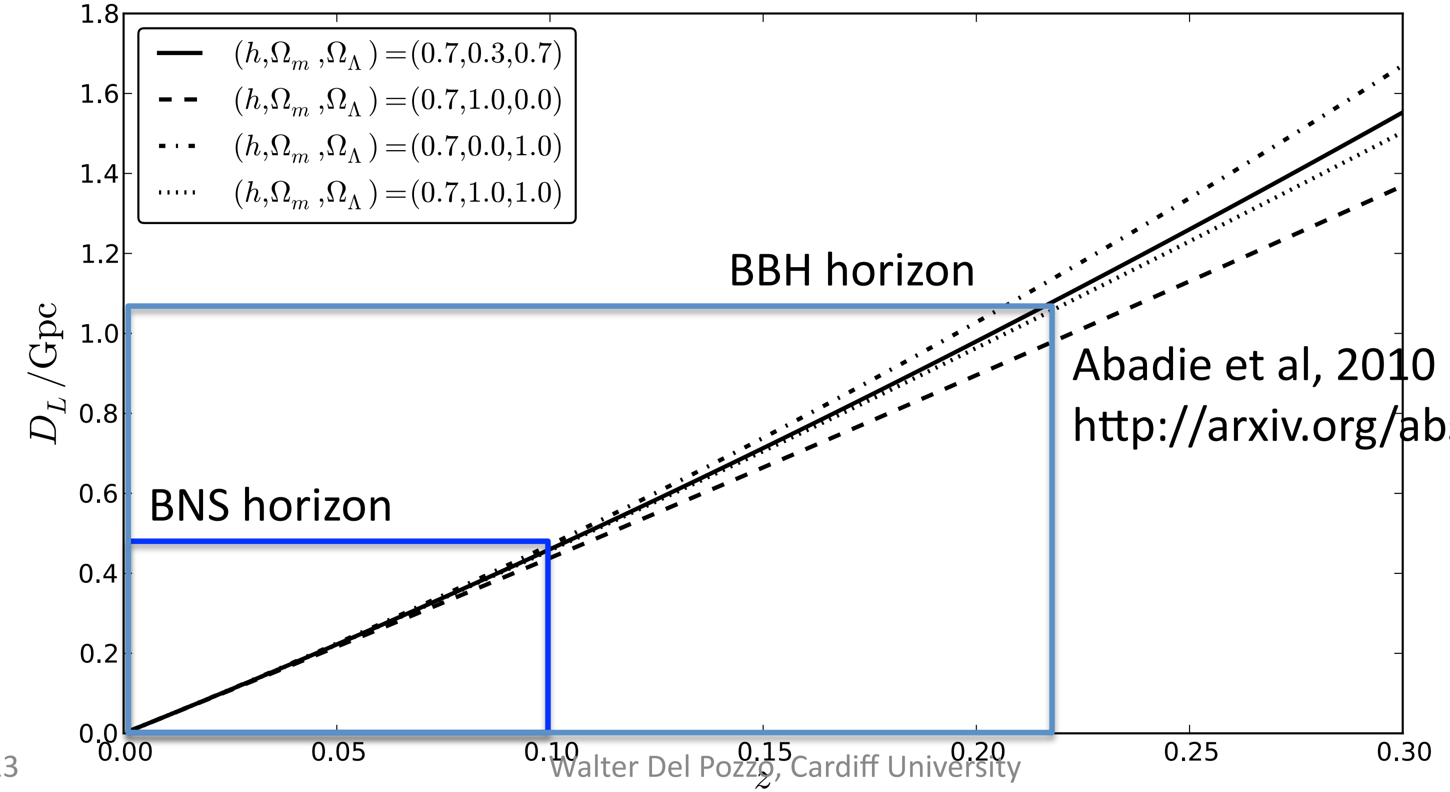
# Sloan Digital Sky Survey

- Map of the SDSS DR8 spectroscopic survey



# What do we expect to measure?

- With 2nd generation IFOs, a measurement of the energy density parameters from stellar mass systems will not be possible



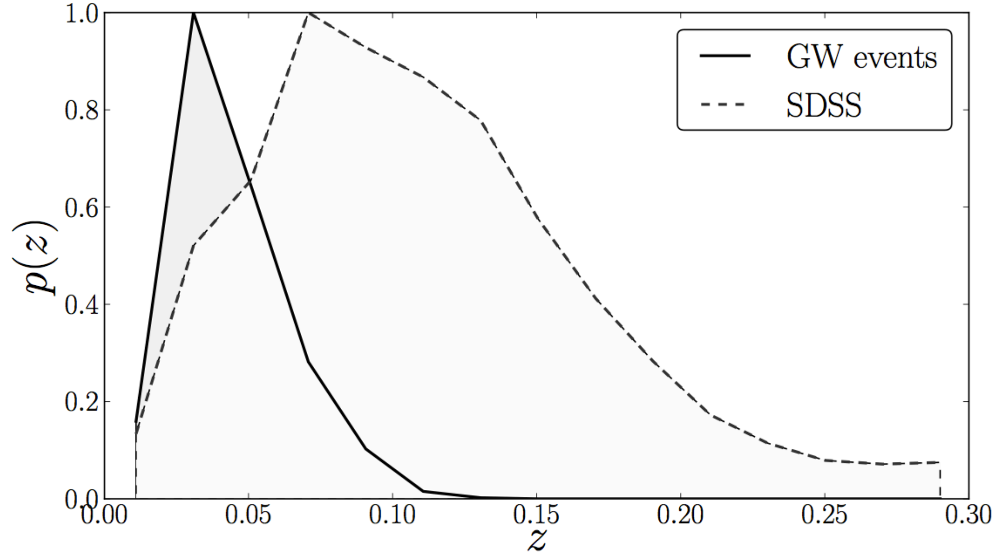

Abadie et al, 2010
http://arxiv.org/abs/1003.2480

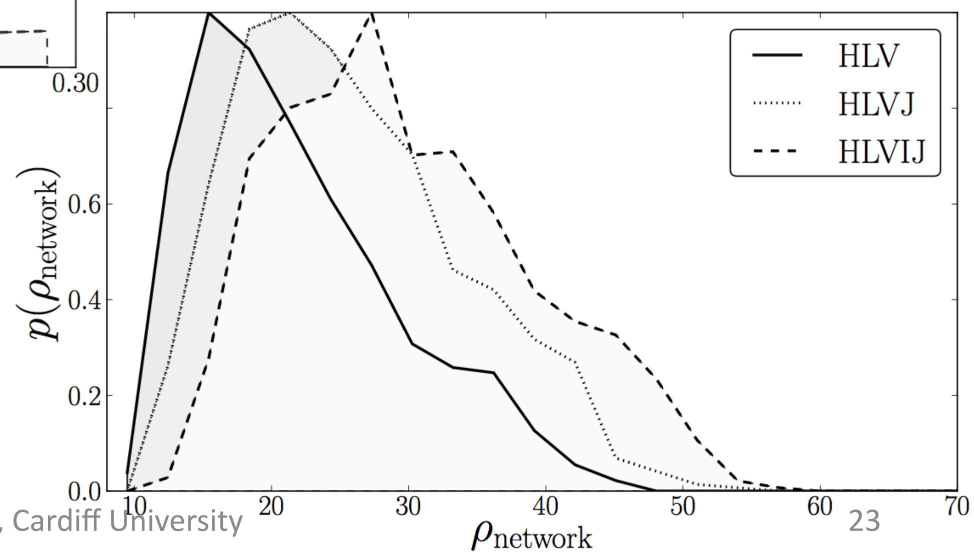
# The GW events catalog

- Generated 1000 GW events uniformly distributed on SDSS (sky position and redshift):
  - m1,m2 in [1.0,15.0] Msun
  - inclination and polarisation uniform on the 2-sphere
  - Frequency domain approximant (TaylorF2)
    - 3.5PN phase
    - 0PN amplitude
    - non-spinning

# The GW events catalog

- SDSS is a reasonable choice for advanced IFOs



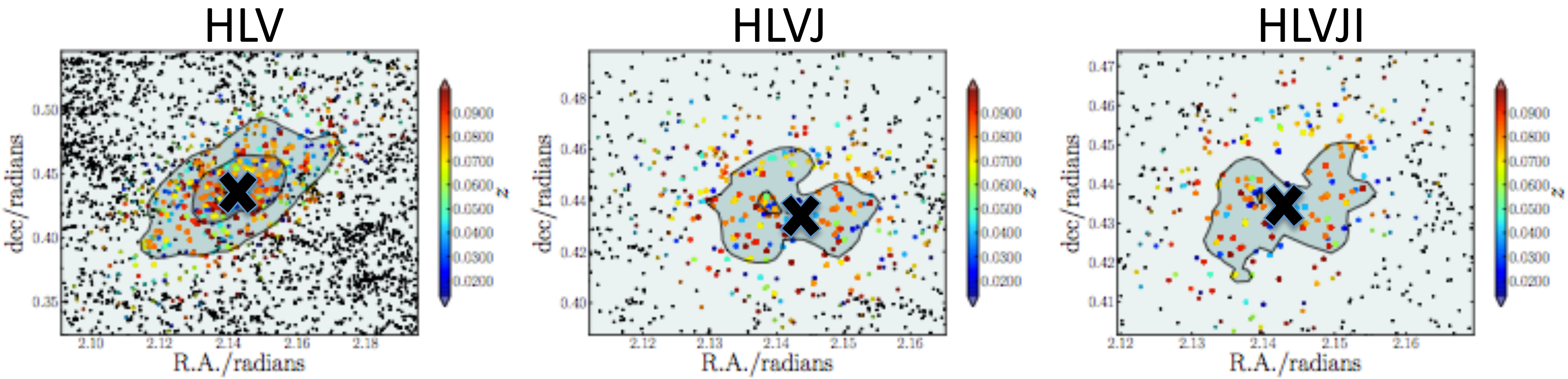



# Analysis details

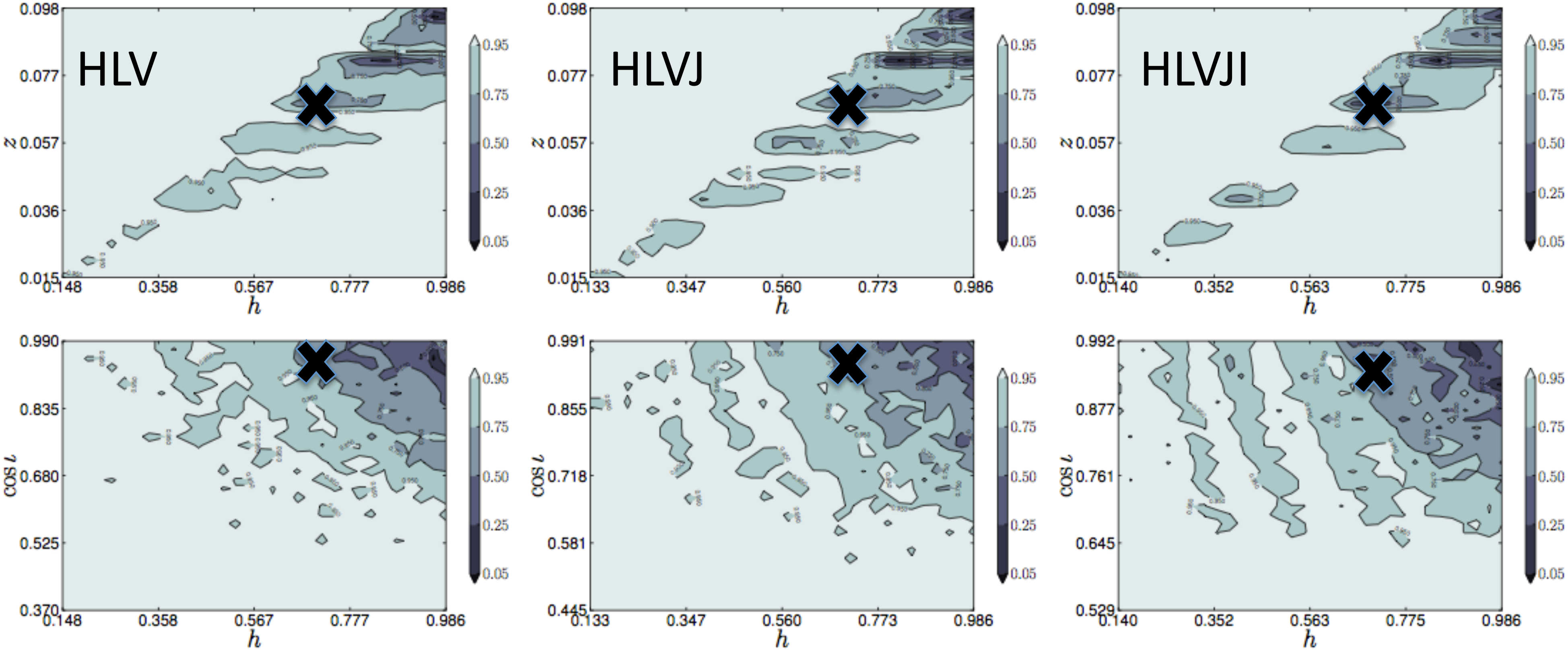
- The analysis has been performed using a Nested Sampling algorithm (Skilling 2004, Veitch & Vecchio 2010)
- Each signal is analysed in 10 independent gaussian noise realisations
  - All pdfs shown are averaged over them
- Priors for $\vec{\theta}$ are standard
- Priors for $h, \Omega_m, \Omega_\Lambda$ are uniform:

$$h \in [0.1, 1.0], \Omega_m \in [0.0, 1.0], \Omega_\Lambda \in [0.0, 1.0], \Omega_k = 1 - \Omega_m - \Omega_\Lambda$$

# Single source analysis




- Single source example:
  - SNR_{HLV,HLVJ,HLVJI} = 13.7,15.1,17.7
  - 95% confidence: 14.8 $deg^2$, 3.9 $deg^2$, 2.2 $deg^2$
  - 600, 339, 230 hosts




- The posteriors from each source show a 3-way degeneracy between redshift, $H_0$ and inclination
- Mitigated with more that 3 IFOs
- Necessity of multiple events for a reliable posterior for $H_0$

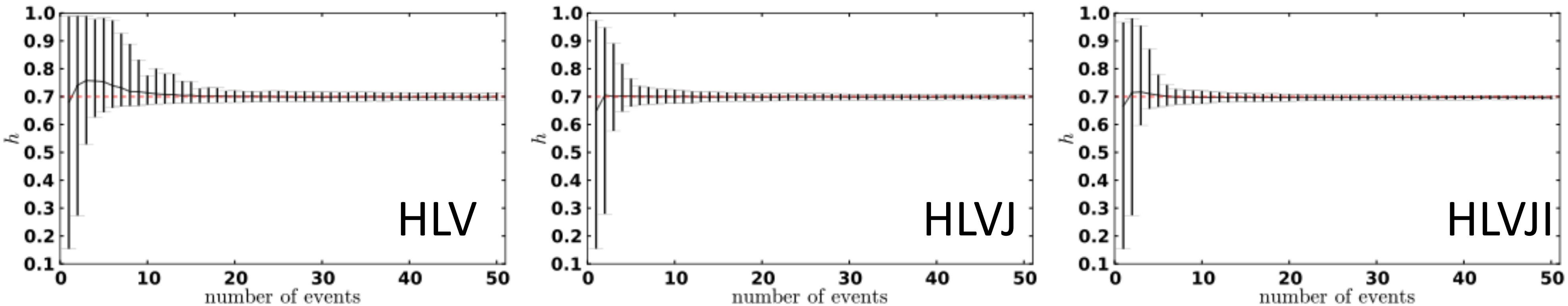
- Some statistics (average 95% confidence widths):

| network | $h$ | $\Omega_m$ | $\Omega_k$ | $\Omega_\Lambda$ | $z$ | dec/rad | R.A./rad | $\cos\iota$ | $\psi$/rad | $t_c$/ms | $\mathcal{M}/M_\odot$ | $\eta$ | $N_{\rm galaxies}$ | $\sqrt{N^2_{\rm galaxies} - \langle N_{\rm galaxies}\rangle^2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HLV | 0.63 | 0.95 | 1.55 | 0.95 | 0.04 | 0.05 | 0.05 | 0.45 | 1.6 | 1.1 | 0.01 | 0.01 | 283 | 332 |
| HLVJ | 0.57 | 0.95 | 1.55 | 0.95 | 0.04 | 0.04 | 0.03 | 0.40 | 1.6 | 0.6 | 0.007 | 0.01 | 171 | 192 |
| HLVJI | 0.54 | 0.95 | 1.55 | 0.95 | 0.03 | 0.03 | 0.02 | 0.34 | 1.6 | 0.4 | 0.006 | 0.01 | 118 | 137 |

- More IFOs in the network improve significantly sky position (e.g. Nissanke et al 2010, Schutz 2011)
  - Reduction of the number of possible hosts
- Note the width for *h*
  - no real measurement of *h* is possible with a single source

# Multiple events analysis

- When the information from multiple sources is combined, the correct value of *h* emerges:



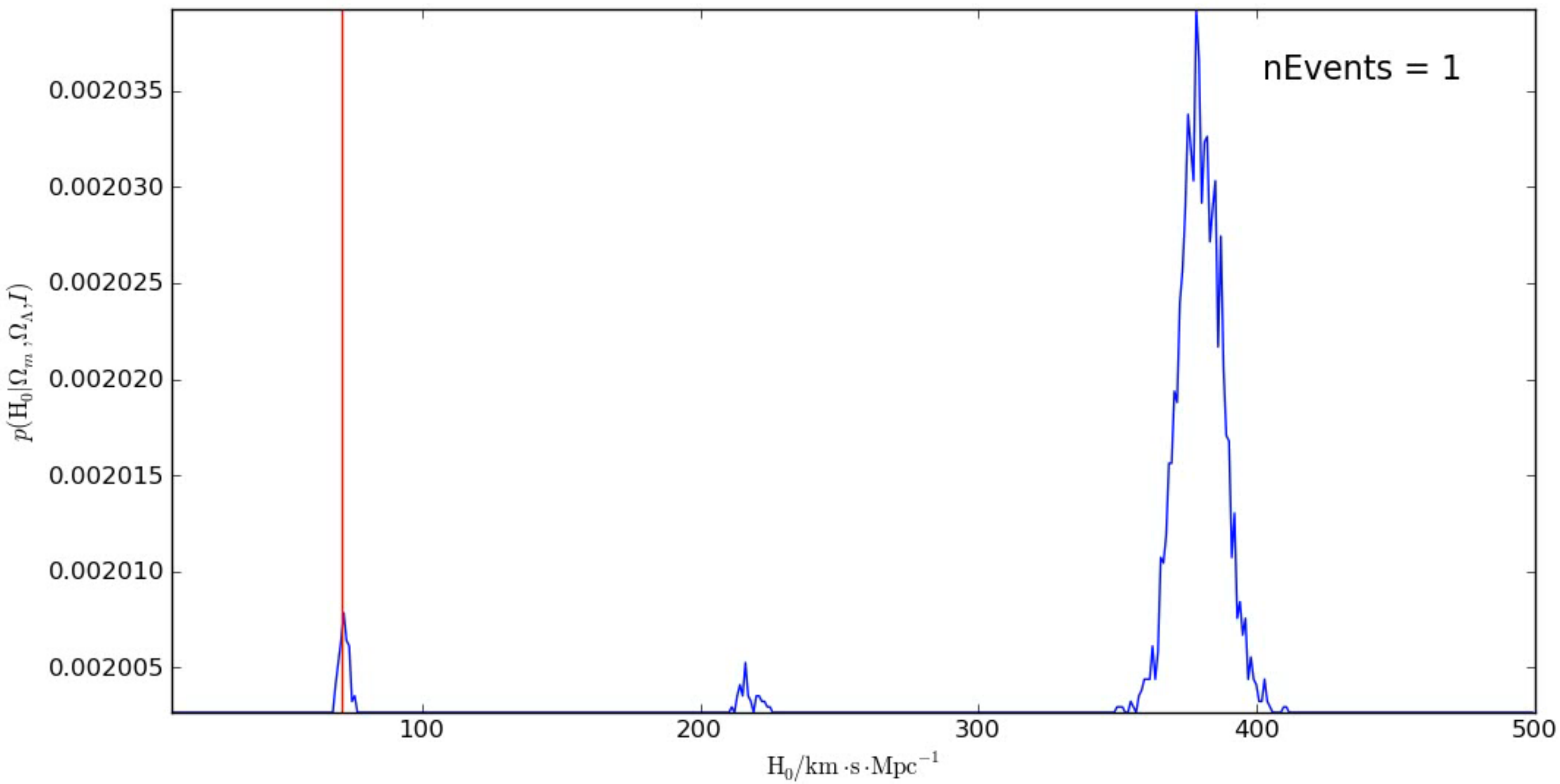

- The larger the network, the faster the convergence (and more accurate the estimate)

# Multiple events analysis - summary

| | HLV | | | HLVJ | | | HLVJI | | |
|---|---|---|---|---|---|---|---|---|---|
| # events | $\langle h_{2.5\%}\rangle$ | $\langle \bar{h}\rangle$ | $\langle h_{97.5\%}\rangle$ | $\langle h_{2.5\%}\rangle$ | $\langle \bar{h}\rangle$ | $\langle h_{97.5\%}\rangle$ | $\langle h_{2.5\%}\rangle$ | $\langle \bar{h}\rangle$ | $\langle h_{97.5\%}\rangle$ |
| 5 | 0.644 | 0.753 | 0.982 | 0.664 | 0.701 | 0.765 | 0.663 | 0.705 | 0.779 |
| 10 | 0.671 | 0.714 | 0.775 | 0.675 | 0.699 | 0.725 | 0.674 | 0.698 | 0.721 |
| 15 | 0.676 | 0.705 | 0.754 | 0.681 | 0.699 | 0.716 | 0.682 | 0.697 | 0.712 |
| 20 | 0.679 | 0.701 | 0.722 | 0.684 | 0.698 | 0.711 | 0.684 | 0.697 | 0.709 |
| 30 | 0.681 | 0.698 | 0.717 | 0.688 | 0.699 | 0.708 | 0.687 | 0.697 | 0.707 |
| 40 | 0.686 | 0.700 | 0.714 | 0.687 | 0.699 | 0.707 | 0.689 | 0.697 | 0.704 |
| 50 | 0.686 | 0.700 | 0.714 | 0.687 | 0.700 | 0.706 | 0.689 | 0.700 | 0.703 |

- Compare with:
  - $H_0$= 73 ± 4(stat) ± 5(syst) km $s^{-1}$ $Mpc^{-1}$(Riess et al. 2005, SnIa)
  - $H_0$= 62.3 ± 1.3(stat) ± 5(syst) km $s^{-1}$ $Mpc^{-1}$ (Freeman et al. 2006, SnIa+Cepheids)
  - $H_0$ = 74.2 ± 3.6 km $s^{-1}$ $Mpc^{-1}$ (Riess et al. 2009, SnIa+Cepheids)

# Multiple events analysis - summary



# Inference of the cosmological parameters with 3rd generation IFOs

Del Pozzo, Li, Messenger, in preparation

# GW from BNS systems

- In the late stages of the inspiral, dissipative forces modify the dynamics of the system via 5PN and 6PN order terms:

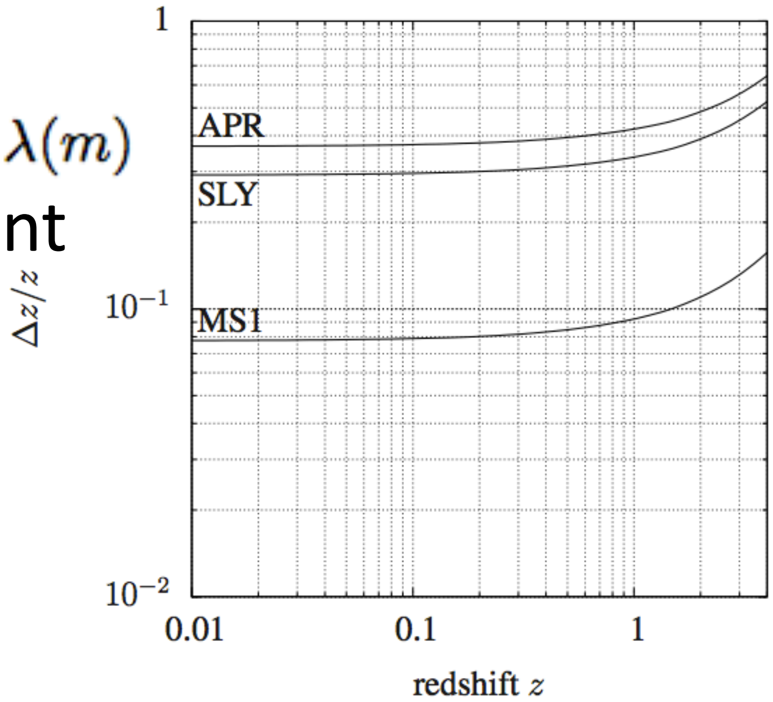
$$\psi_{tidal}(f) = (\pi M f)^{-\frac{5}{3}} \sum_{a=1,2} \frac{3\lambda_a}{128\eta M^5} \left[ -\frac{24}{\chi_a}\left(1+\frac{11\eta}{\chi_a}\right)(\pi M f)^{10/3} - \frac{5}{28\chi_a}\left(3179 - 919\chi_a - 2286\chi_a^2 + 260\chi_a^3\right)(\pi M f)^{12/3} \right]$$

  – with: $\chi_a = m_a/M$ $\lambda_a = \lambda(m_a)$

- Note that $\lambda_a \propto (c^2 R_a/G m_a)^5 \sim 10^5$

# GW from BNS systems

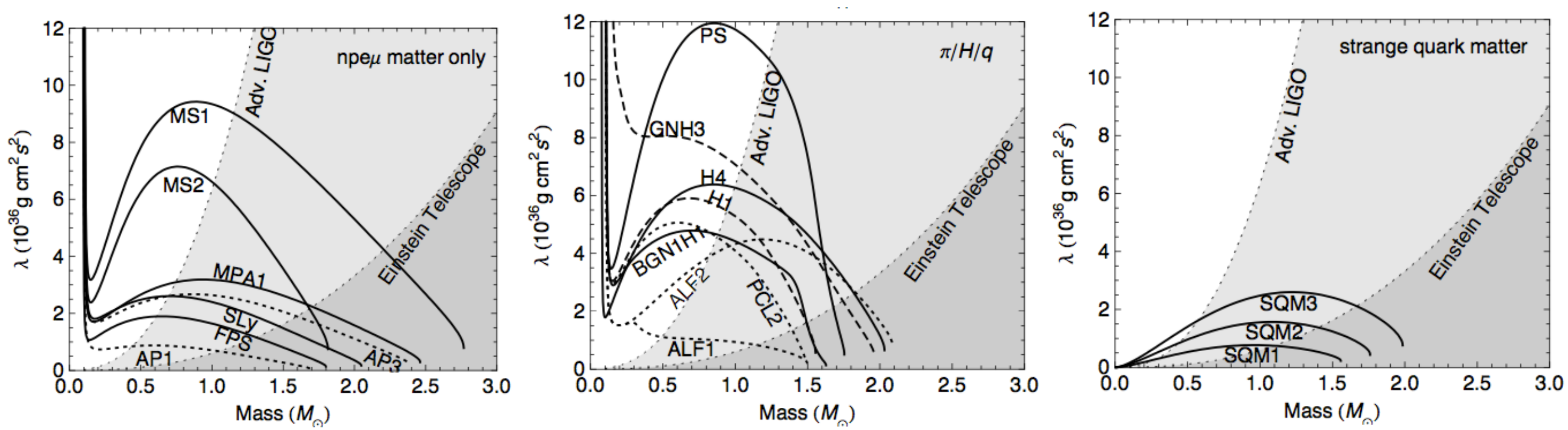
- The tidal terms depend on the rest frame masses:
  - if one knows the function $\lambda(m)$ one can get a measurement of the redshift from GW alone (Messenger, Read 2012)



# Measuring the EOS

- However, there are many models and we don't know what the correct one is

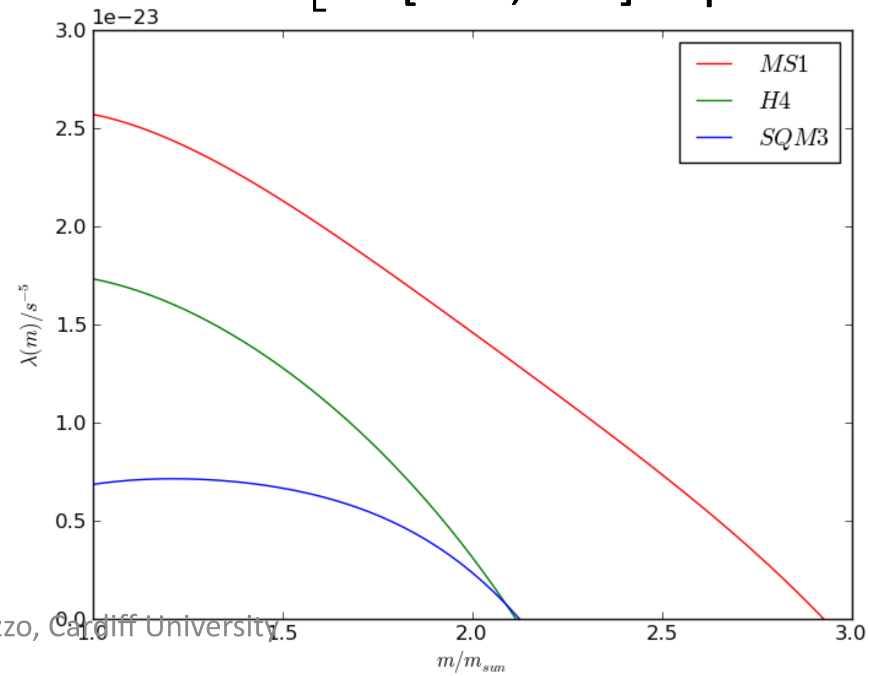
Hinderer et al, 2010




- Can we discriminate among them with advanced LIGO/Virgo?

# Measuring the EOS

- To investigate the measurability of the EOS we generated a set of simulated BNS events embedded in simulated noise for the advLIGO/Virgo network:
  - masses drawn from Normal($1.35M_\odot$, $0.1M_\odot$)
  - uniform in orientation, sky position
  - uniform in comoving volume with $D_L$ in [100,250] Mpc
- Four EOS models:
  - point particle (PP)
  - MS1 (hard EoS)
  - H4 (medium EoS)
  - SQM3 (soft EoS)



# Measuring the EOS

- We investigated two possible routes to infer the EOS (Del Pozzo et al, submitted to PRL):
  1. Expand $\lambda(m)$ in Taylor series and estimate the coefficients of the expansion

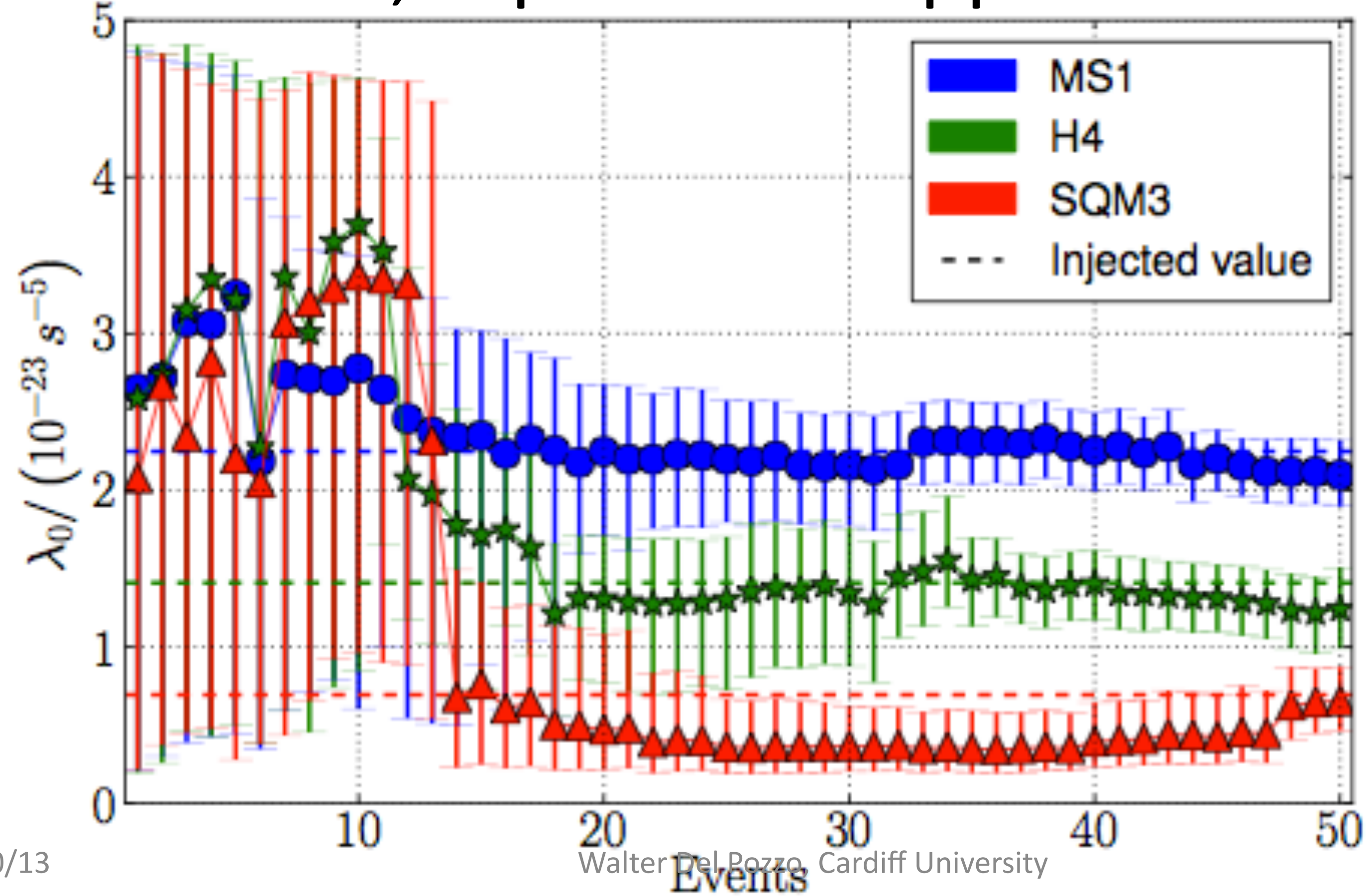
     $$\lambda(m) \simeq \lambda_0 + \lambda_1\,(m - 1.4\,M_\odot)/M_\odot$$

  2. Compare the predictions from different EOS models and rank then according to the odds ratio:

     $$O_{M_1,M_2} = \frac{p(M_1)}{p(M_2)}\frac{p(d|M_1)}{p(d|M_2)}$$

# 1 - Linearised approximation

- $\lambda(1.4 M_{\odot})$ is measurable after 10-15 detections, separation happens for n>30



# 2 – Model selection

- If we define a "detection" threshold of $\log(O_{ij}) \geq 30$, few tens of sources are necessary. Not unreasonable, given the expected rates.

testing model

true model

| Model | PP | MS1 | H4 | SQM3 |
|---|---|---|---|---|
| PP | - | 5 | 10 | 15 |
| MS1 | 5 | - | 20 | 10 |
| H4 | 10 | 45 | - | 15 |
| SQM3 | 15 | 15 | >50 | - |

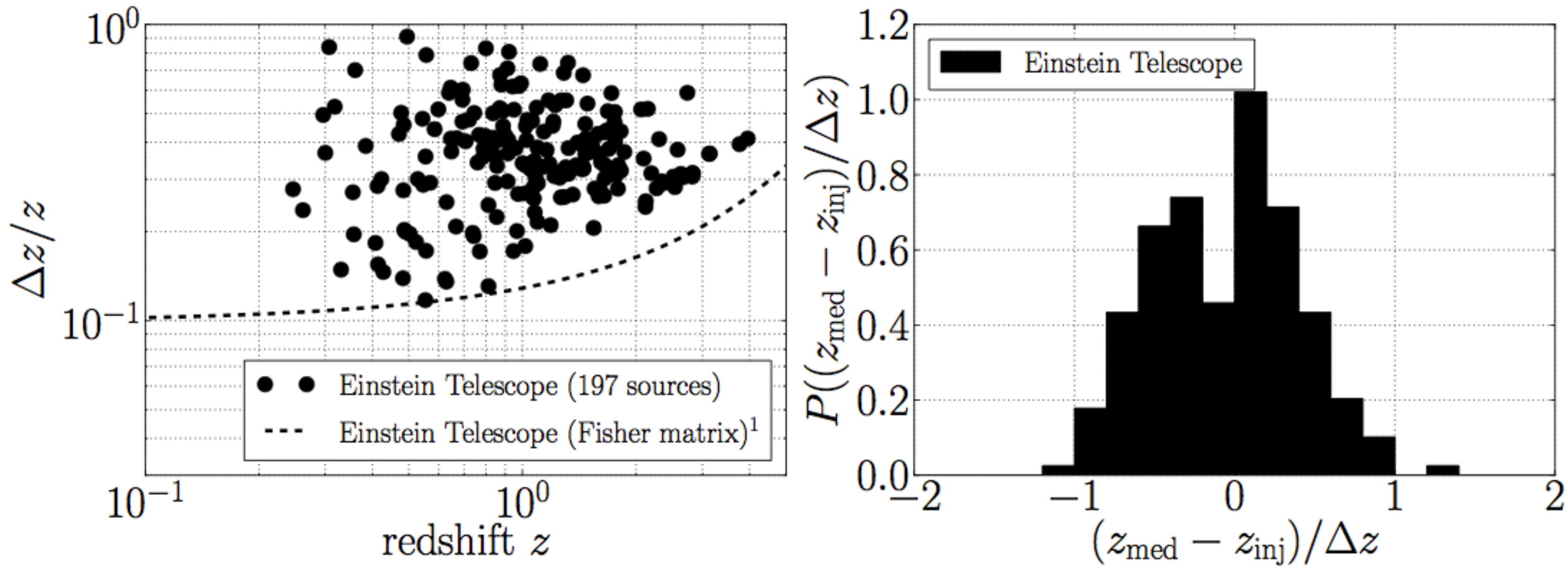
Number of sources necessary to distinguish among EOS models.

# Measuring the redshift

- Let's assume that we know the EOS, how well can we measure the redshift with ET?
- Generated 200 BNS sources uniformly distributed in comoving volume in a concordance cosmology with max(z) = 4 observed by ET-B
- Assumed MS1 EOS (most optimistic case)
- Analysis done using a Nested Sampling algorithm

# Measuring the redshift

- Typical uncertainty of 40%, but no systematic bias

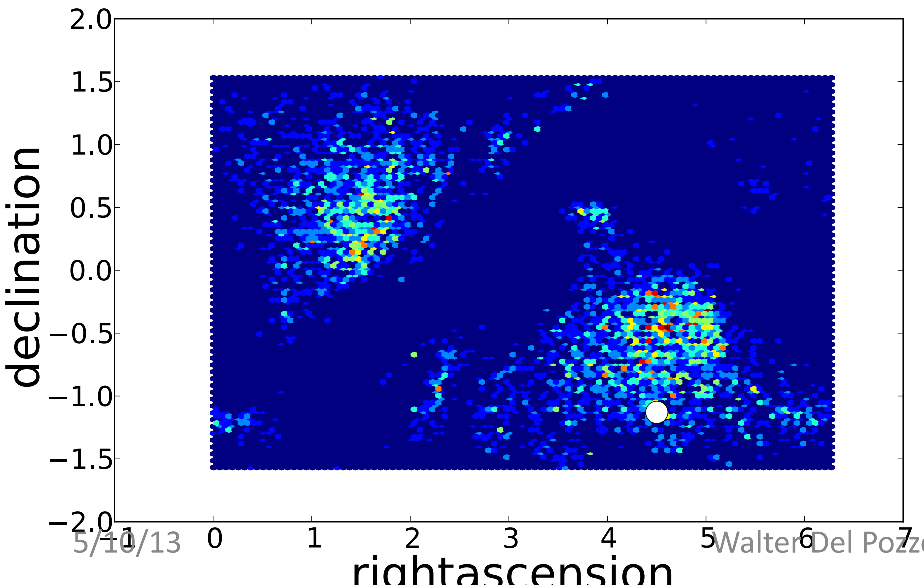


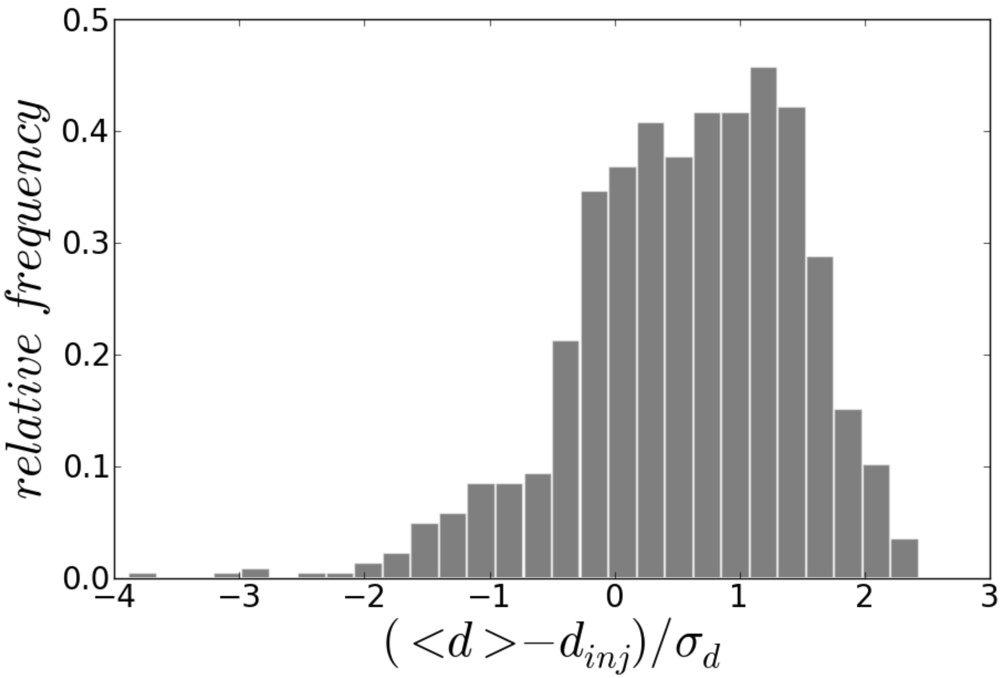

Li, Del Pozzo, Messenger, 2013

# Measuring the distance

- The distance shows a systematic bias

- Due to the poor sky resolution of ET



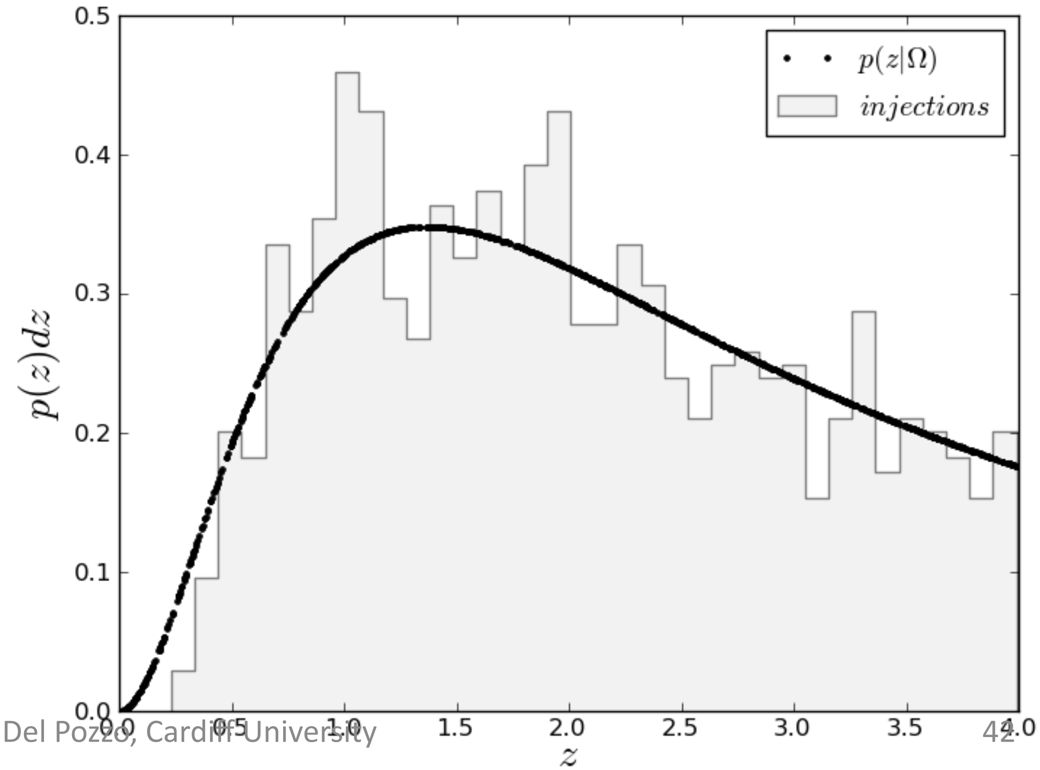



# Measuring the cosmological parameters with ET

- We seem to have all necessary ingredients
- Simulated 1000 BNS events, distributed in co-moving volume up to z=4

$$p(z|\vec{\Omega}) = \frac{1}{R(z_{\max})}\frac{dR}{dz}$$

$$\frac{dR}{dz} = \frac{D_c^2}{1+z}\frac{r_0 e(z)}{H(z)}$$




- We fixed

$$r_0 = e(z) = const$$

# Measuring the cosmological parameters with ET

- Since thresholds would introduce biases in our estimates (e.g. Messenger & Veitch 2013), we take into account every source, irrespective of their optimal SNR
- This is critical because of the redshift prior

# The redshift prior

- The natural choice for the prior for the redshift is (e.g. Coward & Burman 2005):

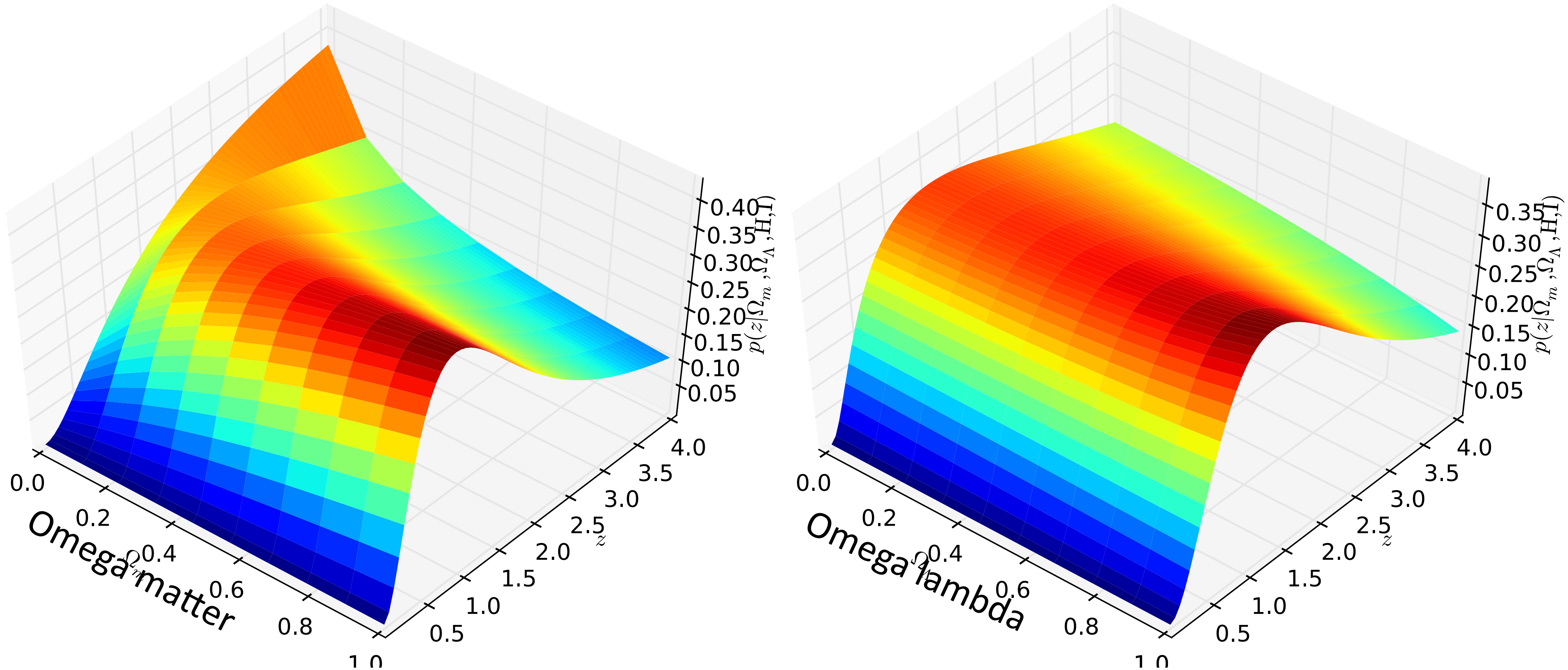
$$p(z|\vec{\Omega}) = \frac{1}{R(z_{\max})}\frac{dR}{dz}$$

$$\frac{dR}{dz} = \frac{D_c^2}{1+z}\frac{r_0 e(z)}{H(z)}$$

$$r_0 = e(z) = const$$

- Note that, differently from most inference problems, the prior depends on the parameters we are trying to estimate

# The redshift prior




- This dependence has some interesting consequences

# The redshift prior

- The prior acts such that:
  - quiet sources tend to select empty universes

    $\Omega_m \to 0, \Omega_\Lambda \to 1$
  - loud sources do the opposite

    $\Omega_m \to 1, \Omega_\Lambda \to 0$
- It is thus necessary take them all into account to converge to the right answer

# Analysis

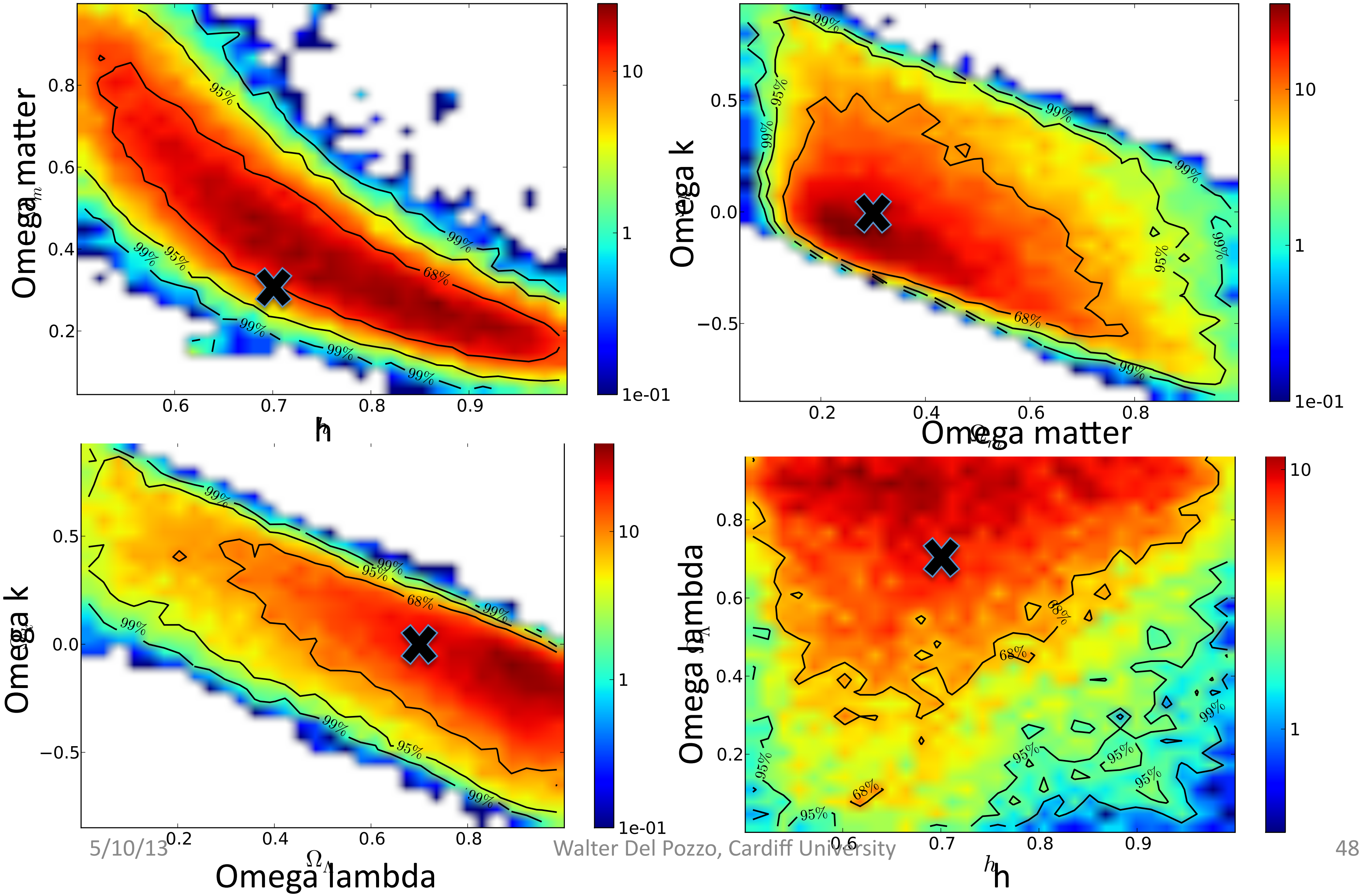
- We found that a direct approach is prone to errors due to poor sampling
- Two stage approach:
  - For each source:
    - marginalise the likelihood over all parameters except redshift and distance using a Nested Sampling
    - model the resulting 2D quasi-likelihood using a 7 components Gaussian mixture model
- Sample with an MCMC the posterior

$$p(\vec{\Omega}|\epsilon_1,\ldots,\epsilon_n,\mathcal{H},\mathcal{I}) = p(\vec{\Omega}|\mathcal{H},\mathcal{I})\prod_{i=1}^{n}\int dD_{L,i}dz_i\, p(z_i|\vec{\Omega},\mathcal{H},\mathcal{I})p(D_{L,i}|z_i,\vec{\Omega},\mathcal{H},\mathcal{I})p(\epsilon_i|D_{L,i},z_i,\mathcal{H},\mathcal{I})$$

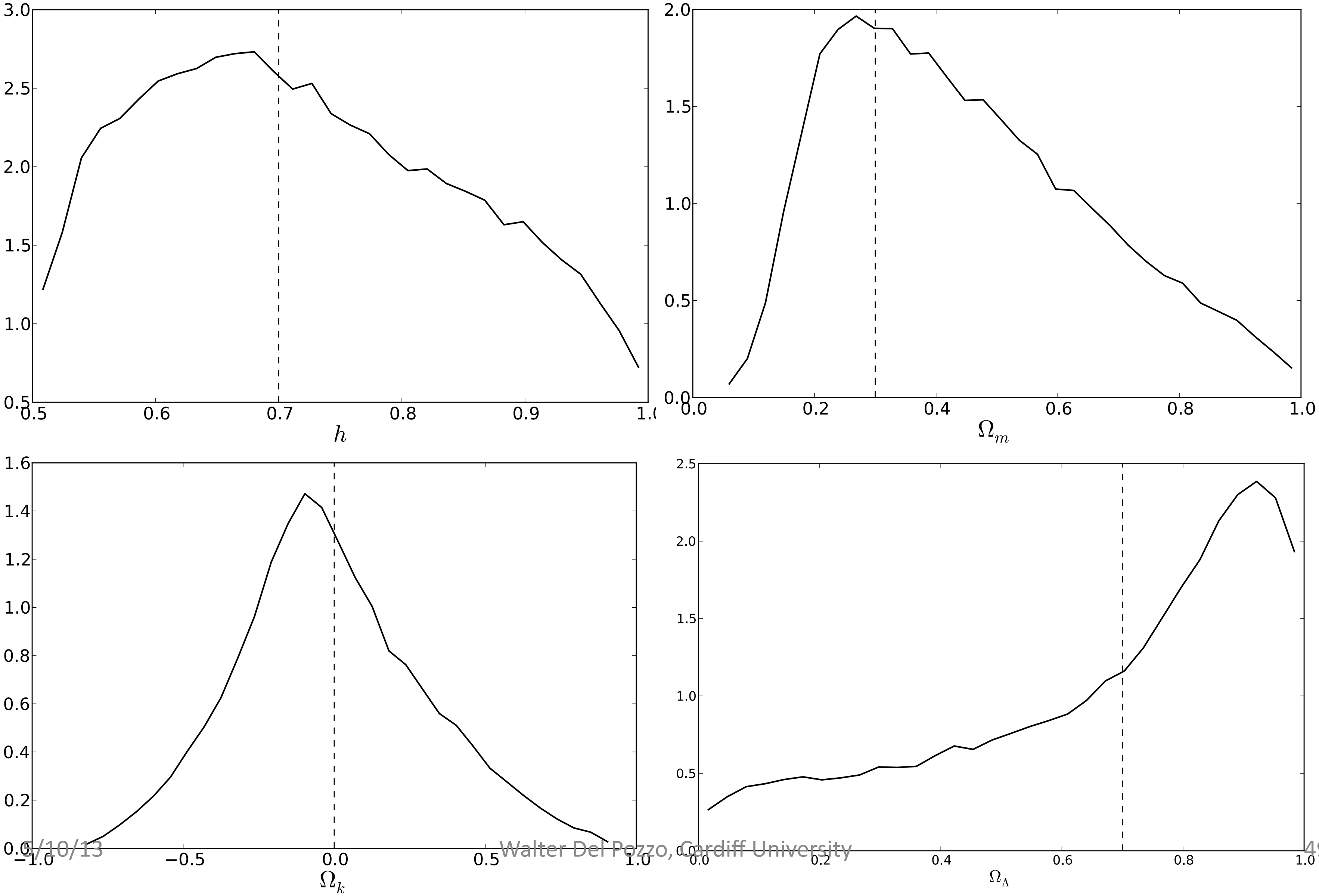
$$p(D_L|z,\vec{\Omega},\mathcal{H},\mathcal{I}) = \delta(D_L - D(\vec{\Omega},z))$$

$$h \in [0.5,1.0], \Omega_m \in [0.0,1.0], \Omega_\Lambda \in [0.0,1.0], \Omega_k = 1 - \Omega_m - \Omega_\Lambda$$

# Results – 100 events



# Results – 100 events



# Results - extrapolation

- Assume that, for a large number of sources (>100), the 68% CL width scale approximately like the square root of the number of sources

| # sources | Δh | ΔOmega matter | ΔOmega k | ΔOmega Lambda |
|---|---|---|---|---|
| 1 | 0.16 | 0.3 | 0.4 | 0.32 |
| 10 | 0.15 | 0.23 | 0.35 | 0.3 |
| 100 | 0.13 | 0.18 | 0.31 | 0.28 |
| $10^3$ | 0.04 | 0.06 | 0.1 | 0.1 |
| $10^4$ | 0.01 | 0.02 | 0.03 | 0.03 |
| $10^5$ | 0.004 | 0.006 | 0.01 | 0.01 |
| $10^6$ | 0.001 | 0.002 | 0.003 | 0.003 |

WMAP7 (Komatsu et al 2011)

extrapolated

# Dark energy equation of state

- The acceleration of the expansion has sparkled interest into a varying equation of state for Dark Energy
- The Hubble parameter is modified to:

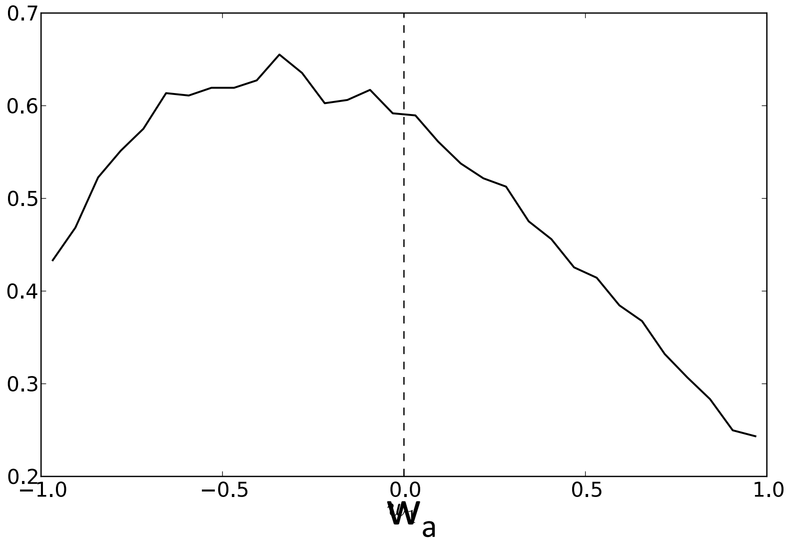
$$H(z) = H_0 \left[\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + (1 - \Omega_m - \Omega_k) E(z)\right]^{1/2}$$

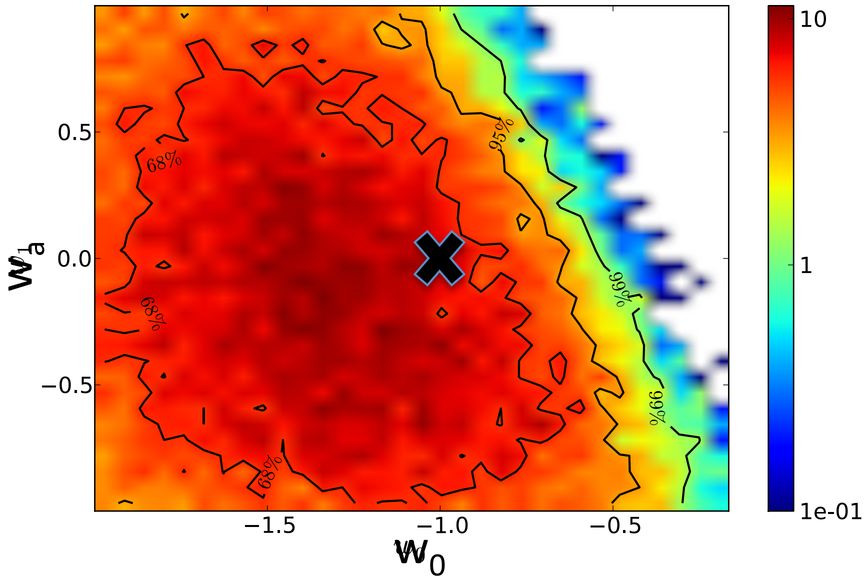
- and in the Chevallier-Polarski-Linder form:

$$E(z) \equiv (1+z)^{3(1+w_0+w_a)} e^{-3 w_a z/(1+z)}$$

- We assumed $\Omega_m, \Omega_k, H_0$ known and tried to infer $w_0, w_a$

# Results – 100 events

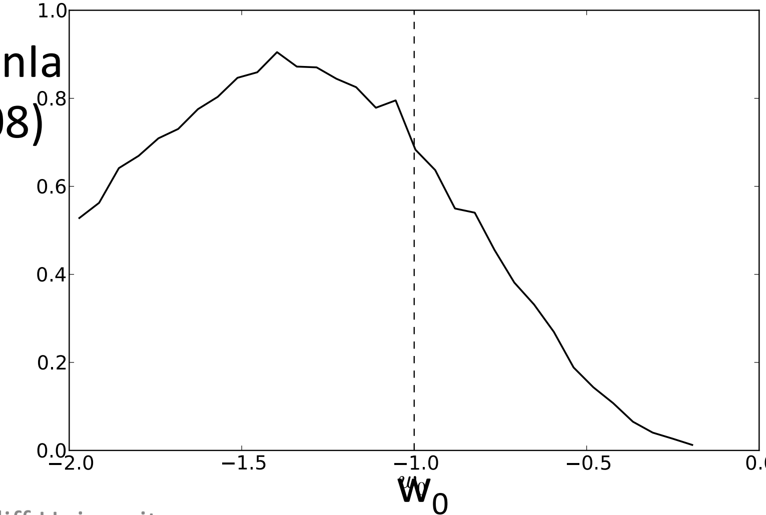



| # | $\Delta w_0$ | $\Delta w_a$ | FoM |
|---|---|---|---|
| 1 | 0.61 | 0.61 | 0.14 |
| 10 | 0.52 | 0.59 | 0.17 |
| 100 | 0.44 | 0.48 | 0.24 |
| $10^3$ | 0.14 | 0.15 | 2.4 |
| $10^4$ | 0.04 | 0.05 | 25.8 |
| $10^5$ | 0.01 | 0.01 | 516 |
| $10^6$ | 0.004 | 0.004 | $3\times10^3$ |

WMAP5+SnIa (Wang 2008)

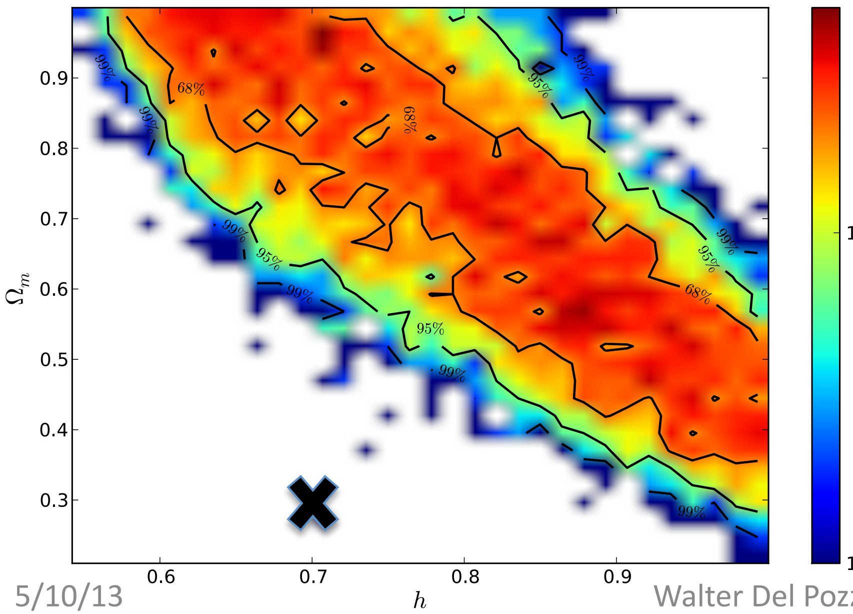
extrapolated



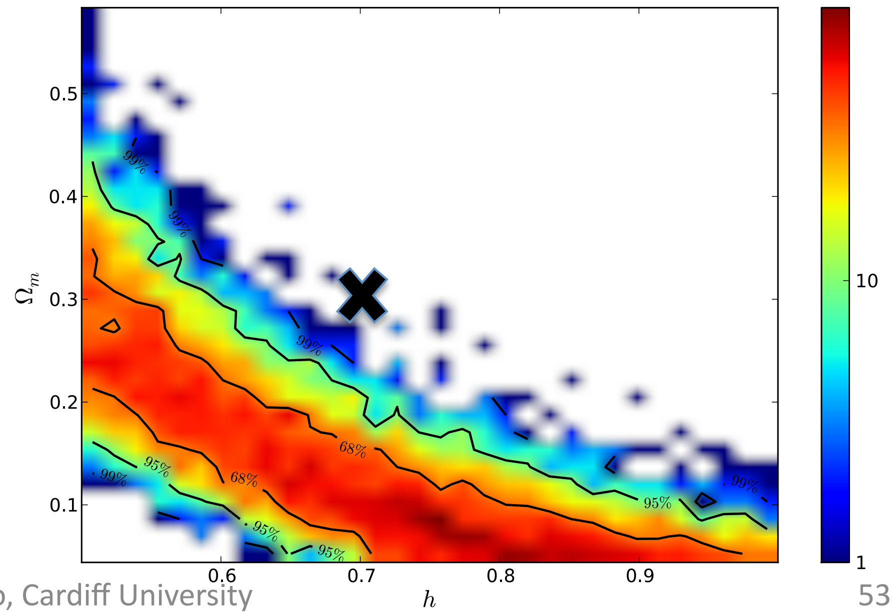
# Why we need "non-detection"

- Messenger & Veitch have shown that an unbiased inference requires to account for "non-detections"

100 sources with logB > 0.0




100 sources with logB < 0.0



# Outlook

- Knowledge of the EOS should allow ET to make cosmological measurements that are comparable to current or future EM mission
  - however, the pdfs we obtained are NOT gaussians, so the central limit theorem for 100 sources is not yet satisfied, more simulations are needed
- The z uncertainty is of order 40%, knowing the mass function should improve it
  - Can we measure it with 2nd generation? (Del Pozzo, Messenger, Veitch, in preparation)
- We assumed a constant rate, how do more realistic models affect our results?

# Summary

- Differently from electromagnetic waves, the Universe is transparent to GW
  - GW offer the unique possibility of independently testing the current cosmological paradigm
- Second generation instruments will constrain $H_0$
- Third generation instruments will constrain the energy content of the Universe